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TO THE UNIVERSITY
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OF BALLIOL COLLEGE.

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THE

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LONDON:

Printed for G. Robinson and R. Baldwin in Pater-noſter Row, 1775.

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THE

MATHEMATICAL PARTS

OF THE

LADIES' DIARIES.

1755.

Eclipſes in 1755, *calculated by Mr. Ra. Hulſe.*

THERE will, this year, happen four eclipſes; two of each luminary, according to the following order.

The firſt is an inviſible eclipſe of the ſun, on March 12th, at 10 at night.

The ſecond is a viſible eclipſe of the moon, on March 27th, beg. 11 h. 17 m. middle 12 h. 34 m. end 13 h. 51 m. total duration 2 h. 34 m. digits eclips'd 7° 7′ on the moon's lower limb.

The third is an inviſible eclipſe of the ſun, on September 6th, at 8 in the morning.

The fourth, and laſt, is of the moon, and likewiſe inviſible, on September 20th, at 11 in the morning.

Mr. Harland Widd (for Whitby) by tables founded on Sir Iſaac Newton's theory, computes the time of the lunar eclipſe on March 27th to be as follows. Beginning 11 h. 21 m. middle 12 h. 38 m. ecliptic oppoſition 12 h. 45 m. end 13 h. 55 m. total duration 2 h. 34 m. digits eclips'd 7° 6′.

Mr. John Ramſay of Morpeth, and Mr. Alexander Rowe of Penzance, alſo favoured us with calculations of this eclipſe.

New Queſtions.

I. QUESTION 391 *by Miſs* Maria Atkinſon.

(*Addreſſed to Mr.* E. P. *who took the liberty to aſk her age.*)

Five times ſeven and ſeven times three
Add to my age, the ſum will be
As many above ſix nines and four
As twice my years exceed a ſcore:
From hence, ſweet Sir, my age explore.

II. QUESTION 392 *by* Sylvius.

A ball, deſcending by the force of gravity from the top of a tower, was obſerved to fall half the way in the laſt ſecond of time: Required the tower's height, and the whole time of deſcent.

III. QUESTION 393 *by Mr.* G. Brownbridge,

(*Addreſſed to the ingenious Mr.* C. T——te.)

In vain you hope to live in quiet
By ſtinting of your vixen's diet;
'Tis five to one that ſcheme won't do;
But I'll exchange a wife with you,
And, if you pleaſe, a daughter too,
If you their ages will make known
From what below is fairly ſhown.

Given $\left\{\begin{array}{l} x^3z + xz^3 = 546560 \\ x^4 + z^4 = 1086992 \end{array}\right\}$ x and z being the two ages required.

IV. QUESTION 394 *by Mr.* W. Kingſton.

The diſtance of the centers of two circles, whoſe diameters are 50 each, being given = 30; 'tis required to find the ſide of the ſquare inſcribed in the interſection, or ſpace common to both circles.

V. QUESTION 395 *by Mr.* Walter Trott.

Sailing on a certain courſe, we obſerved a head-land to bear due Weſt from us; four hours after which, it was ſeen at W. S. W. and ſix hours after this (we ſtill continuing to run at the ſame rate) its bearing was found to be S. S. W. What was our courſe at that time?

VI.

VI. QUESTION 396 *by Mr.* Tho. Moſs.

In the three ſides of an equiangular field ſtand three trees, at the diſtances of 10, 12, and 16 chains from one another; To find the content of the field, it being the greateſt the data will admit of?

VII. QUESTION 397 *by Mr.* J. Aſh.

Standing at the end of a viſto, at the diſtance of 50 yards from each of the parallel ſides, the opening of the ſaid ſides, at a ſylvan ſtatue before me, appeared to be $\frac{3}{2}$ of the opening of the further end; the part, or length, on this ſide the ſtatue being to the part on the other ſide in the ratio of 1000 to 501: From hence I would know the diſtance of the ſtatue from the point of obſervation, and the whole length of the viſto?

VIII. QUESTION 398 *by Mr.* Richard Gibbons.

Being on a journey, I took a guide at Modbury for Dartmouth; with whom having travelled 66 minutes, I aſked him how far we were come? who replied, Juſt half ſo far as we are now from Totneſs. Having jogged on together ſeven miles farther, I aſked him how far we had now to travel? whoſe anſwer was ſtill the ſame, Juſt half as far as we are from Totneſs: Theſe indirect anſwers I did not like, tho' I found when we arrived at Dartmouth, which we reached in 55 minutes more, that all the fellow had ſaid was ſtrictly true, and that the two roads leading from Totneſs to Dartmouth and Modbury formed a right angle. From hence I would know the true diſtances of the three towns, and the rate at which we travelled which was uniform?

IX. QUESTION 399 *by Mr.* H. Watſon.

The latitudes and longitudes of three places on the earth's ſurface, ſuppoſe London, Moſcow, and Conſtantinople, being given, as below; required the latitude and longitude of that place which is equidiſtant from the former three?

The latitude of London is 51° 30′ the latitude and longitude of Moſcow 55° 45′ and 38° 0′, and thoſe of Conſtantinople 41° 30′ and 29° 15′, reſpectively.

X. QUESTION 400 *by Mr.* Hugh Brown.

A lends B 1000l. for which B repays him as follows, viz. at the end of three months 180l. of five months 150l. of ſix months 140l. of eight months 100l. of nine months 90l.

of ten months 120l. and at the year's end 250l. The rate of intereſt is required?

XI. QUESTION 401 *by Mr.* E. Rollinſon.

To inveſtigate the value of an annuity, on a life of a given age, according to any table of obſervations on the degrees of mortality of mankind, by dividing the whole extent of life into different periods, during which the decrements, or numbers dying off yearly, may be eſteemed equal; without having any other ſeries to ſum than a common geometrical progreſſion.

XII. QUESTION 402 *by Mr.* W. Bevil.

Suppoſe the ends of a thread, ten feet long, be faſtened to two tacks, in the ſame horizontal line, at the diſtance of ſix feet: I would know where two weights, the one three and the other five ounces, muſt be fixed to the thread, ſo as to hang at reſt in the ſame horizontal line at the diſtance of three feet from the level of the tacks?

XIII. QUESTION 403 *by Mr.* Thomas Moſs.

Suppoſe that, from the top of a mountain, in form of a paraboloid, whoſe perpendicular height is 600 yards and its baſe-diameter three miles, a cannon ball is to be diſcharged with a quantity of powder ſufficient to carry it to the height of 700 yards in a vertical direction; I would know the elevation of the piece ſo that the ball may fall at the greateſt diſtance poſſible from the place of projection?

XIV. QUESTION 404 *by Mr.* H. Watſon.

To determine the center of attraction of a ſemi-ſpherical body, or that point in the axis where a corpuſcle may be placed to remain in equilibrio by the equal and contrary action of the matter of the hemiſphere ſurrounding it.

XV. QUESTION 405 *by Mr.* Patrick O'Cavanah.

Given $\dot{x}\dot{y} - x\ddot{y} - a\ddot{y} - \frac{x\dot{y}^2}{b} = 0$, to find the general relation of the fluents x and y.

PRIZE QUESTION *by Mr.* E. Rollinſon.

Three ſhips, A, B, C, ſail at the ſame time from three different ports: The ſhip A, from the ſouthermoſt port, runs due eaſt, at the rate of five knots: The ſhip B ſails S. E. by S. at the rate of three knots, her port lying N. N. E. from that of A, and at the diſtance of twelve miles: The other ſhip, C, always bears down upon (or ſteers directly towards) the two former, which ſhe keeps conſtantly in a line. I would know how far ſhe muſt run before ſhe comes up with B, together with the diſtance of her port from the other two, and the path ſhe deſcribes?

A PARADOX *by* Perpendicularius.

Whatever angle any two right lines can poſſibly form that meet with each other, a third line may nevertheleſs be drawn in ſuch manner as to be a perpendicular to them both.

1756.

Queſtions anſwered.

I. QUESTION 391 *anſwered by Miſs* E. S. *to the Propoſer Miſs* Atkinſon.

I happily congratulate
With you in joys of dear EIGHTEEN.
Oh! might we ever theſe partake,
Nor age, nor trouble intervene:
But years, alas! too ſoon will fly,
And all the gaudy ſcene deſtroy.

The ſame anſwered by Mr. W. Litſon.

By Falſe-Poſition it appears,
Your age, fair Miſs, is eighteen years.

Thus it was alſo anſwered by *Emilia*, Mr. *Joſ. Briſcall*, Mr. *J. Wood*, Mr. *M. Hitchins*; and Mrs. *Priſ. Miſterton*, who ſent the proceſs wrote out at length.

Mr. B. Lydal, *addreſſing himſelf to the Propoſer, anſwers it thus:*

Let x denote your age, then will $56+x-58=2x-20$; and conſequently $x=18$. A very good age for matrimony, Miſs.

In this manner it was alſo anſwered by Mr. *J. Bereſford*, Mr. *Joſ. Briſcall*, Mr. *J. Carr*, Mr. *Joſ. Dawſon*, Mr. *John Dodſon*, Mr. *G. Dunn*, Mr. *D. Haſtings*. Mr. *T. Knight*, Mr. *G. Langley*, Mr. *Joſ. Lord*, Mr. *T. Love*, Mr. *W. Richardſon*, Mr. *Joſ. Scott*, Mr. *T. Sharp*, Mr. *G. Stapley*, Mr. *J. Taylor*, Mr. *W. Tombs*, Mr. *T. Wilkin*, and many others.

II. QUESTION 392 *anſwered by Mr.* W. Stoker, *of* Fatfield Staiths.

The ſquare roots of the diſtances being as the times we have (per queſt.) as $\sqrt{1}$ to $\sqrt{2}$, ſo is the time of falling through the firſt half, to the time of falling through the whole required height: And therefore as $\sqrt{2}-1$ is to $\sqrt{2}$, ſo is 1 ſecond (the time of deſcent through the latter half) to $\frac{\sqrt{2}}{\sqrt{2}-1}$ $(=2+\sqrt{2})=3{\cdot}414$ the time of deſcent through the whole height: Whence the height itſelf is found = 187·48 feet.

The ſame anſwered by Mr. J. Nichols.

Let $t=$ the whole time of deſcent, ſo will $t-1=$ the time of deſcent through the firſt half of the tower's height; and therefore (the ſpaces deſcended being always as the ſquares of the times) we have $tt:\overline{t-1}\vert^{2}::2:1$; whence $tt-2t+1=\frac{1}{2}tt$: From which $t=2+\sqrt{2}=3{\cdot}414$, and the tower's height = 187·48 feet.

The ſame anſwered by Mr. J. Boſton.

Let $a=16\frac{1}{12}$ feet (the ball's deſcent in the firſt ſecond of time) and $x=$ the ſeconds the ball was falling; then (by the queſtion and the law of the deſcent of heavy bodies) the tower's height will be $=ax^2$, and the half of that height $=a\times\overline{x-1}\vert^{2}$; hence we have $\frac{1}{2}ax^2=a\times\overline{x-1}\vert^{2}$: conſequently $x=2+\sqrt{2}=3{\cdot}41421$; and $ax^2=187{\cdot}4806$ feet = the tower's height.

In the very ſame manner it was anſwered by Mr. *W. Bacon*, Mr. *S. Bamfield*, Mr. *Ja. Bereſford*, Mr. *W. Bevil*, *Birchoverenſis*, Mr. *G. Brownbridge*, Mr. *R. Butler*, Mr. *Abr. Botham*, Mr. *Lionel Charlton*, Mr. *Tim. Drury*, Mr. *R. Flitcon*, Mr. *P. George*, Mr. *R. Gibbons*, Mr. *E. Griffiths*, Mr. *J. Honey*, Mr. *T. Hopkinſon*, Mr. *S. King*, Mr. *T. Peart*, Mr. *W. Phipps*, Mr. *Ja. Robinſon*, Mr. *Joſ. Scott*, Mr. *J. Shipman*, Mr. *W. Spicer*, Mr. *W. Trott*, Mr. *H. Watſon*, Mr. *R. Weſton*, Mr. *C. Wildbore*, and Mr. *R. Young*.

III. QUESTION 393 *anſwered by Mr.* Rich. Gibbons.

Put $p = xz$, and $s = xx + zz$, $a = 546560$, and $e = 1086992$: Then, by the queſtion, $sp = a$, and $s^2 = e + 2p^2 = \frac{a^2}{p^2}$. Hence, by completing the ſquare and reduction, p is found $= 448$, and s $(= \frac{a}{p}) = 1220$. Conſequently $x + z = \sqrt{s + 2p} = 46$, and $x - z = \sqrt{s - 2p} = 18$, and therefore $x = 32$ and $z = 14$.

The ſame anſwered by Mr. W. Phipps.

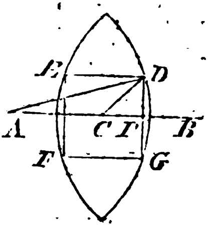
Putting $a = 546560$, and $b = 1086992$, we have, by the firſt equation, $x^2 + z^2 = \frac{a}{xz}$; from the ſquare of which take the ſecond, ſo ſhall $2x^2z^2 = \frac{a^2}{x^2z^2} - b$: whence $x^4z^4 + \frac{1}{2}bx^2z^2 = \frac{aa}{2}$, and conſequently $x^2z^2 = \frac{1}{4}\sqrt{8a^2 + b^2} - \frac{1}{4}b = 20054$; which call m^2, and then $xz = m = 448$; whence, by ſubſtitution, $x^2 + z^2 = \frac{a}{m}$: From which and $xz = m$, we have

$$\left.\begin{matrix} x \\ z \end{matrix}\right\} = \frac{1}{2}\sqrt{\frac{a}{m} + 2m} \pm \frac{1}{2}\sqrt{\frac{a}{m} - 2m} = \left\{\begin{matrix} 32 \text{ the wife's age.} \\ 14 \text{ the daughter's age} \end{matrix}\right.$$

In this manner it was anſwered by Mr. *Abr. Botham*, Mr. *R. Butter*, Mr. *Lionel Charlton*, Mr. *R. Flitcon*, Mr. *J. Hemingway*, Mr. *J. Honey*, Mr. *Step. King*, Mr. *S. Koit*, Mr. *J. Nichols*, Mr. *C. Tate*, Mr. *T. Wilkin*, Mr. *R. Young*, and ſome others.——Mr. *W. Bacon*, Mr. *J. Eaden*, Mr. *P. George*, Mr. *J. Goodhead*, Mr. *E. Griffiths*, Mr. *W. Stoker*, and ſome others, ſolve it by ſubſtituting for the ſum and difference of the two unknown quantities. Thus, making $v + y = x$, $v - y = z$, $546560 = 2a$, and $1086992 = 2b$, the given equations are reduced to $v^4 - y^4 = a$, and $v^4 + y^4 + 6v^2y^2 = b$; and, by ſubtracting the former from the latter, and putting $b - a = 2c$, there comes out $3y^2v^2 = c - y^4$ or $9y^4 \times v^4$ $(= 9y^4 \times \overline{y^4 + a}) = \overline{c - y^4}|^2$, or $8y^8 + \overline{9a + 2c} \times y^4 = c^2$: Which equation, ſolved, gives $y = 9$; whence $v = 23$, $x = 32$, and $z = 14$.

IV. QUESTION 394 *anſwered by Mr.* Cha. Tate.

CONSTRUCTION. Let the given diſtance AB of the two centers, be biſected in C; and make the $\angle BCD = \frac{1}{2}$ a right angle; then a perpendicular DP, let fall from the interſection D, will evidently be half the ſide of the inſcribed ſquare $DEFG$.



CALCULATION. The radius AD being drawn, in the $\triangle ACD$ will be given $AD = 25$, $AC = 15$, and the $\angle ACD = 135°$; whence DAC is found $= 19° \; 54'$; and from thence $DP = 8\cdot5078$, the double of which $17\cdot0156$, is the ſide of the ſquare.

In this manner it was conſtructed and ſolved by Mr. *P. George*, Mr. *R. Gibbons*, Mr. *J. Milbourn*, Mr. *T. Moſs*, Mr. *J. Nichols*, Mr. *J. Shipman*, and Mr. *Abr. Stone.*

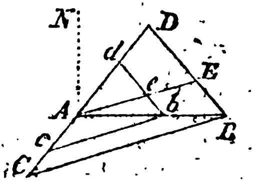
The ſame anſwered algebraically by Mr. W. Harriſon, *of* Wigan.

Putting the radius $AD = a = 25$, $AC \; (= BC) = b = 15$, and $CP \; (= PD) = x$, we have (per Euc. 47. 1.) $\overline{b+x}|^2 + x^2 = a^2$; whence $x^2 + bx = \frac{1}{2}a^2 - \frac{1}{2}b^2$, and conſequently $x = \sqrt{\frac{1}{2}a^2 - \frac{1}{4}b^2} - \frac{1}{2}b = 8\cdot5078$: Therefore $ED \; (= DG) = 17\cdot0156$. *W. W. R.*

In the ſame manner it was anſwered by Mr. *J. Boſton*, Mr. *G. Brownbridge*, Mr. *D. Haſtings*, Mr. *Step. King*, Mr. *S. Koit*, Mr. *B. Lydal*, Mr. *W. Phipps*, and many others.

V. QUESTION 395 *anſwered by Mr.* Philip George.

CONSTRUCTION. Draw the meridian AN; and, ſuppoſing A to be the place of the headland, draw AB, AE, and AD to repreſent the given bearings of the ſhip therefrom; in DAC take Ac to Ad in the given ratio of the times, or as 4 to 6, and draw cb parallel to AE, meeting AB in b; then draw bed, to which the ſhip's courſe BED (let AB be what it will) muſt be parallel: Becauſe (by Euc. 6. 4.) $be : ed \; (:: Ae : Ad :: 4 : 6) :: BE : ED$.



CALCULATION. In the triangle Abc are given all the angles and the ſide Ac (= 4), to find $AB = 7{\cdot}391$; and then from Ab, Ad (= 6) and the contained angle dAb, we get Abd (= ABD) = 47° 25′, which is the complement of the courſe. *W. W. R.*

The ſame anſwered by Mr. J. Shipman, *of* Hull.

Let A repreſent the headland, and AN the meridian; and through A draw the given bearings AB, AE, and AD; make $AC = 4$, $AD = 6$, and draw CB parallel to AE; then from B draw BD, ſo will ABD be the ſhip's courſe from the weſt. For, becauſe of the parallel lines, $AC : AD :: EB : ED$. From hence the courſe is found, by calculation, to be N. 42° 35′ weſterly.

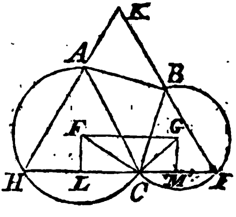
The ſame anſwered by Mr. J. Milbourn.

Let A repreſent the headland, and B, E, and D the places of the ſhip at the three obſervations. Put the ſine of BAE (22° 30′) = a, and that of DAE (45°) = b; then will $AB = \dfrac{BE \times \text{ſin.}\, E}{a}$, and $AD = \dfrac{DE \times \text{ſin.}\, E}{b}$ (by trig.): whence we have $AB : AD \left(:: \dfrac{BE}{a} : \dfrac{DE}{b} :: \dfrac{4}{a} : \dfrac{6}{b}\right) :: 4b : 6a$. From which known ratio of the ſides, and the included angle BAD, the courſe ABD, from the weſt, is found to be 47° 25′.

Mr. *G. Dunn*, Mr. *R. Gibbons*, Mr. *D. Haſtings*, Mr. *T. Moſs*, Mr. *T. Peart*, Mr. *Abr. Stone* (Land Surveyor), Mr. *W. Trott* (the propoſer), and two or three others, likewiſe ſent very neat conſtructions of this problem.

VI. QUESTION 396 *anſwered by Mr.* W. Bevil.

CONSTRUCTION. The given points A, B, C being joined, upon AC and BC let two ſegments of circles be deſcribed, each to contain an angle of 60°; join their centers by the line FG, and parallel thereto draw HCI cutting the two circles in H and I; then through A and B draw HK and IK, ſo ſhall HIK be the triangle required.



For, ſuppoſing FL and GM to be perpendicular to HI, and HI (always terminated by the circles) to revolve about the point C, it is plain that, when HI is a maximum, LM, being the half thereof, muſt alſo be a maximum; and this will evidently be when LM is parallel to FG.

CALCULATION. In the triangle ABC are given all the ſides, to find the angle $C = 38° 38'$; then in the triangle FCG are given the angle $C = 98° 38'$, and the ratio of the containing ſides, as AC to BC (or 16 to 12); whence is found the angle F ($= HCF$) $= 33° 41'$, and the angle G ($= ICG$) $= 47° 4'$; therefore $HCA = 63° 41'$, $ICB = 77° 4'$, $CAH = 56° 19'$, $CBI = 42° 56'$, HI ($= HC + CI$) $= 24{\cdot}7$, and the area $= 26.42 = 26$ acres, 1 rood, 27 perches.

Algebraic Solution to the ſame by Mr. W. Spicer.

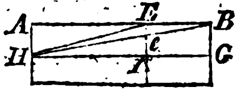
Put $a = BC = 12$, $b = BA = 10$, $s =$ ſine of 60°, and p and $-q$ equal to the ſine and coſine of $\angle BCI + \angle BAK = 152° 51' 58''$, and x and y thoſe of $\angle BAK$: Then will $py + qx =$ ſine $\angle BCI$; and (per trig.) $s : b :: x : \frac{bx}{s} = BK$; and $s : a :: py + qx : \frac{apy + aqx}{s} = BI$: Therefore $\frac{bx + aqx + apy}{s} = IK$, a max. In fluxions, $b\dot{x} + aq\dot{x} + ap\dot{y} = 0$: But $\dot{y} = -\frac{x\dot{x}}{y}$; whence $b\dot{x} + aq\dot{x} - \frac{apx\dot{x}}{y} = 0$: Solved, $\frac{x}{y}$ ($=$ tang. $\angle BAK$) $= \frac{b + ag}{ap}$. Hence $\frac{1}{s}\sqrt{aa + 2abq + bb} = IK = 24{\cdot}7002$; and $\sqrt{\frac{1}{3}} \times \overline{aa + 2abq + bb} = 26{\cdot}42$, the area required.

According to the former of the two preceding methods it was alſo anſwered by Meſſ. *Moſs* (the propoſer), *Peart*, *Rollinſon*, *Trott*, and *Watſon*; and according to the latter, by Meſſ. *Botham*, *Charlton*, *George*, *Nichols*, and ſome others. ——Many contributors have ſolved this queſtion upon a ſuppoſition that one ſide of the required triangle is parallel to the longeſt ſide of the given one; which, though very near, is not ſtrictly true; the ſaid ſides, when the area is the greateſt poſſible, being inclined to each other in an angle of 3° 41'.

VII. QUES-

VII. QUESTION 397 *anſwered by Mr.* Abr. Botham.

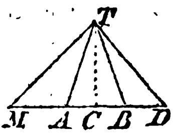
Suppoſing *HG* to be the length of the viſto, and *I* the place of the ſtatue; let *t* expreſs the tang. of $\frac{1}{3}$ of $\angle EHI$; which (by the queſt.) is $\frac{1}{2}$ of $\angle BHG$; alſo let $a = 1501$, and $b = 1000$: Then the tangents of *EHI* and *BHG*, being expreſſed by $\frac{3t - t^3}{1 - 3tt}$ and $\frac{2t}{1 - tt}$, (vide theor. p. 41, Diary 1755) and theſe tangents being to each other in the given proportion of *a* to *b*, we therefore have $\frac{3bt - bt^3}{1 - 3tt} = \frac{2at}{1 - tt}$; which, properly reduced, becomes $t^4 + \frac{6a}{b} - 4 \times t^2 = \frac{2a}{b} - 3$; and this, ſolved, gives $t = {}^{\cdot}019987$: From whence the length of the viſto is found $= 1250$ yards.



According to the ſame method it was alſo anſwered by Meſſrs. *Aſh* (the propoſer), *Butler*, *Charlton*, *Griffiths*, *Moſs*, *Peart*, *Rollinſon*, *Spicer*, and *Watſon*.——Some very ingenious Gentlemen have taken a good deal of pains, in order to convince us, that this queſtion was not properly limited; and others have hinted the ſame thing; not conſidering, that, from the ratio of any two angles (*EHI*, *eHI*) and the ratio of their tangents (which ratios are here given) both the angles and tangents may, always, be determined. ——The ſmallneſs of the angles in the caſe propoſed (where the tangents are nearly in the ſame ratio with them) is, we preſume, what embarraſſed theſe gentlemen: The method of trial-and-error (as it is called) whereby moſt of them attempted the ſolution, being of little or no uſe here.

VIII. QUEST. 398 *anſwered by Mr.* G. Brownbridge.

The places of the three towns being repreſented by *M*, *D*, and *T*, let the three parts of *MD* (as ſpecified in the problem) be denoted by $6x$, a, and $5x$, reſpectively; then $a : 22x :: 2x : \frac{44xx}{a} = AC - BC$; $\therefore$ $AC = \frac{a}{2} + \frac{22xx}{a}$, $BC = \frac{a}{2} - \frac{22xx}{a}$,



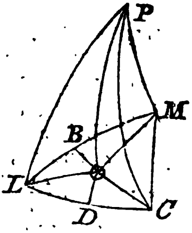
$MC = \frac{a}{2} + 6x + \frac{22xx}{a}$, and $DC = \frac{a}{2} + 5x - \frac{22xx}{a}$. Now $MC \times DC$ ($= CT^2$) $= AT^2 - AC^2$; whence we have $44x^3 + 184ax^2 - 11a^2x = a^3$, or $44x^3 + 1288x^2 - 539x = 343$; which, ſolved, gives $x = \cdot 75284$. Therefore $MD = 15\cdot281$, $MT = 12\cdot236$, and $DT = 9\cdot153$.

The ſame anſwered by Mr. R. Butler.

Suppoſing M, D, and T to repreſent the three towns, let $MA = 6x$, $TA = 12x$, $BD = 5x$, $BT = 10x$, and AB ($= 7$) $= a$: Then (by trig.) $a : 22x :: 2x : \frac{44x^2}{a} =$ diff. ſegments AC and BC; whence $CA = \frac{a}{2} + \frac{22x^2}{a}$, and $CB = \frac{a}{2} - \frac{22a^2}{a}$. But (by Euc. 12. 2.) $MA^2 + AT^2 + 2MA \times AC + DB^2 + BT^2 + 2BD \times BC$ ($= MT^2 + DT^2$) $= MD^2$ (by Euc. 47. 1.); that is, in ſpecies, $305x^2 + 11ax + \frac{44x^3}{a} = \overline{a + 11x}|^2$, or $44x^3 + 184ax^2 - 11a^2x = a^3$. Whence x is found $= .752846$: Therefore $MD = 15,2813$, $MT = 12,2365$, $TD = 9,1534$, and the rate at which he travelled was 4,1064 miles per hour.

To this queſtion upwards of 30 true anſwers have been received.

IX. QUESTION 399 *anſwered by Mr.* H. Watſon, *the Propoſer.*



Let P repreſent the pole of the earth, and L, C, and M the three places propoſed: From the latitudes and longitudes of which the diſtances LM, LC, and the contained angle CLM, will (by common proportions) be found 22° 38′, 22° 17′, and 40° 55′, reſpectively. Then ſuppoſing OB and OD perpendicular to LM and LC (O being the required place) we have given LB ($= \frac{1}{2} LM$) $= 11° 19′$, LD ($= \frac{1}{2} LC$) $= 11° 8\frac{1}{2}′$, with the ſum of the angles BLO, DLO, at the baſes (the hypothenuſe LO being common); whence it will be (*per ſpherics*) ſin. $\overline{LB + LD}$

$\overline{LB+LD}$: Sin. $\overline{LB-LD}$ :: co-tang. $\frac{DLO+BLO}{2}$ (20° 27½′) : tang. of $\frac{DLO-BLO}{2}$ = 1° 13½′; whence BLO = 19° 14′; from which, and LB, we have LO (= MO = CO) = 11° 58′, and then, in the triangle LOP, will be given LO, LP, and the angle OLP (= 83° 27′ = $PLM + BLO$); whence OP = 38° 43′, and ∠ LPO = 19° 13′, being the complement of the latitude, and the longitude from London, reſpectively.

In the very ſame manner it was anſwered by Mr *T. Moſs*.

Algebraic Solution to the ſame by Mr. R. Young.

Having found (by common ſpherics) LB, LD, and the angle DLB, put the tang. of $LB = a$, that of $LD = b$, the ſine and co-ſine of DLB equal to m and n, and thoſe of BLO equal to x and y, reſpectively: Then, the co-ſine of DLO being expreſſed by $mx + ny$, it will be, *per ſpherics*, $\left\{\begin{matrix} y : a :: 1 \text{ (rad.)} : \text{tang. } LO \\ mx + ny : b :: 1 : \text{tang. } LO \end{matrix}\right\}$; whence $\frac{a}{y} = \frac{b}{mx+ny}$; and conſequently $\frac{x}{y} = \frac{b-na}{ma}$ = tang. BLO = 19° 14′: From which the latitude and longitude of the place are found to be 51° 17′, and 19° 13′, reſpectively.

Meſſ. *Bamfield*, *Birchoverenſis*, *Botham*, *Charlton*, *Hopkinſon*, *Spicer*, and ſome others, have alſo given very neat algebraic ſolutions to this problem. Mr. *P. George*, by producing the perpendicular DO to meet the ſide LM, ſolves it by common ſpherics: And Meſſ. *Peart* and *Rollinſon*, after finding LM, LC, and the angle CLM (as above) compute the difference of the other two angles LMC, LCM, which (becauſe $OMC = OCM$) will alſo be the difference of OML and OCL, or of their equals OLB and OLD; whence both theſe angles are known, and from thence LO.

X. QUESTION 400 *anſwered by Mr.* Hugh Brown, *the Propoſer.*

Putting $z = \frac{1}{12}$ of the intereſt of 1 l. for one year, we ſhall (by diſcounting at ſimple intereſt) have $\frac{250}{1+12z} + \frac{120}{1+10z} + \frac{90}{1+9z} + \frac{100}{1+8z} + \frac{140}{1+6z} + \frac{150}{1+5z}$ +

$+ \frac{180}{1+3z} = 1000$. The fractions reduced into ſeries, and properly ordered, give $z - \frac{3605z^2}{397} + \frac{36133z^3}{397}$, &c. $= \frac{3}{794}$; whence $z = {\cdot}0037783 + {\cdot}0001296 + {\cdot}0000040 = {\cdot}003912$: Therefore the required rate is 4·694, or 4 l. 13 s. 10½ d. per cent.

The ſame anſwered by Mr. John Honey.

Let x^{12} = rate per annum; then x (according to compound intereſt, will be the rate for 1 month, x^2 for 2 months, &c. whence the preſent value of 180 l. to be received at the end of 3 months will be $\frac{180}{x^3}$ &c. Hence we have $\frac{180}{x^3} + \frac{150}{x^5} + \frac{140}{x^6} + \frac{100}{x^8} + \frac{90}{x^9} + \frac{120}{x^{10}} + \frac{250}{x^{12}} = 1000$ (per queſt.); and therefore $x^{12} - {\cdot}18x^9 - {\cdot}15x^7 - {\cdot}14x^6 - {\cdot}1x^4 - {\cdot}09x^3 - {\cdot}12x^2 = {\cdot}25$: Solved, $x = 1{\cdot}003852$; and $x^{12} = 1{\cdot}047216$ and conſequently 4 l. 14 s. 5 d. the rate per cent. required.

In the very ſame manner it was anſwered by Meſſ. *W. Bevil*, *E. Rollinſon*, and *H. Watſon*.

Mr. R. Flitcon *anſwers it thus:*

Let x, $2x$, and $3x$ be the intereſt of one pound for 1, 2, and 3 months, reſpectively; then the intereſt of 1000 l. for 3 months will be $3000x$, and its amount $1000 + 3000x$, from which ſubtracting 180 l. then paid, the remainder $820 + 3000x$ will be a new principal; which, in 2 months more, amounts to $820 + 4640x + 6000x^2$; from whence ſubtracting 150 l. then paid, the remainder $670 + 4640x + 6000x^2$ will be the principal (or debt) at the end of 5 months: And by proceeding in the ſame manner with all the reſt of the payments, the equation reſulting (according to the queſtion) will, in its leaſt numbers, be $-3 + 758x + 4871x^2 + 14503x^3 + 24016x^4 + 22692x^5 + 11456x^6 + 2400x^7 = 0$. Hence $x = {\cdot}0038609$, and the rate, per cent. per annum, 4 l. 12 s. 7¾ d.

According to this laſt method it was likewiſe ſolved by Meſſ. *Bamfield, Butler, Goodhead*, and *Trott*, whoſe equations exactly agree.——A great number of other ſolutions have been received, upon principles ſomewhat different, which however come very near the truth.

XI. QUEST.

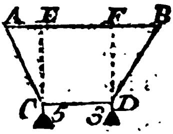
XI. Queſt. 401 *anſwered by the Propoſer, Mr.* Rollinſon.

Let a denote the number of the living correſponding, in the table, to the given age; and let the ſucceeding decrements, or the numbers that die off yearly (for as long as they continue equal) be repreſented, each, by b; alſo let the next ſucceeding decrements (for as long as they continue equal) be denoted, each, by c; and the next after thoſe by d; and ſo on: Putting r to repreſent the rate, and P the value of the perpetuity correſponding: Moreover, ſuppoſe that, after the deceaſe of the propoſed life A, the eſtate, or annuity, is to go to another perſon B, and his heirs, for ever. Then the probability that the life A fails the firſt year being $\frac{b}{a}$, the value of B's expectation on that contingency will therefore be $P \times \frac{b}{a}$: And the probability of A's dropping the ſecond year being alſo $\frac{b}{a}$, the expectation of B thereon will be $\frac{P}{r} \times \frac{b}{a}$, ($\frac{P}{r}$ being the value of the perpetuity diſcounted for one year). In the ſame manner the expectation of B on the third year will appear to be $= \frac{P}{r^2} \times \frac{b}{a}$, and on the fourth year $= \frac{P}{r^3} \times \frac{b}{a}$, &c. Whence it is evident, that B's whole expectancy, on the contingency of the life's failing in the firſt interval, (m) during which the decrements are equal, each, to b, is truly defined by $rP \times \frac{b}{a} \times : \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} \ldots\ldots + \frac{1}{r^m}$, or its equal $rP \times \frac{b}{a} \times \frac{1 - r^{-m}}{r - 1}$, which is alſo $= rP^2 \times \frac{b - br^{-m}}{a}$, becauſe $P = \frac{1}{r - 1}$. Again, the decrements during the ſecond interval (n) being each $= c$, the probability of the life's failing in any aſſigned year of this interval will therefore be denoted by $\frac{c}{a}$; and the value of B's expectation, on the whole interval, by $\frac{P}{r^m} \times \frac{c}{a} + \frac{P}{r^{m+1}} \times \frac{c}{a} + \frac{P}{r^{m+2}} \times \frac{c}{a}$, &c. $= \frac{rP}{r^m} \times \frac{c}{a} \times : \frac{1}{r} + \frac{1}{r^2} +$

$+\frac{1}{r^3}\ldots\ldots+\frac{1}{r^n}=rP^2\times\frac{c-cr^{-n}}{ar^m}$. In the very ſame manner the expectancy on the third interval (p) appears to be $=rP^2\times\frac{d-dr^{-p}}{ar^{m+n}}$; and on the fourth ($q$) $=rP^2\times\frac{e-er^{-q}}{ar^{m+n+p}}$, &c. &c. Therefore, by collecting all theſe values together, the whole expectation of B and his heirs comes out $=\frac{rP^2}{a}\times : b+\frac{c-b}{r^m}+\frac{d-c}{r^{m+n}}+\frac{e-d}{r^{m+n+p}}+$, &c. which, ſubducted from P, the value of the perpetuity, leaves $P-\frac{rP^2}{a}\times : b+\frac{c-b}{r^m}+\frac{d-c}{r^{m+n}}+\frac{e-d}{r^{m+n+p}}$, &c. for the true value of the life A: But rP is $=P+1$, and $\frac{1}{r^m}$, $\frac{1}{r^{m+n}}$, &c. are the preſent values of 1 l. to be received at the end of m, $m+n$, &c. years; which being found (from the tables) and repreſented by M, N, Q, &c. reſpectively, the value of the annuity will be $P+\frac{PP+1}{a}\times : b + \overline{c-b}\times M+\overline{d-c}\times N+\overline{e-d}\times Q+\overline{f-e}\times R$, &c. where the ſeries is to be continued till it terminates, and where the co-efficient of the laſt term will be the laſt of the quantities b, c, d, &c. with a negative ſign, the next letter in order being equal to nothing.

XII. QUESTION 402 *anſwered by Mr.* S. Bamfield.

Suppoſing CE and DF to be perpendicular to the horizontal line AB, it is evident, from mechanics, that $AE : BF :: 3 : 5$. If therefore $EC (= FD) = b = 3$, $AB = a = 6$, $AC + CD + DB = c = 10$, and $AE = 3x$, then will $BF = 5x$; $EF (= CD) = a - 8x$; and $\sqrt{bb+9xx}\,(AC) + \sqrt{bb+25xx}\,(BD) + a - 8x = c$. From the reſolution of which equation x is found $= \cdot 31786$; and from thence $AC = 3\cdot1479150$, and $CD = 3\cdot3951$.



The

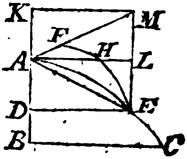
The ſame anſwered by Birchoverenſis.

Put AB $(=6)=l$; CE $(=3)=b$, $AC+CD+DB$ $(=10)=s$, and $AC=x$; then $AE=\sqrt{xx-bb}$, and, by the property of the center of gravity, $BF=\frac{5}{3}\sqrt{xx-bb}$: Conſequently CD $(=EF)=l-\frac{8}{3}\sqrt{xx-bb}$, and $BD=\frac{1}{3}\sqrt{25xx-16bb}$. Whence we have $x+l-\frac{8}{3}\sqrt{xx-bb}+\frac{1}{3}\sqrt{25xx-16bb}=s$. This equation, properly reduced, will give $x^3-3{\cdot}35x^2-10{\cdot}8x+36=0$; where x is found $=3{\cdot}1479$.

Meſſ. *Bevil* (the propoſer), *Botham*, *Moſs*, *Peart*, *Rolkinſon*, *Watſon*, and *Young*, ſent elegant ſolutions to this problem.

XIII. QUESTION 403 *anſwered by Mr.* Tho. Peart.

From the given height of the mountain AB, and its ſemi-diameter BC, the parameter of the parabola will be found $=11616$ yards; from whence the curve AEC may be conſtructed. Take $AK=$ twice the height of the perpendicular ſhot, and draw KM perpendicular thereto; in KM take the point M, ſo that a perpendicular let fall from thence, meeting the curve in E, ſhall be equal to the ſtraight line AE; then join A, M, which will be the line of direction.



CALCULATION. Let $d=$ the height of the perpendicular ſhot, and $a=11616=$ the parameter, $AD=x=LE$; then ME $(=AD+AK)=2d+x$, $DE=\sqrt{ax}$, and $AE=\sqrt{ax+xx}=2d+x$ $(=ME)$; therefore $x=\frac{4dd}{a-4d}=222{\cdot}323$ yards, and AE $(2d+x)=1622{\cdot}323$; from whence the angle AED is found $=7^\circ\ 52'\ 36''$; the half of which being taken from 45°, there remains $41^\circ\ 3'\ 42''$ for the angle of elevation LAM. *W.W.R.*

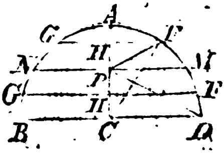
The ſame anſwered by Mr. Walter Trott.

Put the height AB of the mountain $(=600)=a$, the ſemi-diameter BC of the baſe $(=2640)=b$, and the double impetus (= 1400) = c: Then, ſuppoſing E to be the place where the ball (whoſe path is AHE, and firſt direction AF) impinges on the ſurface, let AE and ED be drawn, the latter parallel to CB, and let $AD = x$; and then, by the property of the parabola, $DE^2 = \frac{b^2x}{a}$, and therefore $AE = \sqrt{\frac{bbx}{a} + x^2}$. Hence becauſe the diff. is to be a max. we have (by Simpſon's Exerciſes, p. 199) $\sqrt{\frac{bbx}{a} + xx} - x$ (= $AE - AD = c$; from which $x = \frac{acc}{bb - 2ac} = 222{\cdot}323 = AD$. Whence the angle EAD = 82° 8′; the half of which (= 41° 4′) is the true angle of elevation (HAF). *W. W. R.*

Meſſ. *Botham*, *Moſs* (the propoſer), *Rollinſon* and *Watſon* anſwer it in the ſame manner. Meſſ. *Bumfield* and *Charlton* give the inveſtigation by a neat fluxional proceſs, which room will not admit.

XIV. QUESTION 404 *anſwered by Mr.* Lionel Charlton, *of* Whitby.

Let BAD be the given hemiſphere, P the required point, NPM and GHF ſections of the ſolid perpendicular to the axis AC; let PF, CF, and PD be drawn; putting $AC = r$, $PC = a$, $AP = e$, $PM = f$, $PD = g$, $PH = x$, and $PF = z$. Then the effect of the attraction of all the particles in the plane of the circle GHF, on a corpuſcle at P, will, it is well known, be as $1 - \frac{x}{z}$; and conſequently the fluxion of the attraction of the ſegment $NGFM$ as $\dot{x} - \frac{x\dot{x}}{z}$: Which (becauſe $a^2 + z^2 + 2ax = r^2$, and therefore $x = \frac{ff - zz}{2a}$, and $\dot{x} = -\frac{z\dot{z}}{a}$) will be reduced to $\dot{x} + \frac{f^2 - z^2}{2a} \times \frac{\dot{z}}{a}$, whereof the corrected fluent is $x + \frac{3f^2z - z^3 - 2f^3}{6a^2}$: Which, when



$\dot{x} =$

$x = e = z$, becomes $e + \frac{3ef^2 - e^3 - 2f^3}{6aa}$ for the force of the upper ſegment *NAM* in the direction *PA*.

In the ſame manner, taking *GF* below *NM*, the fluxion of the attraction of the part *NGFM* will be, *ſtill*, expreſſed by $\dot{x} + \frac{f^2 - z^2}{2a} \times \frac{\dot{z}}{a}$, and the corrected fluent by $x + \frac{3f^2z - z^3 - 2f^3}{6aa}$, which, by taking $x = a$ ($= PC$) and $z = g$ ($= PD$) gives $a + \frac{3f^2g - g^3 - 2f^3}{6aa}$ for the whole force of the lower ſegment *NBDM* in the oppoſite direction *PC*. Hence we have $e + \frac{3ef^2 - e^3 - 2f^3}{6aa} = a + \frac{3f^2g - g^3 - 2f^3}{6aa}$, or $6a^2 \times \overline{e - a} + 3ef^2 - e^3 = g \times \overline{3ff - gg}$. But $e = r - a$, $ff = rr - aa$, and $g = \sqrt{rr + aa}$, ſo that, by ſubſtitution, $2r^3 - 8a^3 = \sqrt{rr + aa} \times \overline{2r^2 - 4a^2}$; which, order'd, gives $12a^4 - 8r^3a + 3r^4 = 0$; where, taking $r = 1$, we have $a = 0{\cdot}4230428$. Therefore the ratio of *AP* to *CP* will be that of 0·5769572 to 0·4230428, or as 4 to 3, nearly.

The author of a very neat ſolution to this proplem (whom we ſhould be ſorry to diſoblige by an improper ſtep) will, we hope, candidly excuſe our giving preference to the above; as room would not poſſibly admit of both, and as the fluents, here, may be comprehended by common readers, for whoſe improvement we are ſolicitous.

XV. QUESTION 405 *anſwered by Mr.* T. Moſs.

Divide the given equation by $\frac{\dot{y}\dot{y}}{\dot{x}}$, and you will have $\frac{\dot{x}^2\dot{y} - x\dot{x}\ddot{y}}{\dot{y}\dot{y}} - \frac{a\dot{x}\ddot{y}}{\dot{y}\dot{y}} - \frac{x\dot{x}}{b} = 0$; which, by taking the fluent, becomes $\frac{x\dot{x}}{\dot{y}} + \frac{a\dot{x}}{\dot{y}} - \frac{xx}{2b} = d$ (a conſtant quantity); whence $\dot{y} = 2b \times \frac{a\dot{x} + x\dot{x}}{2bd + x^2}$, and conſequently $y = a$

$\sqrt{\frac{2b}{d}} \times$ arc whoſe tangent is $\frac{x}{\sqrt{2bd}}$ (to radius 1) $+ b \times$ hyp. log. $\frac{2bd + x^2}{e}$; where d and e may denote any conſtant quantities at pleaſure.

The ſame anſwered by Mr. Patrick O'Cavanah, *the Propoſer.*

In the given equation ($\dot{x}\dot{y} = x\ddot{y} - a\ddot{y} - \frac{x\dot{y}^2}{b} = 0$) let $-\frac{\dot{y}\ddot{x}}{\dot{x}}$* be wrote in the place of $\ddot{y}$ (ſo that $\dot{y}$ may become the quantity flowing uniformly) then will $\dot{x}\dot{y} + x \times \frac{\dot{y}\ddot{x}}{\dot{x}} + a \times \frac{\dot{y}\ddot{x}}{\dot{x}} - \frac{x\dot{y}^2}{b} = 0$, or $\dot{x}\dot{x} + x\ddot{x} + a\ddot{x} - \frac{x\dot{x}\dot{y}}{b} = 0$; whoſe fluent, it is evident, will be $x\dot{x} + a\dot{x} - \frac{x^2\dot{y}}{2b} =$ ſome conſtant quantity, let it be $c\dot{y}$, ſo ſhall $\dot{y} = 2b \times \frac{a\dot{x} + x\dot{x}}{2bc + xx}$; and conſequently $y = a\sqrt{\frac{2b}{c}} \times A + b \times L$; A being $=$ the arch whoſe radius is 1, and tangent $\frac{x}{\sqrt{2bc}}$; $L =$ the hyp. log. of $\frac{2bc + xx}{dd}$; and c and d any conſtant quantities.

Mr. *R. W.* (whom for particular reaſons, we ſhould be glad to oblige) will not, we hope, take it amiſs that his ſolution to this problem was not inſerted, as his reaſoning upon the correction of the fluent (tho' very ingenious) is, nevertheleſs, defective.

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* That the ſubſtitution of $-\frac{\dot{y}\ddot{x}}{\dot{x}}$ for $\ddot{y}$, in any fluxionary equation of the ſecond (or any higher) degree, cauſes no difference in the equation of the fluents, may be thus demonſtrated. Suppoſe the relation of the fluents to be expreſſed by the general equation $y = Ax^m + Bx^n + Cx^p$, &

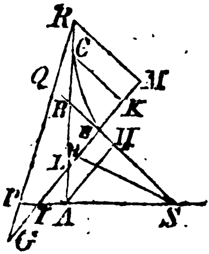
&c. then 'tis evident that $\frac{\dot{y}}{\dot{x}} = mAx^{m-1} + nBx^{n-1} + pCx^{p-1}$, &c. And, if $\dot{x}$ be made conſtant, we ſhall, by taking the fluxion a ſecond time, have $\frac{\ddot{y}}{\dot{x}} = m.\overline{m-1}.Ax^{m-2}\dot{x} + n.\overline{n-1}Bx^{n-2}\dot{x}$, &c. But if $\dot{y}$ be made conſtant, we ſhall then have $-\frac{\dot{y}\ddot{x}}{\dot{x}\dot{x}} = m.\overline{m-1}.Ax^{m-2}\dot{x} + n.\overline{n-1}.Bx^{n-2}\dot{x}$, &c. Therefore, ſeeing $\frac{\ddot{y}}{\dot{x}}$ and $-\frac{\dot{y}\ddot{x}}{\dot{x}^2}$ are both equal to one and the ſame quantity $(m.\overline{m-1}.Ax^{m-2}$, &c.) they muſt neceſſarily be equal to each other, and conſequently $\ddot{y} = -\frac{\dot{y}\ddot{x}}{\dot{x}}$.

In like manner it will appear that $\dddot{y} = \frac{3\dot{y}\ddot{x}^2}{\dot{x}\dot{x}} - \frac{\dot{y}\dddot{x}}{\dot{x}}$. For, by taking again, the fluxions of the preceding equations, $\frac{\ddot{y}}{\dot{x}\dot{x}} = m.\overline{m-1}.Ax^{m-2} + n.\overline{n-1}.Bx^{n-2}$, &c. and $-\frac{\dot{y}\ddot{x}}{\dot{x}^3} = m.\overline{m-1}.Ax^{m-2} + n.\overline{n-1}.Bx^{n-2}$, &c. (making $\dot{x}$ conſtant in the former and $\dot{y}$ in the latter) there ariſes $\frac{\dddot{y}}{\dot{x}^3}$ $(= m.\overline{m-1}.\overline{m-2}.Ax^{m-3}\dot{x}$, &c.) $= -\frac{\dot{y}\dddot{x}}{\dot{x}^3} + \frac{3\dot{y}\ddot{x}^2}{\dot{x}^4}$; and from thence $\dddot{y} = \frac{3\dot{y}\ddot{x}^2}{\dot{x}\dot{x}} - \frac{\dot{y}\dddot{x}}{\dot{x}}$.

M. H.

PRIZE

PRIZE QUESTION *anſwered by* Κυβερνήτης.



Let P, Q, and R repreſent the three ports, and A, B, and C the ſhips ſailing from thence, the former two in the right lines PS, QS, and the latter C in a curve line RCE, whoſe nature muſt be ſuch that a right line ABC paſſing through A and B, may always touch the curve in that very point where the ſhip C then is.——Let E be the place of the ſhip B when A arrives at S, the interſection of the courſes, and C is come up with B: Let alſo SI be to SE in the ratio of the given celerities, or as PS to QE: Draw $GIEM$, and make AH parallel thereto, and CK and RM parallel to QES.

It is evident, that SH and EB will be always equal to each other, being each to AS in the conſtant ratio of SE to SI, or of the celerity in BS to that in AS; whence BH and ES muſt likewiſe be equal: But, by ſimilar triangles, $AH : BH\,(ES) :: EL : BE$, and $EI : AH :: ES : SH\,(BE)$: From the compoſition of which propoſitions we have $BE^2 = \frac{ES^2}{EI} \times EL = En \times EL$ (Sn being drawn to make the $\angle ESn = EIS$) which is a known property of the parabola: For in the parabola (ſuppoſing p to denote the parameter of any diameter EM) it is well known that $CK^2 = p \times EK$, and that $EL = EK$, and conſequently $EB = \frac{1}{2}CK$; ſo that BE^2 $(= \frac{1}{4}CK^2) = \frac{1}{4}p \times EL$; whence, it not only appears that the curve is a parabola, but that En is $\frac{1}{4}$ of the parameter thereof, correſponding to the diameter EKM.

To determine now the length of the arch ECR, &c. we have, in the $\triangle ESI$, the ratio of SI to SE (as 5 to 3), and the contained $\angle S$. $(= 56° 15')$: Whence the $\angle SEI$ (or MKC) which the ordinate makes with the diameter EM, is found $= 86° 56\frac{1}{2}'$. Again, ſince $\angle S = \angle Q$, we have $PS = PQ = 12$, and QE $(= \frac{3}{5}PS) = 7{\cdot}2$; therefore RM $(= 2QE) = 14{\cdot}4$, and QR $\left(= GQ = QE \times \frac{\text{ſin. } E}{\text{ſin. } G}\right)$ $= 14{\cdot}086 =$ the required diſtance of the port R from the port Q. Again, (by trigonometry) $QS = 13{\cdot}3537$, SE $(= QS - QE) = 6{\cdot}1337$, and $En = 4{\cdot}42$; which laſt put $= \frac{1}{4}b$,

$=\frac{1}{2}b$, and let the coſine of the angle MKC ($86° 56\frac{1}{2}'$) $= m$: Then (ſuppoſing $EK = x$, and $CK = y$) we have (by the property of the parabola) $2bx = y^2$; whence $\dot{x} = \frac{y\dot{y}}{b}$, and conſequently $\sqrt{\dot{y}^2 + 2my\dot{x} + \dot{x}^2}$) the flux. of the arch EC $= \frac{\dot{y}}{b}\sqrt{b^2 + 2bmy + y^2}$; which, by putting $v = mb + y$ and $b = b\sqrt{1 - mm}$, will be transformed to $\frac{\dot{v}}{b}\sqrt{b^2 + v^2}$; whereof the (corrected) fluent will be $\frac{v}{2b}\sqrt{bb + vv} - \frac{1}{2}mb + \frac{b^2}{2b} \times$ hyp. log. $\frac{v + \sqrt{bb + vv}}{mb + b}$: From whence (by taking $y = RM = 14{\cdot}4$) the diſtance RE run by the ſhip C is found $= 19{\cdot}77$ miles.

Mr. L. *Charlton*, *Micromegas*, Mr. *E. Rollinſon*, (the propoſer), and Miſs *Frances Harris* (whoſe performances are an honour to her ſex) have given elegant ſolutions to this problum, by a fluxionary calculus founded on the properties of tangents; ſome of which ſolutions we ſhould have inſerted, could we poſſibly have found room for them.—— Mr. *Bamfield* truly determines the diſtance QR, and, by an ingenious approximation, makes the diſtance ſailed by the ſhip C to be 19·89 miles.

The Prize of 12 Diaries was won by Miſs Frances Harris.

The PARADOX *anſwered by Mr.* C. Wildbore.

A right line perpendicular to the plane of the two given lines, at the point of their concourſe, will be perpendicular to them both.

Eclipſes in 1756, calculated by Mr. Ra. Hulſe.

There will be only two eclipſes this year, both of the ſun, and both inviſible to the inhabitants of Great Britain.

1. March the 1ſt, at 2 in the morning, the ſun will be eclipſed in ♓ 11 degrees. This will be a great eclipſe, total and central, in the Indian ſea.

2. Auguſt the 25th, at 7 at night, the ſun will be totally eclipſed in ♍ 3 degrees; viſible at New Spain, Jamaica, Terra-Firma, and places adjacent.

New Queſtions.

I. QUESTION 406, *by* Ruſticus.

Two quondam friends at market meet,
(John ſheep had bought; Hodge pigs and geeſe;)
Who, whilſt a chearful pot they took,
A change propos'd, and bargain ſtruck:
For ev'ry ſheep, they did agree,
A pig and gooſe the price ſhould be.
Hodge had ten pigs leſs than he'd geeſe,
Which laſt were charg'd, ſix groats a-piece:
By computation ſhrewd 'twas found,
That pigs and geeſe were worth five pound;
And that they were in number more
Than all John's ſheep, by four times four.
——Now ſirs, from hence, pleaſe to declare
What number of each ſort there were:
Suppoſing that, for John's whole flock,
Hodge, in exchange, gave all his ſtock.

II. QUESTION 407, *by* Philo-Peſos.

In a given triangle, to inſcribe a rhombus, having one of its angular points coincident with a given point in the baſe.

III. QUESTION 408, *by Mr.* R. Young, *Writing-maſter, in* Cheſter.

The ſides of a quadrangular field are known to be 9, 10, 11, and 12 chains, reſpectively: And, if a diagonal be therein drawn, the part included by it and the two ſhorteſt ſides will be to the remaining part in the given proportion of 3 to 5: From hence the content of the field is required.

IV. QUESTION 409, *by Mr.* H. Watſon.

To determine a point in a given triangle (whoſe ſides are 16, 20, and 24 inches) from whence perpendiculars being let fall on all the ſides, the ſolid (or continual product) contained under them, ſhall be the greateſt poſſible.

V. QUESTION 410, *by Mr.* Walter Trott.

The diſtances of the three corners of a right-angled triangular field, from a watering-place within the field, are known to be 38, 50, and 62 perches, reſpectively: To find, from thence, the content of the field, it being the greateſt the data will admit of.

VI. QUESTION 411, *by Mr.* J. Aſh.

Being on a journey, in lat. 52° 30′ N. the third of April, 1755, in the afternoon, I obſerved the two interſections of the inner part of the interior rain-bow with the horizon, to form an angle, at the place where I ſtood, of 60°: From which the time of obſervation is required.

VII. QUESTION 412, *by Mr.* Chr. Maſon.

The human ſtage is three-ſcore years and ten;
So ſhort the ſpan! yet ſeldom reach'd by men.
As man's quietas ought to be his care,
'Tis now high time for Maſon to prepare.
The rolling years that o'er my head have flown,
If you would know, they're in the margin * ſhown.

* Viz. $v+x+y+z=1734$, $v^2+x^2+y^2+z^2=2850372$, $vxyx=3240960$, and $x=z$; v being the year of my birth, x the month, y the day, and z the hour *P.M.* all which are required.

VIII. QUEST. 413, *by Mr.* Pat. O'Cavanah, *of* Dublin.

(*Addreſſed to the ingenious Authors of the* 378*th and* 393*d Queſtions.*)

Ah! much, my friends, I mourn your lot,
Who have ſuch woeful help-mates got:
Both fluts and ſcolds! oh, dreadful curſe!
Bad daughters too! What can be worſe?
Yet hear me, and to eaſe your grief,
I'll teach you what will yield relief; *
But like rare noſtrums, little known,
In myſtic terms it muſt be ſhown—
For brother Philomaths alone.

This powerful ſpecific is denoted by a word conſiſting of five letters, and theſe have their places in the alphabet expreſſed by the values of u, w, x, y, and z, in the ſubjoined equation: † By means whereof the important myſtery may be diſcloſed, and that, without having the root of any equation to extract, higher than a quadratic.

† Viz. $w^2+z^2=89$; $wz+w+z=53$; $\frac{xx+yy}{x+y}=18\frac{1}{10}$; $\frac{x^3+y^3}{x^2+y^2}=18\frac{172}{181}$; $\frac{u^5+x^5}{ux\times\overline{u+x}^3}=2\frac{1}{20}$.

IX. QUESTION 414, *by Mr.* E. Rollinſon.

A beam *BC* (*ſee fig. to the ſolution*) is to be ſupported in a given poſition, by means of a prop *DE*, of a given length, inſiſting on the horizontal beam *AB*: 'Tis propoſed to determine the poſition of the prop, ſo that the beam *AB* whereon it ſtands may be the leaſt ſubject to break, or ſo that the force, whereby it actually tends to break, may be to the whole force it can ſuſtain in *E* at the leaſt ratio poſſible, the thickneſs of the beam being every-where the ſame.

X. QUESTION 415, *by Mr.* Hugh Brown.

An uſurer lends 2000l. at 5 per cent. per annum compound intereſt, which the borrower is to clear off by quarterly payments, viz. 1l. at the end of the firſt quarter, 2l. at the end of the ſecond, 3l. at the end of the third, and ſo on. Quere, At what time will the debt be the greateſt poſſible?

XI. QUESTION 416, *by Mr.* Tho. Moſs.

Suppoſe 12 half-pence to be thrown up, and thoſe that come up heads to be taken away, and the remaining ones to be thrown up again, and ſo on, in the ſame manner, till all the half-pence have been thrown up heads: 'Tis propoſed to find in what number of throws, according to an equality of chances, this may be effected.

XII. QUESTION 417, *by Mr.* W. Bevil.

Suppoſe that a chain is to be ſuſpended, at its extremes, by two tacks, in the ſame horizontal line, at the diſtance of 10 feet: To find the length of the chain, ſuch that the ſtreſs or force thereof, upon the tacks, may be a minimum.

XIII. QUESTION 418, *by Mr.* Tho. Peart.

Suppoſe a mountain, formed by the rotation of the catenaria, whoſe ſuperficial content is equal to the ſquare of its baſe diameter; and ſuppoſe two equal pendulums, one at the foot, and the other at the vertex of the mountain, to be put in motion the ſame inſtant; and that at the end of 24 hours, meaſured by the former, a cannon ball is there diſcharged in ſuch a direction, as to fall the fartheſt poſſible upon the mountain, and arrives at the vertex at the ſame inſtant the pendulum there has meaſured 24 hours: It is required to find the mountain's height, and the velocity with which the ball is diſcharged.

XIV. QUEST. 419, *by Mr.* R. Weſton, Diſcip. Landenii.

From the equation $a^3\ddot{y} = a^2\dot{y}^2 - y^2\dot{x}^2$ it is propoſed to find x in terms of y, without firſt finding y in terms of x, and then reverting the ſeries.

XV. QUESTION 420, *by Mr.* Pat. O'Cavanan, *of* Dublin.

To find the ſum of the infinite ſeries $1 - \frac{z^3}{2.3} + \frac{z^6}{2.3.4.5.6} - \frac{z^9}{2.3.4.5.6.7.8.9}$, &c. by means of circular arcs and logarithms.

PRIZE QUESTION, *by Mr.* H. Watſon.

A perſon being to paſs from one place A to another B, at the diſtance of three miles, between which places there lies a moraſs, is deſirous to know the path he muſt deſcribe, and alſo how far he muſt travel, to perform his journey in the ſhorteſt time poſſible; the way, by reaſon of the moraſs, being rendered ſo bad, that he can only move with a celerity every-where proportional to his diſtance from the center thereof, lying one mile diſtant from A, and two from B.

1757.

Queſtions anſwered.

I. QUESTION 406 *anſwered by Maſter* Jonath. Kimbell, *Of* Leiceſter.

John chang'd ten ſheep, at ten * a-piece, * *Shillings*,
For eight ſmall pigs and eighteen geeſe.

The ſame anſwered by Mr. T. Barker, *of* Weſthall, *in* Suſſex.

Let x denote the number of pigs, and y the price of a pig; then will $x + 10$ be the number of geeſe, and $2x - 6$ the number of ſheep; and therefore, by the queſtion, $xy + \overline{x+10} \times 2 = 100$, and $\overline{2x-6} \times \overline{y+2} = 100$. From the firſt equation $y = \frac{80}{x} - 2$; and from the ſecond $y =$

$\frac{100}{2x-6}-2$; ∵ $80\times\overline{2x-6}=100x$: whence $x=8$, and $y=8$. Therefore there were 8 pigs, 10 ſheep, and 18 geeſe.

The ſame anſwered by Mr. Tho. Wilkin.

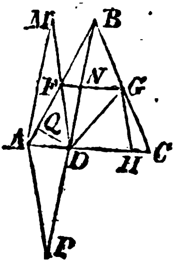
Let $x=$ number of pigs; then $x+10=$ number of geeſe, and $2x-6=$ number of ſheep; whence $2x+20=$ price of all the geeſe; $100-2x-20$ $(=80-2x)=$ price of all the pigs; and $\frac{80-2x}{x}$ $(\frac{80}{x}-2)$ the price of one pig; and conſequently $\frac{80}{x}$ (= the price of one ſheep) which multiplied by the number of ſheep gives $\frac{80}{x}\times\overline{2x-6}=100$: Hence $160x-480=100x$, and $x=8$. Therefore there were 8 pigs at 8 ſhillings each, 18 geeſe at 2 ſhillings each, and 10 ſheep at 10 ſhillings each.

In the ſame manner it was alſo anſwered by Mr. *G. Armſtrong*, Mr. *W. Baxter*, Mr. *J. Bereſford*, Mr. *Turner Boſton*, Mr. *G. Crabbe*, *Darcy Donought*, Mr. *Joſ. Farrington*, Mr. *T. Fletcher*, Mr. *Ja. Giles*, Mr. *E. Griffiths*, Mr. *J. Hampſon*, Mr. *W. Harvey*, Mr. *G. Hicks*, Mr. *J. Hudſon*, Mr. *T. Hudſon*, *Juvenis*, Mr. *W. Lee*, Mr. *W. Litſon*, Mr. *Joſ. Lord*, Mr. *B. Lydal*, Mr. *R. Marſh*, Mr. *R. Morris*, Mr. *W. Patrick*, Mr. *W. Rippon*, Mr. *Alex. Rowe*, Mr. *T. Sandling*, Mr. *W. Stoker*, Miſs *S. T.* Mr. *W. Terrell*, Mr. *R. Terry*, Mr. *W. Thompſon*, Mr. *J. Vicary*, Mr. *R. Walton*, Mr. *J. Woolcott*, and ſeveral others.

II. QUESTION 407 *anſwered by Mr.* Abr. Botham.

CONSTRUCTION. From the vertex *B*, of the triangle, through the given point *D*, draw *BDP*; to which, from *A*, apply *AP* equal to the baſe *AC*; draw *DF*, *FG*, *GH* parallel to *AP*, *AC*, and *FD*, reſpectively; and the thing is done.

For, by ſim. Δs, *AP* : *FD* (:: *BA* : *BF*) :: *AC* : *FG*; but *AP* = *AC* (by conſtruction); whence *FD* = *FG*.



The ſame anſwered by Mr. T. Peart.

Draw *BD* from the vertex to the given point *D*; and make *AM* parallel thereto, and equal to the baſe AC of the triangle; draw MD cutting AB in F; then draw FG and GH parallel to AC and DF, for the other ſides of the rhombus.

DEMONSTRATION. Let BD cut FG in N. The triangles ADM and DFN (becauſe of the parallel lines) will be ſimilar; whence AM $(AC) : DF :: AD : FN :: AC : FG$; and conſequently $DF = FG$.

METHOD OF CALCULATION. From AB, AD, and the $\angle DAB$, the $\angle BDC$ $(= MAD)$ will be known; from which and the given ſides AD and AM, the $\angle ADF$ will alſo be known, and conſequently the ſide DF.

An algebraic Solution to the ſame by Mr. W. Smith, *of* Irthlingborough.

Put $AB = b$, $AC = c$, DQ (perp. to AB) $= d$, $BQ = e$, and DF $(= FG) = x$: Then $c : b :: x$ $(FG) : BF = \frac{bx}{c}$; whence $FQ = e - \frac{bx}{c}$; and conſequently $\overline{e - \frac{bx}{c}}|^2 + d^2 = x^2$.

Which ſolved gives $x = \frac{bce}{cc-bb} + \sqrt{c^2 \times \frac{dd+ee}{cc-bb} + \overline{\frac{bce}{cc-bb}}|^2}$.

According to the former of theſe methods the problem was conſtructed by Mr. *O'Cavanah*, *Philo-Peſos*, Mr. *E. Rollinſon*, Mr. *Walter Trott*, and Mr. *H. Watſon*.——Algebraic ſolutions to the ſame have alſo been received from Mr. *S. Bamfield*, Mr. *W. Baxter*, *Birchoverenſis*, Mr. *L. Charlton*, Mr. *J. Vicary*, and many others.

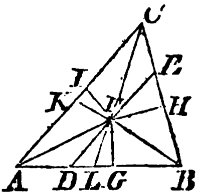
III. QUESTION 408 *anſwered by Mr.* Wm. Kingſton, *of Bath.*

Let $DFGH$ be the quadrangular field (ſee laſt figure); then, ſince by a well-known theorem, the area of the triangle DFG is $\frac{1}{4}\sqrt{\overline{DF+FG}|^2 - DG^2 \times DG^2 - \overline{DF-FG}|^2}$, if DG be put $= x$, we ſhall have (becauſe FG is given $= 9$, and $DF = 10$) $DFG = \frac{1}{4}\sqrt{361 - x^2 \times x^2 - 1}$. And in the very ſame manner, the area of DHG will be $= \frac{1}{4}\sqrt{529 - x^2 \times x^2 - 1}$: which being to that of DFG as 5 to 3 (by the queſt.) it is evident that $25 : 9 :: 529 - x^2 : 361 - x^2$; whence $xx = 266{\cdot}5$; and the area $DFG = \frac{1}{4}\sqrt{94{\cdot}5 \times 265{\cdot}5} = 39{\cdot}5994$: and conſequently, $\frac{8}{3} \times 39{\cdot}5994 = 105{\cdot}5984$ (= 10ac. 2 r. 9½p.) the content of the field.

In the very ſame manner it is anſwered by Mr. *Ja. Bereſford*, Mr. *J. Boſton*, Mr. *Abr. Botham*, and Mr. *W. Trott*.—It was alſo truly and conciſely anſwered by Mr. *W. Allen*, Mr. *S. Bamfield*, Mr. *T. Barker*, Mr. *W. Baxter*, Mr. *Joſ. Farrington*, Mr. *J. Fletcher*, Mr. *R. Flitcon*, Mr. *Ja. Giles*, Mr. *J. Hampſon*, Mr. *G. Hicks*, Mr. *J. Hudſon*, Mr. *W. Smith*, Mr. *W. Stoker*, Mr. *W. Terril*, Mr. *R. Terry*, Mr. *W. Thompſon*, Mr. *J. Vicary*, Mr. *T. Wilkin*, Mr. *J. Woolcott*, and ſeveral others.

IV. QUESTION 409 *anſwered by* Birchoverenſis.

When $FG \times FH \times FI$ is a maximum, the product thereof by the conſtant quantity $\frac{1}{2}AB \times \frac{1}{2}BC \times \frac{1}{2}AC$, will alſo be a maximum; that is, the product of the three parts ABF, BCF, and ACF of the given triangle, will be a maximum, and conſequently thoſe parts equal among themſelves; ſince it is evident (from Euc. 5. 2.) that the continual product of the parts of any given quantity (whatever their number is) will be the greateſt when the parts are all equal: Therefore, ABF being $= \frac{1}{3}ABC$, it is evident that FG will be $= \frac{1}{3}$ of the perpendicular falling from C upon AB, &c. and conſequently, that F will be the center of gravity of the given triangle ABC. Hence the three required perpendiculars are found to be 5·2915, 4·4095, and 6·6143, and their continual product = 154·33. *W. W. R.*



The ſame anſwered by Mr. Walter Trott.

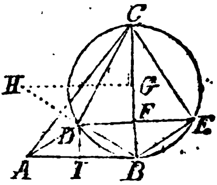
From any point F, in $DE \parallel$ to one ſide AC of the triangle, conceive $\perp$s FG, FH, and FI to be let fall; then FG being in a conſtant ratio to DF, and FH to FE, it is evident, that, when $DF \times FE$ is a maximum, $FG \times FH$ or $FG \times FH \times FI$ (becauſe FI is ſuppoſed to continue the ſame) will likewiſe be a maximum; which therefore is known to be when $FD = FE$, or when $AK = CK$, ſupoſing the right line BFK drawn meeting AC in K. Hence it appears that (let the diſtance between DE and AC be what it will) the quantity $FG \times FH \times FI$ cannot be a maximum, unleſs $AK = CK$: Neither can it be a maximum (by the very ſame argument) unleſs AL (ſuppoſing CFL drawn) is $= BL$; therefore it muſt be ſo when $AK = CK$, and $AL = BL$: Whence the conſtruction is manifeſt; the point F required being the center of gravity of the triangle, and the three required perpendiculars equal to $\frac{5}{3}\sqrt{7}$, $2\sqrt{7}$, and $\frac{5}{2}\sqrt{7}$, or 4·4096, 5·2915, and 6·6144, and the content of the ſolid contained under them $= \frac{175}{3}\sqrt{7} = 154{\cdot}33$ cubic inches.

A Fluxionary Solution to the ſame, by Mr. W. Allen.

Put $AB = a$, $BC = b$, $AC = c$, $FG = x$, $FH = y$, $FI = z$, and the area ABC (= 158·745) $= d$; then $ax + by + cz = 2d$; and xyz a max. which laſt, when z is exterminated by means of the former, will give $2dxy - ax^2y - bxy^2$, a max. From which, ſuppoſing y invariable, we have $2dy\dot{x} - 2ayx\dot{x} - by^2\dot{x} = 0$; whence $y = \frac{2d - 2ax}{b}$. Again, by making x invariable, we get (after reduction) $y = \frac{2d - ax}{2b}$. Therefore $\frac{2d - ax}{b} = \frac{2d - ax}{2b}$; whence $x = \frac{2d}{3a} = 4{\cdot}40958$; alſo $y = \frac{2d}{3b} = 6{\cdot}61437$; $z = \frac{2d}{3c} = 5{\cdot}2915$; and $xyz = 154{\cdot}33$.

According to the firſt of the three foregoing methods the problem is reſolved by Mr. *E. Rollinſon*, and Mr. *H. Watſon*; and according to the laſt of them, by Mr. *W. Bevil*, Mr. *R. Flitcon*, Mr. *T. Peart*, *Penovius*, Mr. *W. Smith*, Mr. *W. Spicer*, and ſome others.

V. QUESTION 410 *anſwered by Mr.* Pat. O'Cavanah.



Let ABC repreſent the field, and D the watering-place. Let DE be = and ∥ to AB, interſecting CB in F, and let BE and CE be drawn: Then will the trapezium $BDCE$ ($= DE \times \frac{1}{2} BC$) and the $\triangle ABC$ ($= AB \times \frac{1}{2} BC$) be mutually equal to each other. Moreover, ſince $CE^2 - CD^2$ ($= EF^2 - DF^2$) $= BE^2 - BD^2$, it is evident that $CE^2 = BE^2$ (AD^2) $- BD^2 + CD^2$; from which CE is given (= 70). Therefore, all the four ſides of the trapezium $DCEB$ being given, the area will be a maximum when the figure is inſcribed in a circle. But the rectangle of the two diagonals of any trapezium inſcribed in a circle is equal to the ſum of the rectangles of the oppoſite ſides: Therefore $DE \times BC = DC \times BE + DB \times CE$; the half of which is (= 2880 perches, or 18 acres) the true area ſought. The geometrical conſtruction from hence is very eaſy; for (by ſim. △'s) *DF* is to *BF* in the given ratio of *CD* to *BE* (or *AD*). Therefore, having firſt drawn two ⊥ lines at pleaſure, in the one of them take $BG = AD$, and ∥ to the other draw $GH = CD$; in *BH* (when drawn) take *BD* of the given length; and from *D*, to *BA* and *BC*, apply *DA* and *DC* alſo of the given lengths; from whence the points *A* and *C*, and conſequently the △ *ABC* itſelf, will be determined.

A Fluxionary Solution to the ſame, by Mr. J. Honey.

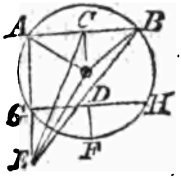
Let $AD = 62 = a$, $BD = 38 = b$, $CD = 50 = c$, and $AB = x$; then $x : a + b :: a - b : \frac{aa - bb}{x} =$ diff. ſegments of the baſe; whence, putting $aa - bb = n$, we get $\frac{xx - n}{2x} = IB = DF$; and (p. 47. 1 Euclid) $BF = \frac{1}{2x}\sqrt{4b^2x^2 - x^4 + 2nx^2 - n^2}$; alſo $CF = \frac{1}{2x}\sqrt{4c^2x^2 - x^4 + 2nx^2 - n^2}$; and conſequently $BC = \frac{1}{2x}\sqrt{rx^2 - x^4 - n^2} + \frac{1}{2x}\sqrt{mx^2 - x^4 - n^2}$, by putting $r = 4bb + 2n$, and $m = 4cc + 2n$. Hence $\sqrt{rx^2 - x^4 - n^2} + \sqrt{mx^2 - x^4 - n^2} = 4$ times the area *ABC*; whoſe fluxion being taken and made $= 0$, we thence get $x^4 - \frac{mr + 4nn}{m + r} \times x^2 = -n^2$; ſolved, $x = 78{\cdot}3435$ perches: Whence $AC = 73{\cdot}5223$, and the area $ABC = 18$ acres.

By this laſt method, and the ſame ſubſtitution, the problem is alſo anſwered by Mr. *W. Allen*, Mr. *Abr. Botham*, Mr. *L. Charlton*, *Penovius*, and Mr. *W. Spicer*; whoſe ſolutions are equally neat with that above.——Mr. *Peart*, (who gives the ſolution without fluxions) ſays the angles *BAD* and *BCD* muſt be equal to each other; the truth of which is evident from the foregoing conſtruction: For *BED*, which is $= BAD$, muſt neceſſarily be $= BCD$, ſtanding on the ſame arch *BD*.

VII.

VI. QUESTION 411 *anſwered by Mr.* L. Charlton.

Let O be the center of the Iris, E the eye of the ſpectator, and $EACB$ the plane of the horizon: Then, if AE be taken as the radius, it is evident (from the writers on optics) that OA will be the ſine of an $\angle(AEO)$ of $40^\circ\ 17' = \cdot 6465$; from whence and AC, which is given $=$ ſin. $30^\circ = \cdot 5$, OC is found $= \cdot 4099$. Then in the $\triangle\ EOC$ (right-angled at O) it will be, as EO ($= \cdot 7628 =$ coſ. $40^\circ\ 17'$) is to OC, ſo is rad. (1) to the tang. of $OEC = 28^\circ\ 15' =$ the depreſſion of the center of the bow, or the ſun's altitude: From which the time is found 3 h. 15 m. afternoon.



The ſame anſwered by Mr. Samuel Bamfield.

The arch (AO or BO) of a great circle of the ſphere, drawn from the interior ſide of the bow ABF to the center thereof, is (according to the writers on optics) $= 40^\circ\ 17''$ Therefore in the iſoceles ſpherical $\triangle\ AOB$ (where AB repreſents the horizon) we have given $AO = OB = 40^\circ\ 17'$, and $AB = 60^\circ$; whence the perpendicular OC ($=$ the ſun's altitude) is found $= 28^\circ\ 15'$; and from thence the time of obſervation 3 h. 15 m. afternoon.

VII. QUESTION 412 *anſwered.*

Making $a = 1734$, $b = 2850372$, and $c = 3240960$, we have $v + y = a - 2x$, $vv + yy = b - 2xx$, and $vy = \frac{c}{xx}$. From the ſquare of the firſt of which equations ſubtract the double of the laſt, ſo ſhall $v^2 + y^2 = a^2 - 4ax + 4x^2 - \frac{2c}{xx} = b - 2x^2$; and conſequently $x^4 - \frac{2ax^3}{3} + \frac{aa - b}{6} \times x^2 = \frac{c}{3}$; that is, $x^4 - 1156x^3 + 26064x^2 = 1080320$: Whence $x = 8$, $v = 1688$, $y = 30$, and the time of the propoſer's birth Oct. 30, 1688, 8 hours P.M.

Thus the problem was anſwered by Mr. *W. Allen*, Mr. *G. Armſtrong*, Mr. *S. Bamfield*, Mr. *T. Barker*, Mr. *W. Baxter*, Mr. *Tho. Baxtonden*, Mr. *J. Bereſford*, *Birchoverenſis*, Mr. *Abr. Botham*, Mr. *J. Boſton*, Mr. *L. Charlton*, Mr. *G. Crabbe*, Mr. *R. Flitcon*, Mr. *T. Fletcher*, Mr. *J. Goodhend*, Mr. *E. Griffiths*, Mr. *W. Harriſon*, Mr. *G. Hicks*, Mr. *J. Honey*, Mr. *J. Hudſon*, *Juvinis*, Mr. *W. Kingſton*, Mr. *B. Lydal*, Mr. *T. Peart*, *Penovius*, Mr. *Alex. Rowe*, Mr. *W. Smith*, Mr. *W. Spicer*, Mr. *W. Stoker*, Mr. *W. Terrill*, Mr. *R. Terry*, Mr. *W. Thompſon*, Mr. *W. Trott*, Mr. *M. Ward*, Mr. *T. Wilkin*, and Mr. *J. Woolcott*; to none of whom, in particular, we have a right to aſcribe the ſolution here put down.

VIII. QUESTION 413 *anſwered.*

Meſſ. *Barker*, *Bevil*, *Birchoverenſis*, *Botham*, *Juvenis*, *Honey*, *Peart*, *Penovius*, *Smith*, and *Trott*, ſolve this problem by ſubſtituting for the half-ſum, and the half-difference of the quantities ſought in the ſeveral given equations: Thus, let $w = s + d$ and $z = s - d$; then the two firſt equations ($w^2 + z^2 = 89$ and $wz + w + z = 53$) will become $2ss + 2dd = 89$, and $ss - dd + 2s = 53$; whence $ss + s = 48\frac{3}{4}$, and $s = 6\frac{1}{2}$: Therefore $d\ (= \sqrt{44\frac{1}{2} - ss}) = 1\frac{1}{2}$, $w = 8$, and $z = 5$.

Again, by making $x = s + d$, and $y = s - d$, the 3d and 4th equations ($xx + yy = a \times \overline{x + y}$, and $x^3 + y^3 = b \times \overline{xx + yy}$) will be $2ss + 2dd = 2as$, and $2s^3 + 6sdd = b \times \overline{2ss + 2dd} = b \times 2as$ (a being $= 18\frac{1}{10}$, and $b = 18\frac{172}{181}$): Whence $dd = as - ss = \frac{1}{3}ab - \frac{1}{3}ss$, and $ss - \frac{3}{2}as = -\frac{1}{2}ab$. From which $s = \frac{3}{4}a \pm \frac{1}{4}\sqrt{9a^2 - 8ab} = 10$, and $d\ (= \pm\sqrt{as - ss}) = \pm 9$; therefore $x = 10 \pm 9$, and $y = 10 \mp 9$; that is, the greateſt number will be 19, and the leſſer 1; but which of theſe x muſt be, depends on the other given equation $\frac{x^5 + x^5}{ux \times \overline{u + x}^3} = 2\frac{1}{20}$; which equation, by ſubſtituting in like manner ($u = s + d$, and $x = s - d$) becomes $= \frac{s^4 + 10ssdd + 5d^4}{\overline{ss - dd} \times 4ss} = 2\frac{1}{20}$; whence $d^4 + 3{\cdot}64ssdd = 1{\cdot}44s^4$. Now, by compleating the ſquare, and taking the root, $dd = 0{\cdot}36ss$, and $d = \frac{6s}{10}$: Therefore $x\ (= s - d) = s - \frac{6s}{10}$; whence $s = \frac{10x}{4}$; alſo $d\ (\frac{6s}{10}) = \frac{6x}{4}$; and conſequently $u\ (s + d) = \frac{16x}{4} = 4x = 4$ or 78. But, if u be taken as the leſſer of the two numbers, it is evident that, then, $x = 4u$, or $u = \frac{x}{4}$; that is, $u = \frac{1}{4}$, or $\frac{19}{4}$; but of theſe four different values of u, the firſt only can fulfil the conditions of the problem: So that the five numbers required will be 4, 5, 1, 19, and 8; and the letters correſponding D, E, A, T, H.

Another Anſwer to the ſame, by Mr. H. Watſon.

1. By adding the double of the ſecond equation to the firſt, we have $\overline{w+z}|^2 + 2 \times \overline{w+z} = 195$; whence $w+z = \sqrt{196} - 1 = 13$: From which, and $w^2 + z^2 = 89$, the greater number is found $= 8$, and the leſſer $= 5$.

2. By multiplying together the third and fourth equations, we have $\frac{x^3+y^3}{x+y} = ab$; that is, $xx - xy + yy = ab$ (a being $= 18\frac{1}{10}$, and $b = 18\frac{172}{181}$): The double of which, taken from the triple of the third equation, $xx + yy = a \times \overline{x+y}$, gives $xx + 2xy + yy = 3a \times \overline{x+y} - 2ab$, or $\overline{x+y}|^2 - 3a \times \overline{x+y} = -2ab$; whence $x+y = \frac{3a}{2} \pm \frac{1}{2}\sqrt{9aa - 8ab} = 20$: From which, and $xx + yy = 18\frac{1}{10} \times 20$ $(= 362)$, the greater quantity is found $= 19$, and the leſſer $= 1$.

3. It is plain that $\overline{u+x} \times \overline{u^4 - u^3x + uuxx - ux^3 + x^4} = u^5 + x$; whence $\frac{u^5+x^5}{ux \times \overline{u+x}|^3} = \frac{u^4 - u^3x + uuxx - ux^3 + x^4}{ux \times \overline{n+x}|^2} = 2\frac{1}{10}$; and conſequently $u^4 - u^3x + uuxx - ux^3 + x^4 = 2{\cdot}05\,ux \times \overline{uu + 2ux + xx}$; that is, $\overline{uu+xx}|^2 - ux \times \overline{uu+xx} - uuxx = 2{\cdot}05\,ux \times \overline{uu+xx} + 4{\cdot}1\,uuxx$, or $\overline{uu+xx}|^2 - 3{\cdot}05\,ux \times \overline{uu+xx} = 5{\cdot}1\,uuxx$; whence, by compleating the ſquare, and extracting the root, $uu + xx - 1{\cdot}525\,ux = 2{\cdot}725\,ux$; and therefore $uu - 4{\cdot}25\,xu = -xx$; and, by compleating the ſquare again, $u = 2{\cdot}125\,u \pm 1{\cdot}875\,x = 4x$ or $\frac{x}{4}$; that is, $u = 4$, 76, $\frac{1}{4}$, or $\frac{19}{4}$; but the firſt of theſe values muſt be the required one; and the letters, anſwering the conditions of the problem, D, E, A, T, H.

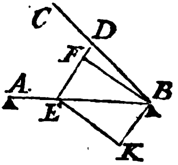
Oh! cruel caſe! how fix'd the grief!
When DEATH alone can yield relief.

Mr. *W. Allen* and Mr. *Lionel Charlton*, by ſubſtituting for the ſum and product in each caſe, bring out the ſame concluſion from the reſolution of quadratics only.——Many contributors have anſwered all the equations, except the laſt, by means of quadratics.

X. QUES-

IX. QUEST. 414 *anſwered by the Propoſer, Mr.* Rollinſon.

Since the ſtreſs or preſſure upon the prop *ED* is (by mechanics) as $\frac{1}{BF}$ (ſuppoſing *BF* perpendicular to *ED*), the force in the perpendicular direction, whereby the beam tends to break at *E*, will be as $\frac{1}{BE}$ (it being to the abſolute force, in the direction *DE*, as *BF* to *BE*). But the ſtrength of (or the whole force neceſſary to break) the beam at *E*, is known be as $\frac{1}{AE \times BE}$; which is to $\frac{1}{BE}$ the force above-mentioned in the proportion of $\frac{1}{AE \times BE}$ to $\frac{AE}{AE \times BE}$, or of 1 to *AE*; whence (by the queſtion) *AE* is to be a minimum, and conſequently *BE* a maximum: But *BE* is to the ſine of the $\angle D$ in the given ratio of *ED* to the ſine of *B*; whence it is evident that *BE* will be a maximum when the ſine of *D* is ſo, or when *D* itſelf is a right angle.——Therefore having made *BK* $\perp$ *BC* and = the given length of *DE*, draw *KE* ‖ *BC*, meeting *AB* in E, ſo ſhall *E* be the place where the end of the prop muſt ſtand.



Much after the ſame manner the problem was ſolved by Meſſ. *Botham*, *Charlton*, *Holliday*, *Trott*, and *Watſon*; all of whom determined the angle *BDE* to be a right one.

X. QUESTION 415 *anſwered by Mr.* W. Bevil.

Let $S = 2000 =$ the ſum propoſed, $r\ (1{\cdot}05^{\frac{1}{4}}) =$ the amount of 1l. in one quarter, and $n =$ the number of quarters required; then the amount of the ſum *S* will be Sr^n, and the amount of all the quarterly payments (excluſive of that due at the end of *n* quarters) $= r^{n-1} + 2r^{n-2} + 3r^{n-3} + 4r^{n-4}$, &c. continued to $n - 1$ terms; the ſum of all which will be found $= \frac{r}{\overline{r-1}|^{2}} \times \overline{r^n - rn + n - 1}$ *; and conſequently the money then owing $= Sr^n - \frac{r}{\overline{r-1}|^{2}} \times \overline{r^n - rn + n - 1}$; whereof the intereſt for one quarter is $\overline{r-1}$

$\overline{r-1}\times Sr^n - \frac{r}{\overline{r-1}|^2}\times\overline{r^n - rn + n - 1}$; and this, by the nature of the queſtion, muſt be equal to rn (the ſum r paid at the end of n quarters, and its intereſt for the ſame quarter): Hence we have $\overline{r-1}|^2 \times Sr^{n} - 1 - r^n + 1 = 0$, and $n = \frac{\log. a}{\log. r} = 28{\cdot}99$ (a being put equal to $\frac{r}{r - S.\overline{r-1}|^2} = 1{\cdot}42361$): Therefore at the end of 29 quarters the debt will be the greateſt poſſible (the moment before the 29th quarter is paid).

* *See* Turner's *Mathematical Exerciſes, No.* 2.

The ſame anſwered by Mr. Hugh Brown.

Put $a = 2000$, $r = 1{\cdot}05^{\frac{1}{4}} = 1{\cdot}0122722$, &c. and z the number of quarters at the end of which the debt will be the greateſt; then (by the queſtion, and known principles) $ar^z - r^{z-1} - 2r^{z-2} - 3r^{z-3} \ldots\ldots\ldots - \overline{z-1}\times r = ar^z + \frac{rz}{r-1} - \frac{r^{z+1}-r}{\overline{r-1}|^2}$ a max. Therefore $a\dot{z}r^z n + \frac{r\dot{z}}{r-1} - \dot{z}r^z n \times \frac{r}{\overline{r-1}|^2} = 0$; n being $=$ (${\cdot}0121975$) the hyperbolic log. of r: Whence $rz = \frac{r}{n}\times\frac{r-1}{r - a\times\overline{r-1}|^2} = 1{\cdot}43233$, &c. and $z = \frac{\log. 1.43233, \&c.}{\frac{1}{4}\log. 1{\cdot}05} = \frac{{\cdot}156042}{{\cdot}005297} = 29{\cdot}45$, &c. Hence, as the anſwer by the nature of the queſtion is reſtrained to a whole number, it is manifeſt, that at the end of 29 quarters (the moment before the payment then due is made) the debt will be the greateſt poſſible.

Meſſ. *Bamfield*, *Birchoverenſis*, *Botham*, *Rollinſon*, *Smith*, *Trott*, *Watſon*, and ſome others, alſo anſwered this queſtion, in a conciſe and elegant manner.

XI. QUESTION 416 *anſwered by Mr.* E. Rollinſon.

The probability of miſſing a head x times together with a ſingle halfpenny being $\frac{1}{2}^x$, the probability of throwing any halfpenny a head, in x trials, will therefore be expreſſed by $1 - \frac{1}{2}^x$; and conſequently that of throwing all 12 aſſigned heads, in x trials, $= \overline{1 - \frac{1}{2}^x}|^{12}$ (for the throwing all the 12 heads may be conſidered as 12 independent events;

ſince the happening of ſome of theſe, ſooner or later, no ways influences the happening of the others). Hence, by the conditions of the problem, we have $\overline{1-\tfrac{1}{2}^{x}}\big|^{12}=\tfrac{1}{2}$; and conſequently $x=\dfrac{\log.\ 1-\tfrac{1}{2}^{\frac{1}{12}}}{\log.\ \tfrac{1}{2}}=4{\cdot}155$, the number of throws required; which, not being an integer, ſhews there can be no exact equality of chance in the caſe propoſed; 4 being a ſmall matter too little, and 5 conſiderably too great.

COROLLARY. It appears from hence that the exact odds of bringing up all the heads of any number (n) of halfpence, in x throws, will be as $\overline{1-\tfrac{1}{2}^{x}}\big|^{n}$ to $1-\overline{1-\tfrac{1}{2}^{x}}\big|^{n}$, univerſally.

In this manner it was alſo anſwered by Mr. *O'Cavanah*, and Mr. *Moſs* the propoſer.

XII. QUESTION 417 *anſwered by Mr.* Lionel Charlton.

If half the length of the chain or curve (GF or HF) be denoted by z [*ſee laſt fig. but one*], then will the abſciſſa $DF=\sqrt{aa+zz}-a$, and the given ſemi-ordinate GD $(=c)=a\times$ hyp. log. $\dfrac{z+\sqrt{aa+zz}}{a}$ (ſee Landen's Mathematical Lucubrations, p. 34). Moreover, by mechanics, the ſtreſs or force upon both the pins, at G and H, will be to the weight of the chain, as the radius is to the ſine of the $\angle G$ or H, that is, as $\dot{z}$ to $\dot{x}$ $\left(\dfrac{z\dot{z}}{\sqrt{aa+zz}}\right)$ or as $2\sqrt{aa+zz}$ to $2z$: Therefore the weight of the chain being as the length $2z$, the ſtreſs upon the pins will conſequently be as $2\sqrt{aa+zz}$; which quantity is to be a minimum, by the queſtion: From whence, and the above equation, $c=a\times$ hyp. log. $\dfrac{z+\sqrt{aa+zz}}{a}$, both the values of a and z will be found for the hyp. log. $\dfrac{z+\sqrt{aa+zz}}{a}$ being $=\dfrac{c}{a}$, ſuppoſing N to denote the number whoſe hyp. log. $=1$, it is evident that $N^{\frac{c}{a}}=\dfrac{z+\sqrt{aa+zz}}{a}$; whence, by reduction, z is found $=\tfrac{1}{2}aN^{\frac{c}{a}}-\tfrac{1}{2}aN^{-\frac{c}{a}}$, and from thence $2\sqrt{aa+zz}$

$2\sqrt{aa+zz}=aN^{\frac{c}{a}}+aN^{-\frac{c}{a}}$: This, in fluxions, &c. gives $\dot{a}N^{\frac{c}{a}}-\frac{c\dot{a}}{a}N^{\frac{c}{a}}+\dot{a}N^{-\frac{c}{a}}+\frac{c\dot{a}}{a}N^{-\frac{c}{a}}=0$; whence $N^{\frac{2c}{a}}=\frac{c+a}{c-a}$, or $N^{2v}=\frac{v+1}{v-1}$ (by putting $a=\frac{c}{v}$); which (in logarithms) becomes $2v=$ hyp. log. $\frac{v+1}{v-1}$; whence $v=1{\cdot}1996$; and from thence $a=c\times{\cdot}8336=4{\cdot}168$, and $2z$ (GFH) $=2c\times1{\cdot}2578=12{\cdot}578$.

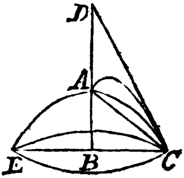
The ſame anſwered by Penovius.

Let (GF) half the length of the chain $=z$, GD (HD) $=b$, and $DF=x$; then will $b=a\times$ hyp. log. $\frac{z+\sqrt{aa+zz}}{a}$, by the property of the curve. Alſo as $\dot{x}$ $\left(\frac{z\dot{z}}{\sqrt{aa+zz}}\right)$: $\dot{z}$ :: z (half the chain) : $\sqrt{aa+zz}=$ the ſtreſs on each pin, which put $=n$; then, by exterminating z, we have $\frac{b}{a}=$ hyp. log. $\frac{n+\sqrt{nn-aa}}{a}=$ hyp. log. $\overline{n+\sqrt{nn-aa}}-$ hyp. log. a; whereof the fluxion, when n is a minimum, will be $-\frac{b\dot{a}}{aa}=\frac{-a\dot{a}}{\sqrt{nn-aa}\times\overline{n+\sqrt{nn-aa}}}-\frac{\dot{a}}{a}$; whence by reduction, $n=\frac{ba}{\sqrt{bb-aa}}$: Therefore $\sqrt{nn-aa}=\frac{aa}{\sqrt{bb-aa}}$, and $n+\sqrt{nn-aa}=\frac{ba+aa}{\sqrt{bb-aa}}=a\sqrt{\frac{b+a}{b-a}}$; which value ſubſtituted above gives $\frac{b}{a}=$ hyp. log. $\sqrt{\frac{b+a}{b-a}}$, or $a\times\overline{\text{hyp. log. } b+a-\text{hyp. log. } b-a}=2b$: Whence $a=4{\cdot}1677$; and from thence $2z$ (GFH) $=b\times1{\cdot}2578=12{\cdot}578$.

COROLLARY. Hence the length of the chain GFH (when the ſtreſs is a minimum) is to the given diſtance of the tacks G, H, as 1·2578 to 1; and the ſtreſs on the tacks is to the weight of the chain, as 1·1996 to 1, or as 6 to 5, nearly.

This queſtion was alſo anſwered, in an elegant manner, by Mr. *O'Cavannah*, Mr. *W. Bevil* (the propoſer), Mr. *E. Rollinſon*, and Mr. *H. Watſon*.

XIII. QUEST. 418 *anſwered by Mr.* Peart, *the Propoſer.*



From the equation $zz = 2ax + xx$ of the generating curve, the ſurface of the mountain *AEBC* is eaſily found to be $4c \times \overline{yz - ax}$ (where $c = \frac{3{\cdot}1416}{2}$), which being given $= 4yy$, we thence have (by compleating the ſquare) $y = \frac{1}{2}cz + \sqrt{\frac{1}{4}c^2z^2 - cax} = \frac{1}{2}c\sqrt{2ax + xx} + \sqrt{\frac{1}{4}c^2 \times \overline{2ax + xx} - cax}$: but y is alſo $= a \times$ hyp. log. $\frac{a + x + \sqrt{2ax + xx}}{a}$ (by property of the curve): From which equal values, by ſubſtituting $ux = a$, we have $\frac{1}{2}c\sqrt{2u + 1} + \sqrt{\frac{1}{4}c^2 \times \overline{2u + 1} - cu} =$ hyp. log. $\frac{u + 1 + \sqrt{2u + 1}}{u}$; whence u is found $= 1{\cdot}779 +$, and from thence $y = 1{\cdot}8078\,x$. Having, therefore, made *BC* to *AB* in the given proportion of 1·8078 to 1, and taken $AD =$ the right-line $AC = 2{\cdot}066\,x$, let *CD* be drawn, which (becauſe the range is a *maximum*) will be the line of direction.

Now let $r =$ the earth's ſemi-diameter, in feet; $b = 16\frac{1}{12}$ feet; and $s =$ the number of ſeconds in 24 hours: Then it will be as $r : r + x :: s : s + \frac{sx}{r} =$ the ſeconds taken up in performing an equal number of vibrations by the pendulum on the top of the mountain: Therefore $\frac{sx}{r}$ is the time of the ball's flight; and conſequently $\frac{s^2x^2}{rr} \times b = 2{\cdot}066\,x$ $(= AD)$; whence x is given $= \frac{2{\cdot}066\,rr}{bss} = 7598$ feet, the mountain's height; and the time of flight $(\frac{sx}{r}) = 31{\cdot}24$ ſeconds: By which number dividing CD $(= 27046)$, the quotient $865\frac{3}{4}$ feet will be the velocity, *per ſecond*, with which the ball is diſcharged.

In this ſolution I have ſuppoſed the gravitation to be proportional to the ſquare of the diſtance from the earth's center, inverſely, without having regard to the attraction of the mountain: But if *this laſt* (which is to the former as the height of the mountain to the ſemi-diameter of the earth, nearly) be alſo taken into the conſideration, the time loſt by the pendulum on the top of the mountain will then be only the half of $(\frac{sx}{r})$ what it is above found to be: Therefore the height of the mountain will come out four times as great here, as when the mountain's attraction is neglected.*

XIV. QUESTION 419 *anſwered by Mr.* R. Weſton, *the Propoſer.*

Since it appears, by what is done in the laſt diary, that $-\frac{\dot{y}\ddot{x}}{\dot{x}}$ may be ſubſtituted for $\ddot{y}$, if ſuch ſubſtitution be made in the given equation, the required value of x may then be readily obtained by the common method of finding fluents by infinite ſeries: Or that value may be found in the following manner:

Aſſume $x = b + fy + py^2 + qy^3 + ry^4$, &c. ſuppoſing that when y is $= a$, x is $= b$, and $\frac{\dot{x}}{\dot{y}} = f$: Then the given equation $(a^3\ddot{y} = a^2\dot{y}^2 - y^2\dot{x}^2)$ being multiplied by the invariable quantity $\dot{x}$, we have $a^3\dot{x}\ddot{y} = a^2\dot{x}\dot{y}^2 - y^2\dot{x}^3$; whence ſubſtituting for $\dot{x}^3$ its value found by the aſſumed equation, we have, after dividing by $\dot{y}^2$, $\frac{a^3\dot{x}\ddot{y}}{\dot{y}\dot{y}} = a^2\dot{x} - f^3y^2\dot{y} - 6f^2py^3\dot{y}$, &c. Hence, by taking the fluents, we find $-\frac{a^3\dot{x}}{\dot{y}} = a^2x - \frac{f^3y^3}{3} - \frac{6f^2py^4}{4} -$

$9f^2q +$

---

* This may be explained thus. Suppoſe the earth's attraction to be 1, and that it is to that of the mountain as r to x; then $\frac{x}{r}$ will be the attraction of the mountain, and $1 + \frac{x}{r}$ the ſum of both; and, becauſe the times of *vibration* are as the roots of the forces reciprocally, $\sqrt{1} : \sqrt{1 + \frac{x}{r}} = 1 + \frac{x}{2r}$ nearly $:: s : x + \frac{sx}{2r}$ nearly, and $\frac{sx}{2r} =$ time loſt nearly.

$\frac{9f^2q+12fp^2}{5}y^5$, &c. But, x being $= b$, and $\frac{\dot{x}}{\dot{y}} = f$, when y is $= 0$, the correct equation of the fluents is — $\frac{a^3\dot{x}}{\dot{y}} = -a^2b - a^3f + a^2x - \frac{f^3y^3}{3} - \frac{6f^2py^4}{4}$ &c. which, multiplying by $-\frac{\dot{y}}{a^3}$ and ſubſtituting for x its aſſumed value, becomes $\dot{x} = f\dot{y} - \frac{fv\dot{v}}{a} - \frac{py^2\dot{y}}{a} + \overline{\frac{f^3}{3a^3} - \frac{q}{a}} \times y^3\dot{y}$, &c. whence, taking the correct fluents, we get $x = b + fy - \frac{fy^2}{2a} - \frac{pv^3}{3a} + \frac{f^3 - 3a^2q}{3\cdot 4a^3}y^4 + \frac{6f^2p - 4a^2r}{4\cdot 5a^3}y^5$, &c. Conſequently, by comparing the two values of x; p, q, r, &c. will be known.

The ſame anſwered by Mr. Henry Watſon.

Put $\dot{y} = z\dot{x}$, then $\ddot{y} = \dot{z}\dot{x}$; which values being ſubſtituted in the given equation, $a^3\ddot{y} = a^2\dot{y}^2 - y^2\dot{x}^2$, and the whole divided by $\dot{x}$, we have $a^3\dot{z} = a^2z^2\dot{x} - y^2\dot{x} = a^2z^2 \times \frac{\dot{y}}{z} - y^2 \times \frac{\dot{y}}{z}$ (becauſe $\dot{x} = \frac{\dot{y}}{z}$) and conſequently $a^3z\dot{z} = a^2z^2\dot{y} - y^2\dot{y} = \overline{a^2z^2 - y^2} \times \dot{y}$: Put $v = a^2z^2 - y^2$; then $zz = \frac{v + yy}{aa}$ and $z\dot{z} = \frac{\frac{1}{2}\dot{v} + y\dot{y}}{qa}$; and ſo, by ſubſtitution, $a \times \overline{\frac{1}{2}\dot{v} + \dot{y}y} = v\dot{y}$, or $\frac{1}{2}a\dot{v} = \overline{v - ay} \times \dot{y}$: Put now $w = v - ay$ ($= a^2z^2 - y^2 - ay$) then $v = w + ay$, and $\dot{v} = \dot{w} + a\dot{y}$; whence again, by ſubſtitution, $\frac{1}{2}a \times \overline{\dot{w} + a\dot{y}} = w\dot{y}$, or $\frac{1}{2}a\dot{w} = w - \frac{1}{2}aa \times \dot{y}$, and therefore $\dot{y} = \frac{\frac{1}{2}a\dot{w}}{w - \frac{1}{2}aa}$; whereof the fluent is $y = \frac{1}{2}a \times$ hyp. log. $\frac{w - \frac{1}{2}aa}{d}$ (*d* being any conſtant quantity at pleaſure) $= \frac{1}{2}a \times$ hyp. log. $\frac{a^2z^2 - y^2 - ay - \frac{1}{2}qa}{d}$. Hence, putting M = the number whoſe hyp. log. = 1, we have $M^{\frac{2y}{a}} =$

$= \frac{a^2z^2 - y^2 - ay - \frac{1}{2}aa}{d}$, and conſequently $z^2 = \frac{y^2 + ay + \frac{1}{2}aa + dM^{\frac{2y}{a}}}{aa}$; from whence $\dot{x}$ $(= \frac{\dot{y}}{z}) = \frac{a\dot{y}}{\sqrt{y^2 + ay + \frac{1}{2}aa + dM^{\frac{2y}{a}}}}$: From which, when $d = 0$, x will be found $= a \times$ hyp. log. $\frac{y + \frac{1}{2}a + \sqrt{y^2 + ay + \frac{1}{2}aa}}{c}$, wherein c may be any conſtant quantity at pleaſure.

In a manner very little different from this laſt *it* was anſwered by Mr. *Rollinſon*, who brings out the very ſame concluſion.

XV. QUESTION 420 *anſwered by* Κυβερνήτης.

Conceive the given ſeries $1 - \frac{z^3}{2.3} + \frac{z^6}{2.3.4.5.6}$, &c. to be compoſed of three others,

$$A \times \overline{1 + pz + \frac{p^2z^2}{2} + \frac{p^3z^3}{2.3} + \frac{p^4z^4}{2.3.4}, \&c.}$$

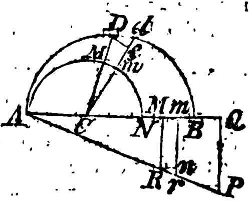
$$B \times \overline{1 + qz + \frac{q^2z^2}{2} + \frac{q^3z^3}{2.3} + \frac{q^4z^4}{2.3.4}, \&c.}$$

$$C \times \overline{1 + rz + \frac{r^2z^2}{2} + \frac{r^3z^3}{2.3} + \frac{r^4z^4}{2.3.4}, \&c.}$$

Then, by taking A, B, C each $= \frac{1}{3}$, and equating the homologous terms, we ſhall have $p + q + r = 0$, $pp + qq + rr = 0$; $p^3 + q^3 + r^3 = -3$, $p^4 + q^4 + r^4 = 0$, &c. &c. Make now $p^3 = -1$, $q^3 = -1$, and $r^3 = -1$; that is, let p, q, and r be the three roots, $(-1, \frac{1}{2} + \sqrt{-\frac{3}{4}}, \frac{1}{2} - \sqrt{-\frac{3}{4}})$ of the cubic equation $x^3 = -1$, or $x^3 + 1 = 0$; then, as both the ſecond and third terms of this equation are wanting, not only the ſum of all the roots $(p + q + r)$ but the ſum of all their ſquares $(pp + qq + rr)$ will vaniſh, or be equal to nothing, as they ought, to fulfil the conditions of the two firſt of the preceding equations. Moreover, becauſe $p^3 = -1$; $q^3 = -1$, and $r^3 = -1$, it is likewiſe evident that $p^4 + q^4 + r^4 = -p - q - r = 0$, $p^5 + q^5 + r^5 = -p^2 - q^2 - r^2 = 0$, and $p^6 + q^6 + r^6 = -p^3 - q^3 - r^3 = 3$; which equations being nothing more than the three firſt repeated, the values of p, q, r, above determined, will equally fulfil the conditions of theſe alſo: So that the ſeries ariſing from the addition of the three aſſumed ones will agree, in every term, with that propounded. But $1 + pz + \frac{p^2z^2}{2} + \frac{p^3z^3}{2.3}$, &c. the firſt of thoſe ſeries is known to expreſs the number whoſe hyperbolic logarithm is pz: Therefore, if N be taken to denote the number (2·71828, &c.) whoſe hyp. log. is unity, then will $1 + pz + \frac{ppzz}{2}$, &c. $= N^{pz}$: And in the ſame manner $1 + qz + \frac{qqzz}{2}$, &c. $= N^{qz}$, &c. and conſequently $1 - \frac{z^3}{2.3} + \frac{z^6}{2.3.4.5.6}$, &c. $= \frac{1}{3}$ into $\overline{N^{pz} + N^{qz} + N^{rz}} = \frac{1}{3}N^{-z} + \frac{1}{3}N^{\frac{1}{2}z} \times \overline{N^{nz\sqrt{-1}} + N^{-nz\sqrt{-1}}}$ (making $n = \sqrt{\frac{3}{4}}$). But $N^{nz\sqrt{-1}} + N^{-nz\sqrt{-1}}$ is known to expreſs the double of the coſine of the arch nz (the radius being 1); which coſine let be denoted by S, and let the number whoſe hyp. log. is z be repreſented by T; then we ſhall have $1 - \frac{z^3}{2.3} + \frac{z^6}{2.3.4.5.6}$, &c. $= \frac{1}{3T} + \frac{2ST^{\frac{1}{2}}}{3}$. *Q.E.I.*

From the ſame methods, and the known roots of the equation $x^n \pm 1 = 0$, the ſeries $1 \pm \frac{z^n}{1.2.3(n)} + \frac{z^{2n}}{1.2.3(2n)}$, &c. may be ſummed, n being any whole poſitive number.

PRIZE QUESTION *anſwered by Mr.* O'Cavanah.



From C, the center of the moraſs, let a ſemi-circle AMN be deſcribed; and from any two points D, d in the required curve ADB, indefinitely near to each other, conceive DC and dC to be drawn, interſecting the ſemi-circle in M and m: Calling CA, 1; CB, b; AM, z; CD, y; Mm, $\dot{z}$; and de, $\dot{y}$. Then $CM\ (1) : Mm\ (\dot{z}) :: CD\ (y) : De = y\dot{z}$. Whence $Dd\ (= \sqrt{De^2 + de^2} = \sqrt{y^2\dot{z}^2 + \dot{y}^2}$. This, divided by y, gives $\sqrt{\dot{z}^2 + \frac{\dot{y}\dot{y}}{yy}}$; which (by the queſtion) is the time of deſcribing *Dd*, and which, by making $\dot{x} = \frac{\dot{y}}{y}$ (or $x =$ hyp. log. y) will be reduced to $\sqrt{\dot{z}\dot{z} + \dot{x}\dot{x}}$.

Make now the right line *AQ* = the ſemi-circumference *AMN*, and *QP* (perpendicular thereto) = hyp. log. b (= the laſt value of x, when $y = b = CB$): And let the line *ARP* be of ſuch a nature, that, taking the abſciſſa *AM* = the arch *AM*, the correſponding ordinate *MR* ſhall be every-where equal to x: Then $R r$ (the fluxion of *AR*) being alſo expreſſed by $\sqrt{\dot{z}\dot{z} + \dot{x}\dot{x}}$, it is evident that *AR* will truly expreſs the time of deſcribing the arch *AD*; and, conſequently, *AP* the whole time of deſcribing the arch *ADB*; which will evidently be the ſhorteſt poſſible, when *AP* is a ſtraight line (as being the ſhorteſt that can poſſibly be drawn between the two given points *A* and *P*).

Hence, putting *AQ* (= 3·1416) = p, and *QP* (= hyp. log. b, or of $\frac{CB}{CA}$ = ·693147) = q, we have, by ſimilar triangles, $p : q :: \dot{z} : \dot{x} = \frac{q\dot{z}}{p} = \frac{\dot{y}}{y}$ (per above); and conſequently $De = y\dot{z} = \frac{p\dot{y}}{q}$; which being to de ($\dot{y}$) in the conſtant ratio of p to q, the ange *Dde* muſt be every-where the ſame, and therefore the curve *ADB*, the proportional, or logarithmical ſpiral; wherein *Dd* being to *ed* in the conſtant ratio of $\sqrt{pp + qq}$ to q, we ſhall therefore have $q : \sqrt{pp + qq} ::$ *NB* (the whole increaſe of the diſtance *CD*) : $\sqrt{\frac{pp}{qq} + 1} \times NB$ = 4·641444 the true length of the ſpiral arch *ABD*. *W. W. R.*

COROLLARY. It appears from hence that the time of deſcribing the ſpiral *ADB* will be to the time of uniformly deſcribing the arch of the ſemi-circle *AMN*, with the firſt velocity at *A*, in the given ratio of *AP* to *AQ*, or of $\sqrt{pp + qq}$ to p.

The prize queſtion was alſo truly and conciſely anſwered by *Birchoverenſis*, Mr. *Abr. Botham*, Mr. *L. Charlton*, Mr. *J. Goodhead*, Mr. *E. Rollinſon*, Mr. *Walter Trott*, Mr. *H. Watſon*, and Mr. *R. Weſton*, from the prob. in p. 496 of Simpſon's Fluxions.——The anſwer by *Plus-Minus* (though a ſmall a ſmall miſtake is therein committed) ſufficiently diſcovers the author to be a man of genius.

The prize of 12 *Diaries was won by Mr.* Pat. O'Cavanah, *of* Dublin.

Eclipſes in 1757, *calculated by Mr.* Ra. Hulſe.

There will happen four eclipſes this year, viz. two of each luminary; whereof thoſe of the moon will only be viſible to the inhabitants of Great Britain.

1. February 4th, the moon will be eclipſed at 7h. 6m. morning, near 7 digits on the upper ſide; the beginning of the eclipſe 5h. 27m. middle 6h. 48m. end 8h. 9m. total duration 2h. 42m.

2. February 18th, at 1h. afternoon, the ſun will be 8 dig. eclipſed, on the ſouth ſide, in ♓ 0° 9′, vertical to the Ethiopian ocean, lat. 11° ſouth, long. 15° weſt.

3. July 30th, the moon will be eclipſed at 11h. 54m. at night; the beginning 10h. 20m. Middle 11h. 54m. End 1h. 18m. total duration 2h. 58m. digits eclipſed 11° 30′.

4. Auguſt 14th, at 11 at night, the ſun will be eclipſed 5 digits on the upper ſide, in ♌, 22 deg. vertical to the great ocean, lat. 14° north, long. 165° weſt.

The times and quantities of the two viſible eclipſes, for the meridian of London are calculated by Mr. *Edward Greenſted*, as below.

1. The moon will be eclipſed the 4th of February, in the morning; beginning 5h. 20m. middle 6h. 45m. end 8h. 10m. duration 2h. 50m. digits eclipſed 6° 41′.
2. The moon will be eclipſed the 30th of July, in the evening; beginning 10h. 16m. middle 11h. 48m. end 1h. 19m. duration 3h. 3m. digits eclipſed 11° 32′.

New Queſtions.

I. QUESTION 421, *by* Juvenis.

Three men to ſhare a ſtock agree, of fifteen hundred pound:
The part of *A* to that of *B*, as four to three was found;
But *C*'s exceeding that of *A* by pounds juſt ten times ſeven.
What each man ſhar'd, pray, ladies, ſay, from what above is given.

II. QUES-

II. QUESTION 422, *by Mr.* William Spry, *Engineer.*

Suppoſe that a cannon ball is diſcharged to hit a target (or other obſtacle) at the diſtance of 500 yards: How far muſt I ſtand from the target, in a perpendicular to the line drawn between it and the cannon, ſo as to hear the report of the ſhot and the exploſion of the piece at the ſame inſtant of time, allowing the velocity of the ball to be to that of ſound in the proportion of 3 to 2?

III. QUESTION 423, *by* Birchoverenſis.

To determine the poſition of a point with reſpect to four given points, ſo that lines being drawn from thence to the given points, the ſum of the four ſquares formed upon them ſhall be the leaſt poſſible.

IV. QUESTION 424, *by Mr.* Tho. Baxtonden, *of* Liverpool.

At a ſtation due ſouth of a tower, I obſerved the altitude of the top of the tower to be 30°, and that of its baſe 12°: Proceeding from thence 100 yards, north-eaſt, down a path making an angle of 5° 30′ with the plane of the horizon, I again took the altitude of the tower's ſummit, which I then found to be 38° 30′. From whence I deſire to know the height of the tower, and the diſtance thereof from my firſt ſtation.

V. QUESTION 425, *by Mr.* Ja. Beresford.

The four ſides of a field, whoſe diagonals are equal, are known to be 25, 35, 31, and 19 perches, in a ſucceſſive order; from whence the content of the field is required.

VI. QUESTION 426, *by Mr.* W. Spicer.

A man laid out 60 pounds in ſheep, of three different ſorts; for the firſt ſort he paid 9 ſhillings a-piece, for the ſecond 12, and for the third 15: And the number he bought of each ſort was ſuch, that the ſum of their three ſquares was leſs than it could poſſibly have been, had he bought more of any one ſort and leſs of another. What number of ſheep did he buy?

VII. QUESTION 427, *by Mr.* T. Moſs.

The rectangle under the two diagonals of any trapezium drawn into twice the coſine of the angle contained by them (the radius being 1) is equal to the difference of the aggregates of the ſquares of the oppoſite ſides of the trapezium;

and the area of the trapezium is equal to one-fourth of the ſame difference drawn into the tangent of the ſaid angle. A demonſtration of this is required.

VIII. QUESTION 428, *by Mr.* Lionel Charlton.

The ſum (200) of the two extremes, and the ſum (300) of the four means of ſix numbers in continued geometrical proportion being given; to find the numbers themſelves, by an equation not exceeding a quadratic.

"This queſtion, Mr Charlton obſerves, was propoſed to "him by a Gentleman at the mathematical college at Edin"burgh."

IX. QUESTION 429, *by Mr.* Walter Trott.

Two ſhips, *A* and *B*, ſail from a certain port, in north latitude, to two other ports lying under the equinoctial line, at the diſtance of $666\frac{2}{3}$ leagues from each other. The ſhip *A*, ſteering full ſouth (which was her direct courſe), made her port in 15 days; but *B*, though ſhe ſteered the ſhorteſt courſe poſſible, and run at the ſame rate as *A*, did not arrive at her port till the end of 25 days. Now I demand the latitude of the port ſailed from, and the true diſtance run by each ſhip.

X. QUESTION 430, *by Mr.* Henry Watſon.

In a given triangle (whoſe three ſides are 40, 50, and 60) to deſcribe the greateſt ellipſis poſible; and to determine the area, and the principal diameters thereof.

"This problem, or one like it, was printed in the Ladies "Diary for 1739, but never was anſwered in any ſucceed"ing Diary, or elſewhere, that I have been able to diſcover. "Your propoſing of it, at this time, will oblige many of "your readers, and particularly your humble ſervant,

"HENRY WATSON."

XI. QUESTION 431, *by Mr.* W. Bevil.

Suppoſe a round poſt one foot in diameter, and fixed perpendicular to the horizon, on which are wound 100 rounds of Mancheſter binding (one upon another) whoſe thickneſs is one twenty-eighth of an inch. Now, if a perſon takes hold of the extremity of the outward end, and moves round the ſaid poſt until he has unwrapt it all, how many yards will he have travelled when he arrives at the end of his journey, always keeping as far from the poſt as the binding will admit him?

XII. QUESTION 432, *by Mr.* Edw. Rollinſon.

From the equation $m x^3 \dot{z}^2 - n x^2 \ddot{z}^2 = \overline{p + zz})^2 \times q \dot{x}^2$ to determine the general relation of x and z; and alſo to find in what circumſtances of the coefficients m, n, p, q, that relation can be expreſſed in finite terms.

'Dr. BROOK TAYLOR, in his INCREMENTS, after hav-
'ing given a ſolution of that caſe where $m = 4$, $n = 4$,
'$p = 1$, and $q = 1$, (in which there ſeems to be a miſtake)
'ſays, that if the coefficients be changed, it does not ap-
'pear to him that x can be determined, in terms of z, by
'a finite equation.'

XIII. QUEST. 433, *by Mr.* R. Weſton, *Diſcip. Landenii.*

If a ſtraight, uniform, ſlender rod, or bar, of heavy metal, of a given length, be left to deſcend after being ſet leaning, in a given poſition, with its lower end (n) on the immoveable horizontal plane AB, [*See the fig. to the ſolut.*] and its upper end (m) full againſt the immoveable vertical plane AC (the lower end being at liberty to ſlide freely along the firſt-mentioned plane, while the upper end is deſcending), what will be the poſition of the rod when it ſhall ceaſe to touch the ſaid vertical plane? how long will it then have been in motion? and how far from the point A will the end (m) ſtrike the horizontal plane?

PRIZE QUESTION, *by Mr.* O'Cavanah, *of* Dublin.

A pert young exciſeman, who boaſted his knowledge
In gauging of veſſels, and taking an ullage,
A wager would lay, his ſkill to make good;
And the caſe we propos'd for his trial thus ſtood:
'Eighteen inches, five tenths, a caſk's length is given:
'The heads, which are equal, are each thirty-ſeven*.
"It likewiſe is known, that ſuch is the make,
'The caſk it is formed the leaſt † wood to take.
To find the content quite baffled his art;
But he hopes you the method next year will impart.

* Each head diam. is 37 inches.

† The ſuperficies is the leaſt poſſible.

1758.

Queſtions anſwered.

I. QUEST. 421 *anſwered by Mr.* W. Bacon, *of* Ipſwich.

Let the ſhare of A be $= x$; then that of B will be $= \frac{3x}{4}$, and that of $C = x + 70$ (by the queſt.); whence $2x + \frac{3x}{4} + 70 = 1500$ (by the queſt.) and conſequently $x = \frac{1430 \times 4}{11} = 520$. Therefore the ſhare of A was 520l. that of B 390l. and that of C 590l.

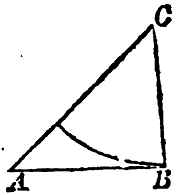
The ſame anſwered by Mr. R. Flitcon.

Let $4x$, $3x$, and $4x + 70$ denote the ſhares of A, B, and C, reſpectively; then will $11x + 70 = 1500$ (per queſtion): Whence $x = 130$; and the three ſhares 520, 390, and 590 pounds.

Thus the problem is alſo anſwered by Mr. *W. Bamfield*, Mr. *Edw. Barraſs*, Mr. *Fra. Bell*, Mr. *Ja. Bereſford*, Mr. *J. Chapman*, Mr. *G. Clarke*, Mr. *E. Cock*, Mr. *T. Corbett*, Mr. *Ja. Fellowes*, Mr. *Ja. Giles*, Mr. *J. Hampſon*, Mr. *E. Hare*, Mr. *W. Harriſon*, Mr. *G. Hickes*, Mr. *W. Honnor*, Mr. *T. Howe*, Mr. *R. Hudſon*, Mr. *T. Jeffery*, Mr. *J. Johnſon*, Mr. *T. Knight*, Mr. *E. Langworth*, Mr. *J. Lewin*, Mr. *W. Litſon*, Mr. *R. Marſh*, Mr. *W. Mathewſon*, Mr. *Herbert Nokes*, Mr. *J. Rennard*, Mr. *R. Richardſon*, Mr. *Joſ. Roſe*, Mr. *Alex. Rowe*, Mr. *T. Sandling*, Miſs *T. S—l*, Mr. *Joſ. Scott*, Mr. *T. Sims*, *Sinbad*, Mr. *G. Stapley*, Mr. *H. Stephens*, Mr. *J. Stothart*, Mr. *R. Walton*, and many others.

II. QUEST.

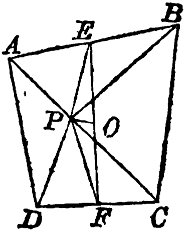
II. QUEST. 422 *anſwered by Mr.* J. Hampſon, *of* Leigh.



Let *A* be the place of the cannon, *B* that of the target, and *C* that of the ſpectator; and in *CA* take $CD = CB$: Then the ſound of the cannon will be at *D*, on the ball's ſtriking the target at *B*; and ſo *AD* will be to *AB* in the given proportion of 2 to 3 (by the queſtion). Putting, therefore, $AB = 3a$, $AD = 2a$, and $BC (= CD) = x$, we have $AC = x + 2a$; and from thence (by Euc. 47. 1.) $xx + 4ax + 4aa = xx + 9aa$: From which $x = \frac{5a}{4} = 208\frac{1}{3}$ yards.

In the very ſame manner it is anſwered by Mr. *W. Allen*, Mr. *J. Bank*, Mr. *Ja. Bereſford*, Mr. *W. Bevil*, *Birchoverenſis*, Mr. *J. Chapman*, Mr. *T. Corbett*, Mr. *J. Herrington*, Mr. *G. Hicks*, Mr. *J. Honey*, Mr. *W. Honnor*, Mr. *T. Howe*, Mr. *R. Hudſon*, Mr. *J. Milbourn*, Mr. *J. Rennard*, Mr. *T. Sims*, Mr. *H. Stephens*, Mr. *W. Stoker*, Mr. *W. Terrill*, Mr. *J. Wilſon*, and ſeveral others.

III. QUESTION 423 *anſwered by Mr.* Will. Kingſton, *of* Bath.



Let *A*, *B*, *C*, and *D* be the four given points: Biſect *AB* in *E*, and *CD* in *F*; then aſſuming *P* for the required point, and putting $AE = BE = a$, $DF = CF = b$, $EP = x$, and $FP = z$, we ſhall (by the 12th of the 2d of Simpſon's Geometry) have $AP^2 + BP^2 = 2aa + 2xx$, and $DP^2 + CP^2 = 2bb + 2zz$: Hence $AP^2 + BP^2 + DP^2 + CP^2 = 2aa + 2bb + 2xx + 2zz$, a minimum: And conſequently $xx + zz$ ($EP^2 + FP^2$) a minimum alſo; which will evidently be when $EP + FP$ is a min. that is, when *EP* and *FP* make a right line; and *EP* will then be $= FP$; becauſe (by Euc. 9. 2.) the ſum of the ſquares of the two parts of a line equally divided is always leſs than the ſum of the ſquares of the parts, when the line is divided unequally.

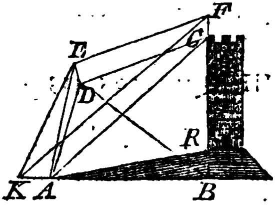
The ſame anſwered by Mr. W. Lucas.

Let A, B, C, and D repreſent the four given points, and P any point taken at pleaſure; let PE and PF be drawn to biſect AB and DC in E and F; then (by a theorem in geometry) $AP^2 + BP^2 = 2AE^2 + 2EP^2$, and $DP^2 + CP^2 = 2DF^2 + 2FP^2$; therefore $AP^2 + BP^2 + DP^2 + CP^2 = 2 \times \overline{AE^2 + DF^2 + EP^2 + FP^2}$, a minimum (per queſt.); and conſequently $EP^2 + FP^2$ a minimum alſo: But, drawing PO to biſect EF, we get $EP^2 + FP^2 = 2EO^2 + 2PO^2$: Therefore PO^2 a minimum, and ſo PO being $= 0$, O muſt be the point required.

In this manner, and from the very ſame property, the problem is elegantly ſolved by Mr. *Rob. Butler*, Mr. *Patrick O'Cavanah*, and Mr. *W. Davies*.

IV. QUESTION 424 *anſwered by Mr.* Tho. Baxtondan, *the Propoſer.*

Let the ∠s BAC and BAR be the given elevations of the top and baſe of the tower above the plane of the horizon AB, at the firſt ſtation A; let D be the ſecond ſtation, and DE its perpendicular diſtance below the plane ABE of the firſt. In BC produced, ſuppoſe CF to be taken $= DE$; let $FK \parallel CA$, and let EF, EB, FK, and AD be drawn. Then, becauſe DE is $=$ and $\parallel FC$, EF will alſo be $=$ and $\parallel DC$, and conſequently the $\angle EFB$ $(= DCB) =$ the complement of the given elevation at the ſecond ſtation D. Now, from AD (100) and the $\angle DAE$ (5° 30′), we have given $AE = 99{\cdot}5396$, and $DE\,(FC) = 9{\cdot}5846$: and it will be as rad. : tang. BCA (60°) $(:: BC : BA) :: FC : KA = 16{\cdot}601$: from which, together with AE and the contained $\angle KAE$ (135°), the $\angle AKE$ will be found $= 38°\ 59'$. Moreover the ∠s FBE and FBK being both right ones, it will be tang. BFE (51° 30′) : tang. BFK (60°) $:: (BE : BK) ::$ ſin. BKE (38° 59′) : ſin. $BEK = 60°\ 5'$: whence ABE is alſo given $= 80°\ 56'$, $AEB = 54°\ 4'$: from which, and AE $(= 99{\cdot}5396)$ will be found $AB = 81{\cdot}61$; and from thence $BC = 47{\cdot}12$, $BR = 17{\cdot}34$, and $RC = 29{\cdot}77 =$ the tower's true height.

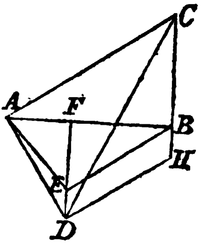


An

An Algebraic Solution to the ſame by Mr. Rich. Mallock.

‘ I have ſome reaſon to believe that this queſtion is wrong ‘ printed, viz. north for ſouth, ‘or north-eaſt for ſouth-eaſt; ‘ which being corrected, I ſolve it thus.’

Let DE be the perpendicular diſtance of the ſecond ſtation D, below the horizontal plane (ABE) of the firſt ſtation A; in CB (the tower's height) produced, take $BH = ED$; join D, H, and upon AB let fall the perpendicular EF.

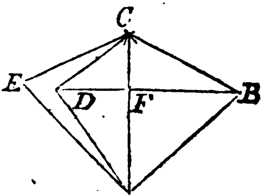


By means of AD ($= 100$) and the given $\angle DAE$ ($= 5^\circ\ 30'$) I find ED ($= BH$) $= 9{\cdot}58458$, and $AE = 99{\cdot}53955$; from which laſt AF (EF) $= 70{\cdot}3853$. Now put $BH = b$, AF ($= EF$) $= c$, and $BC = x$; and let m and n denote the tangents of the complements (BCA and HCD) of the tower's elevation at the two ſtations A and D: Then (by trigonometry) $BA = mx$, and $HD = n \times \overline{x+b} = BE$ (becauſe BH being ∥ and $= ED$, BE will alſo be ∥ and $= HD$). Hence $BF = mx - c$; and conſequently $\overline{mx-c}|^2 + cc = \overline{nx+nb}|^2$ (Eu. 47. 1.): which ordered, gives $xx - \frac{mc+nnb}{mn-nn} \times 2x = -\frac{2cc-nnbb}{mm-nn}$; in numbers, $xx - 193{\cdot}11x = -6877{\cdot}8$; whence $x = 96{\cdot}555 \pm 49{\cdot}44$; that is, the tower's height above AB is either 47·11 yards, or 146 yards; from the former whereof the required diſtance AB is found $= 81{\cdot}5$ yards.

In the very ſame manner the problem is anſwered by Mr. *W. Allen*, Mr. *R. Butler*, *Birchoverenſis*, Mr. *W. Marſhall*, Mr. *J. Milbourn*, Mr. *S. Towndrow*, and Mr. *W. Stoker*.

V. QUESTION 425 *anſwered by Mr.* O'Cavanah.]

In the caſe propoſed, where the ſums of the ſquares of the oppoſite ſides of the trapezium are equal (and where the diagonals do, therefore, cut each other at right angles) the problem may be thus conſtructed.



Having made AB and AE perpendicular to each other, and equal each to any one of the given ſides (as 35) from the centers B and E with radii equal to the two oppoſite ſides (25 and 31) let two arcs be deſcribed, interſecting in C; draw AC, and alſo $BD \perp$ and $=$ thereto; ſo ſhall $ABCD$ (when AD and CD are drawn) be the trapezium ſought.

For the $\angle$s ABD and EAC are equal, being both complements of CAB, to a right angle (by conſtruction), and the ſides AB, BD, AE, AC, containing them, are alſo reſpectively equal (by conſtruction): Whence the remaining ſides AD and EC muſt neceſſarily be equal. Alſo $AB^2 - AD^2$ $(= BF^2 - DF^2) = BC^2 - DC^2$, and therefore $AB^2 + DC^2 = AD^2 + BC^2$, that is, in the preſent caſe, $35^2 + DC^2 = 31^2 + 25^2$, and conſequently $DC = 19$, as it ought to be. The numerical ſolution is from hence very eaſy, whereby the area comes out $= 4$ A. 1 R. $38\frac{1}{4}$ P.*

Mr. *W. Harriſon*, Mr. *J. Honey*, Mr. *R. Mallock*, Mr. *J. Pearce*, Mr. *W. Stoker*, and Mr. *W. Terrill* (whoſe ſolutions agree in almoſt every ſtep) give the following analytical inveſtigation of this problem.

‘ Since the ſums of the ſquares of the oppoſite ſides of the ‘ trapezium $ABCD$ are equal (by the queſtion), the diagonals will therefore cut each other at right angles in F: ‘ ſo that, putting $BC (= 25) = a$, $BA (= 35) = b$, $DA (= 31)$ ‘ $= c$, $CD (= 19) = d$, and $AC (= BD) = x$, we ſhall have ‘ $x : b + a :: b - a : AF - CF = \frac{bb - aa}{x}$; and $x : b + c$ ‘ $:: b - c : BF - DF = \frac{bb - cc}{x}$; whence $AF = \frac{x}{2}$ ‘ $\frac{bb - aa}{2x}$, and $BF = \frac{x}{2} + \frac{bb - cc}{2x}$; and conſequently ‘ $\overline{\frac{x}{2} + \frac{bb - aa}{2x}}\Big|^2 + \overline{\frac{x}{2} + \frac{bb - cc}{2x}}\Big|^2 = bb$; from which equa‘tion $2x^4 - \overline{aa + cc} \times 2xx + \overline{bb - aa}|^2 + \overline{bb - cc}^2 = 0$. ‘ This ſolved, gives $x = \sqrt{\frac{aa + cc}{2} \pm \sqrt{bbdd - \frac{1}{4} \times \overline{cc - aa}|^2}}$ ‘ $= 37{\cdot}9$; and the area ſought $(\frac{xx}{2}) = 4$ A. 1 R. 38 P.’

A general Solution of the ſame Problem by Birchoverenſis.

Let the given ſides DA, AB, BC, CD of the trapezium be denoted by a, b, c, and d reſpectively; and let each of the equal diagonals AC, BD, be called x: Then (ſuppoſing

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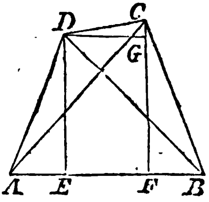
* The numerical calculation at large may be ſeen at p. 129 Hutton's Menſuration.



(ſuppoſing DE and CF to be $\perp AB$, and $DG \parallel AB$) we ſhall have $b : x+a :: x-a : BE-AE = \frac{xx-aa}{b}$; whence $AE = \frac{bb+aa-xx}{2b}$, and $DE = \sqrt{\frac{-x^4+\overline{aa+bb}\times 2xx-\overline{bb-aa}|^2}{4bb}}$. In the ſame manner $BF = \frac{bb+cc-xx}{2b}$, and $CF = \sqrt{\frac{-x^4+\overline{cc+bb}\times 2xx-\overline{bb-cc}|^2}{4bb}}$; and conſequently DG ($EF = AB - AE - BF$) $= \frac{2xx-aa-cc}{2b}$; From which (becauſe $\sqrt{DC^2-DG^2} = GC = CF - DE$) we get $\sqrt{4bbdd-4x^4+\overline{aa+cc}\times 4xx-\overline{aa+cc}|^2} = \sqrt{-x^4+\overline{cc+bb}\times 2xx-\overline{bb-cc}|^2} - \sqrt{-x^4+\overline{aa+bb}\times 2xx-\overline{bb-aa}|^2}$; whence, by involution and proper reduction, there reſults $2x^6 - \overline{aa+bb+cc+dd}\times x^4 - \overline{2aacc+2bbdd-\overline{aa+cc}\times\overline{bb+dd}}\times xx + \overline{aacc-bbdd}\times\overline{aa+cc-bb-dd} = 0$; from which x may be found, let the values of a, b, c, and d be what they will: But, in the caſe propoſed, $aa+cc$ being $= bb+dd$, the laſt term will vaniſh, and we ſhall have $2x^4 - \overline{2aa+2cc}\times xx - \overline{2aacc-bbdd+\overline{aa+cc}|^2} = 0$; which ſolved, gives $x = \sqrt{\frac{aa+cc}{2} \pm \sqrt{bbdd-\frac{1}{4}\times\overline{cc-aa}|^2}} = 37{\cdot}90025$. Whence the area $ABCD$ ($= \frac{1}{2}xx$) $= 718{\cdot}2145$ $=$ 4 A. 1 R. 38·2145 P.

Meſſ. *W. Allen*, *Ja. Bank*, *T. Baxtonden*, *T. Barker*, *Ja. Beresford*, *R. Butler*, *R. Flitcon*, *Joſ. Herrington*, *J. Hampſon*, *R. Hudſon*, *W. Kingſton*, *W. Litſon*, *W. Lucas*, *W. Marſhall*, *W. Mathewſon*, *Penovius*, *R. Richardſon*, *Alex. Rowe*, *W. Spicer*, *J. Thompſon*, and *J. Wilſon*, alſo favoured us with ſolutions to this problem.

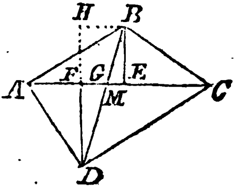
VI. QUESTION 426 *anſwered by Mr.* L. Charlton.

Let x, y, and z repreſent the ſheep bought of each ſort, and a, b, and c the given prices paid for each ſort per head reſpectively; then will $ax+by+cz = 1200$ ($= d$), and $xx+yy+zz$ a minimum (by the queſtion). Now, ſeeing any two of the quantities x, y, z, may be here varied independently of the other, we ſhall (by making x and y to flow, while z remains conſtant) have $a\dot{x}+b\dot{y}=0$, and $2x\dot{x}+2y\dot{y}=0$; from whence $\dot{x}=-\frac{b\dot{y}}{a}=-\frac{y\dot{y}}{x}$; and therefore $y=\frac{bx}{a}$. In the very ſame manner $z=\frac{cx}{a}$: Whence, by ſubſtitution, $ax+\frac{bbx}{a}+\frac{ccx}{a}=d$; and conſequently $x=\frac{ad}{aa+bb+cc}=24$; whence y is alſo given $=32$, and $z=40$.

In the very ſame manner the problem is reſolved by Mr. *T. Baxtonden*, Mr. *R. Butler*, and Mr. *W. Lucas*; and in a manner very little different, by Mr. *W. Allen*, Mr. *W. Bevil*, Mr. *J. Fellows*, Mr. *R. Flitcon*, Mr. *J. Honey*, Mr. *T. Howe*, Mr. *R. Hudſon*, Mr. *W. Kingſton*, *Penovius*, Mr. *Joſ. Scott*, Mr. *W. Spicer*, Mr. *W. Terrill*, and Mr. *J. Thompſon*.

VII. QUESTION 427 *anſwered by Mr.* J. Thompſon, *of* Witherly Bridge.

In the trapezium $ABCD$, put $AG=m$, $BG=n$, $CG=p$, $DG=q$, area $ABCD=A$, x and y for the ſine and coſine of the $\angle AGD$; then, by a known theorem, $\begin{cases} AB^2=mm+nn+2mny \\ DC^2=pp+qq+2pqy \end{cases}$ And $\begin{cases} AD^2=mm+qq-2mqy \\ BC^2=nn+pp-2npy \end{cases}$



Whence, by equal ſubtraction, $\overline{AB^2+DC^2}-\overline{AD^2-BC^2}=\overline{mn+pq+mq+np}\times 2y=\overline{m+p}\times\overline{n+q}\times 2y=AC\times BD\times 2y$. Again, by a known theorem, $mnx+npx+pqx+qnx=2A$, or $\overline{m+p}\times\overline{n+q}\times x^{*}=2A$; hence $\frac{2A}{x}=\frac{AB^2+DC^2-AD^2-BC^2}{2y}$, and $\overline{AB^2+DC^2-AD^2-BC^2}\times\frac{x}{4y}=A$. *Q.E.D.*

COROLLERY. From the equation * the area of any trapezium is equal to half the rectangle under the two diagonals drawn into the ſine of the angle contained by them.

The

The ſame anſwered by Mr. Moſs, *the Propoſer.*

Suppoſe the diagonal AC to be biſected in M; and having let fall the $\perp$s BE and DF, draw $BH \parallel AC$, meeting DF, produced, in H. It is well known, that $AB^2 - BC^2 = 2AC \times ME$, and $CD^2 - AD^2 = 2AC \times MF$: Whence, by equal addition, $AB^2 + CD^2 - BC^2 - AD^2 = 2AC \times \overline{ME + MF} = 2AC \times BH$: But $BD = \frac{BH \times \text{rad.}}{\text{coſ.}\, AGD}$; whence $AC \times BD = AC \times BH \times \frac{\text{rad.}}{\text{coſ.}\, G} = \overline{AB^2 + CD^2 - BC^2 - AD^2} \times \frac{\frac{1}{2}\,\text{rad.}}{\text{coſ.}\, G}$. Again, $DH = BH \times \frac{\text{tang.}\, G}{\text{rad.}}$; whence $\frac{1}{2} AC \times DH$ (= area $ABCD$) $= \frac{1}{2} AC \times BH \times \frac{\text{tang.}\, G}{\text{rad.}} = \overline{AB^2 + CD^2 - BC^2 - AD^2} \times \frac{\frac{1}{4}\,\text{tang.}\, G}{\text{rad.}}$. *Q. E. D.*

Meſſ. *Ja. Bank, W. Bevil, Birchoverenſis, R. Butler, W. Davies, W. Kingſton, W. Lucas, Penovius, W. Spicer,* and *J. Wilſon,* have alſo obliged us with anſwers to this problem; all whereof have a near reſemblance to the one, or the other, of thoſe above exhibited.

VIII. QUESTION 428 *anſwered by Mr.* Robert Butler.

Let a = the firſt term, r = common ratio; then $a + ar + ar^2 + ar^3 + ar^4 + ar^5$ $\left(= a \times \frac{r^6 - 1}{r - 1}\right) = 500$, and $a \times \overline{r^5 + 1} = 200$ (per queſtion). Multiply theſe two equations croſs-wiſe; whence $2 \times \overline{r^6 - 1} = 5 \times \overline{r^5 + 1} \times \overline{r - 1}$. Now put $2x = r + 1$, and $2z = r - 1$; then will $x + z = r$, and $x - z = 1$; therefore, by ſubſtitution, $\overline{x + z}^6 - \overline{x - z}^6 = 5z \times x + z \;\; + x - z$; which, being expanded, gives $19z^4 = x^4 - 30xxzz$; whence $xx = \overline{15 + \sqrt{244}} \times zz$; therefore $z = \frac{1}{\sqrt{\overline{15 + \sqrt{244}}} - 1} = {\cdot}22057$. Hence $r = 1{\cdot}4412$, and $a = 27{\cdot}529$: Therefore the ſix numbers anſwering the queſtion are 27·527, 39·94, 57·56, 82·95, 119·55, and 172·473.

In this manner the problem is reſolved by Mr. *W. Lucas.*

The ſame anſwered by Meſſ. Ja. Bank *and* J. Wilſon.

Let x and z repreſent the two middle terms, and put $a = 200$, and $c = 300$: Then, by the queſtion, $x + z + \frac{xx}{z} + \frac{zz}{x} = c$, and $\frac{x^3}{z^2} + \frac{z^3}{x^2} = a$, or (by reduction) $\overline{x+z} \times xz + x^3 + z^3 = cxz$, and $x^5 + z^5 = ax^2 z^2$; which, by putting $s = x + z$ and $p = xz$, will become $s^3 - 2ps = cp$, and $s^5 - 5ps^3 + 5p^2 s = ap^2$; from the former whereof we have $p = \frac{s^3}{2s+c}$; which value, ſubſtituted in the latter, gives $s^5 - \frac{5s^6}{2s+c} + \frac{5s^7}{\overline{2s+c}|^2} = \frac{as^6}{\overline{2s+c}|^2}$: Whence $s^2 + \overline{a+c} \times s = c^2$, and conſequently $s = \frac{\sqrt{4c^2 + \overline{a+c}|^2} - a - c}{2} = 50\sqrt{61} - 250 = 140{,}51248$. Hence $p \left(= \frac{s^3}{2s+c}\right) = \frac{2590000 - 320000\sqrt{61}}{19}$, and $x - z \left(= \sqrt{ss - 4p}\right) = \frac{\sqrt{805000\sqrt{61} - 6275000}}{19} = 25{\cdot}39269$; from whence the numbers are found to be 27·714, 39·939, 57·559, 82·952, 119·55, and 172·286.

The ſame Queſtion anſwered by Mr. William Davies.

Put $2a = 200$ the ſum of the two extremes, $3a = 300$ the ſum of the four means, $a - ax =$ the firſt term, $a + ax =$ the laſt term, and $y =$ the common ratio: Then $\overline{a - ax} \times y^5 = a + ax$, and therefore $y^5 = \frac{1+x}{1-x}$; alſo $\overline{a-ax} \times \overline{y + y^2 + y^3 + y^4}$, or $\overline{a - ax} \times \frac{y - y^5}{1 - y} = 3a$; whence $y^5 = \frac{4y - 3 - xy}{1-x} = \frac{1+x}{1-x}$; and conſequently $y = \frac{4+x}{4-x} = \frac{c+x}{c-x}$ (putting $c = 4$.) Hence we have $\frac{\overline{c+x}|^5}{\overline{c-x}|^5} = \frac{1+x}{1-x}$; from which, by reduction, $\overline{5c-1} \times x^4 + \overline{10c^3 - 10c^2} \times x^2 = 5c^4 - c^5$, or $19x^4 + 480x^2 = 256$; ſolved $x = \sqrt{\frac{32\sqrt{61} - 240}{19}}$; whence the reſt may be found.

In this manner the ſolution is alſo given by Mr. *Richard Mallock*.

The ſame anſwered by Mr. T. Barker, *of* Weſthall, *in* Suffolk.

Let $x-y$ and $x+y$ denote the third and fourth numbers (putting $a=200$ and $b=300$). Then (by the queſtion) we ſhall have $\frac{\overline{x-y}|^3}{\overline{x+y}|^2}+\frac{\overline{x+y}|^3}{\overline{x-y}|^2}=a$, and $2x+\frac{\overline{x-y}|^2}{x+y}+\frac{\overline{x+y}|^2}{x-y}=b$; which, cleared of fractions, give $2x^5+20x^3y^2+10xy^4=a\times\overline{x^4-2x^2y^2+y^2}$, and $4x^3+4xy^2=b\times\overline{xx-yy}$; from the laſt whereof we obtain $y=\frac{\frac{1}{4}bx^2-x^3}{\frac{1}{4}b+x}$. Now ſubſtitute the value of yy in the other equation; from whence, after proper reduction, there will come out $xx+\frac{a+b}{2}\times x=\frac{bb}{4}$; which, ſolved, gives $x=\frac{\sqrt{aa+2ab+5bb}-a-b}{4}=70{,}256$; therefore $y=12{\cdot}696$; and the ſix numbers ſought are 27·72, 39·964, 57·564, 82·952, 119·44, and 172·28.

In this laſt way, and by the very ſame ſubſtitution, the problem is alſo anſwered by Mr. *W. Spicer*; and in a manner very little different, by Mr. *W. Bevil*, *Birchoverenſis*, and Mr. *W. Kingſton*.

The ſame anſwered by Mr. Hugh Brown.

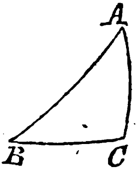
Let the two extremes be denoted by $a-x$ and $a+x$; then, putting the given ſum of the four means $=b$, the ſum of the 1ſt, 2d, 3d, 4th, and 5th terms will be $=b+a-x$, and that of the 2d, 3d, 4th, 5th, and 6th terms $=b+a+x$: Whence (by the nature of proportionals) it is manifeſt, that the firſt term is to the ſecond as $b+a-x$ to $b+a+x$, or as 1 to $\frac{b+a+x}{b+a-x}$: But the ratio of the firſt term to the ſixth is the quintuplicate of that of the firſt to the ſecond, that is, $\frac{a+x}{a-x}=\overline{\frac{b+a+x}{b+a-x}}|^5$. From whence, putting $c=b+a=400$, we get $x\times\overline{c+x}|^5+\overline{c-x}|^5=a\times\overline{c+x}|^5-\overline{c-x}|^5$, or $x\times\overline{2c^5+20c^3x^2+10cx^4}=a\times$

$10c^4x$

$\overline{10c^4x+20c^2x^3+2x^5}$; and conſequently $x^4+\frac{c-a}{5c-a}\times 10c^2x^2=\frac{5a-c}{5c-a}\times c^4$: Whence x will be found, and, from thence, all the other quantities ſought.

IX. QUESTION 429 *anſwered by Mr.* W. Spicer.

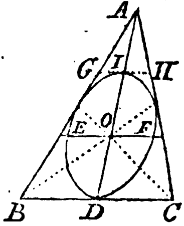
Let A be the port ſailed from, and B and C the ports to which the two ſhips are bound. Then, in the right-angled ſpherical triangle ABC, we have given the baſe BC, and the ratio of AB to AC, as 5 to 3: Therefore, putting $a=$ coſine BC (33° 20′) and $x=$ coſine $\frac{1}{5}$ AC (or $\frac{1}{5}$ AB) we ſhall (by a well known theorem) have $4x^3-3x=$ coſine of AC, and $16x^5-20x^3+5x=$ coſine of AB; and therefore (per ſpherics) 1 (rad.) : a :: $4x^3-3x : 16x^5-20x^3+5x$: and conſequently $16x^4-20x^2+5=4ax^2-3a$; which ſolved, gives $x=\sqrt{\frac{5+a+\sqrt{24+\overline{1-a}|^2}}{8}}=0{\cdot}990091$; anſwering to an arch of 8° 4′ 21″. Therefore $AC=24°\ 13'\ 3''$, or 1453·05 miles; and $AB=40°\ 21'\ 45''$, or 2421·75 miles.



Mr. *O'Cavanah*, *Penovius*, and Mr. *W. Stoker*, anſwer it exactly in the ſame manner.

X. QUESTION 430 *anſwered by* Penovius.

Draw the tangent GH, and the conjugate diameter EF, parallel to BC. Put $BD=DC=20=b$, $AD=51{\cdot}478=a$, the ſine of the angle at D (74° 30′ 30″) $=s$, and $DI=x$; then will $AI=a-x$, $GI=IH=\frac{b}{a}\times\overline{a-x}$, by ſimilar triangles; and, by the property of the ellipſis, $EO=OF=b\sqrt{\frac{a-x}{a}}$; whence $bx\sqrt{\frac{a-x}{a}}$ is a maximum; in fluxions $2ax\dot{x}-3x^2\dot{x}=0$, and $x=\frac{2a}{3}$; whence $EF=\frac{2b}{\sqrt{3}}$, and the area of the ellipſis $=\frac{3{\cdot}1416\,sab}{3\sqrt{3}}=599{\cdot}8582$.



Now put the tranſverſe axis $= y$, and the conjugate $= z$; then, by conics, $yz = \frac{4ab}{3\sqrt{3}} = 2n$, and $yy + zz = \frac{4aa}{9} + \frac{4bb}{3} = 4m$; from which $y + z = 2\sqrt{m+n}$, and $y - z = 2\sqrt{m-n}$; whence $\sqrt{m+n} \pm \sqrt{m-n} = \left\{ \begin{matrix} y = 35\cdot229236 \\ z = 21\cdot67976 \end{matrix} \right\}$ the principal diameters required.

COROLLARY 1. All the ſidas of the triangle are biſected by the points of contact; and the center of the ellipſis coincides with the center of gravity of the triangle.

COROLLARY 2. The area $\triangle$: the area of the ellipſe :: $3\sqrt{3} : 3\cdot1416$. Hence, the areas of all triangles circumſcribing the ſame ellipſis, having their ſides biſected by the points of contact, are equal: When $s = 1$, the triangle is iſoſceles; and equilateral, when the ellipſis becomes a circle.

Mr. *W. Kingſton* ſolves it in a manner very little different.

Mr. *O'Cavanah*, after demonſtrating geometrically, that the leaſt triangle that can be deſcribed about any oval figure, will have all its ſides biſected by the points of contact, derives, by a different method, the very ſame concluſions above exhibited; and farther adds, by way of note, 'That the greateſt parabola that can be deſcribed in a given 'triangle, may be determined by the ſame method; the area 'thereof being to that of the triangle, as $\sqrt{3}$ is to 2; which 'proportion is general, whatever the ſpecies of the triangle 'is, or which ever of its ſides the baſe of the parabola is 'ſuppoſed to ſtand upon.' 'Thoſe perſons (continues he) 'are therefore miſtaken, who make the area to be greater 'or leſs, according as the baſe is made to coincide with this, 'or that ſide of the triangle.——The axis of the parabola 'will not be perpendicular to the baſe of the triangle, but 'parallel to a line drawn from the vertex to biſect the baſe.'

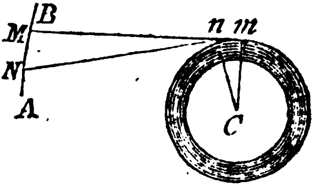
XI. QUESTION 431 *anſwered by the Propoſer, Mr.* Bevil.

Let $a =$ diameter of the poſt, $b =$ twice the thickneſs of the binding, $n =$ number of rounds, $f = a + nb$, and $c = 3\cdot14159$ &c. then will f, $f - b$, $f - 2b$, $f - 3b$, &c. be the diameter of the poſt and binding, after 0, 1, 2, 3, &c. rounds are diſengaged; and cf, $c \times \overline{2f - b}$, $c \times \overline{3f - 3b}$, &c. will be the length diſengaged in 1, 2, 3, &c. rounds reſpectively. But, if d be taken to denote the diameter, and s the length of the part diſengaged, at the end of any number (r) of rounds; then (the length unwrapped at the end of the next round after, being $= s + cd$) it appears (by p. 163 of Simpſon's Fluxions) that the diſtance moved over in unwrapping that round, will be $= \frac{\overline{s+cd}^2 - ss}{d} = 2cs + ccd$. Now, when $r = 0$, then $d = f$, and $s = 0$; therefore, in the firſt round, $2cs + ccd = ccf$. When $r = 1$, then $d = f - b$, $s = cf$, and conſequently $2cs + ccd = 2ccf + ccf - ccb = cc \times \overline{3f - b}$. When $r = 2$, then $d = \overline{f - 2b}$, $s = c \times \overline{2f - b}$; and therefore $2cs + ccd = cc \times \overline{5f - 4b}$: In the ſame manner, when $r = 3$, then $2cs + ccd = cc \times \overline{7f - 9b}$, &c. Now, by collecting all theſe values of $2cs + ccd$ together, we get the two following ſeries, viz. $ccf \times : 1 + 3 + 5 + 7 +$ &c. and $- ccb \times : 0 + 1 + 4 + 9$, &c. where each ſeries is to be continued to n terms: Theſe ſerieſes being ſummed, we have $ccf \times nn - ccb \times \frac{1}{6}n.\overline{n-1}.\overline{2n-1} = ncc \times \overline{nf - \frac{1}{6}b.\overline{n-1}.\overline{2n-1}} =$ 26 M. 1 F. = the diſtance ſought.

In this manner the ſolution is alſo given by Mr. *R. Butler*.

The ſame anſwered by Mr. H. Watſon.

Let $a =$ the ſemi-diameter of the poſt, $b =$ the thickneſs of the binding wrapped thereon, $n =$ the whole number of rounds at firſt, $x =$ the number of rounds unwrapped at the end of any time, and $u =$ the length of the part diſengaged at that time: And, ſuppoſing N and M to be two poſitions of the end of the line indefinitely near to each other, to the points of contact n and m, from the center C, let Cn and Cm be drawn.



It is evident, that $a + nb$ will be the ſemi-diameter of the poſt and binding together, and conſequently that the length of the firſt round (computed at the middle of the binding's thickneſs) will be $= p \times \overline{a + nb - \frac{1}{2}b}$ (p being put $= 2 \times 3{\cdot}1416$). In the ſame manner the length of the laſt of x rounds will be $= p \times \overline{a + nb - \overline{x - 1}.b}$. Therefore we have here the firſt and laſt terms of an arithmetical progreſſion, whereof the number of terms is x: whereby the ſum of the progreſſion is found $= p \times \overline{2a + \overline{2n - x}.b} \times \frac{x}{2} = u$.

Now, ſince the angles *NnC* and *Mmc* are both right ones, the angle *NmM* will be $=$ *nCm*; and therefore the figures *NmM* and *nCm* being ſimilar, we have as *Cn* $(a+\overline{n-\dot{x}}.b) : nm\,(\dot{u}) :: Nm\,(u) : NM = \frac{u\dot{u}}{a+n-x.b}$: But $\dot{u} = p\dot{x} \times \overline{a+\overline{n-x}.b}$; and conſequently $NM = p\dot{x} \times u = pp \times \overline{a+n}.x\dot{x} - \frac{1}{2}bx^2\dot{x}$; whoſe fluent $(pp \times \overline{a+nb}.\frac{1}{2}xx - \frac{1}{6}bx^3)$ when $x = n$, will be $= nnpp \times \overline{\frac{1}{2}a+\frac{1}{3}nb} = 1654320$ inches ($=$ 26 M. 1 F. nearly) $=$ the diſtance ſought.

N. B. If the curve *AMB* be conſidered as the involute of *Archimedes*'s (or any other kind of) ſpiral, the length thereof will be always found, by multiplying *Nn* and its fluxion together, dividing the product by the radius of the curvature of the evolute (at *n*), and then taking the fluent of the quotient.

XII. QUESTION 432 *anſwered by Mr.* O'Cavanah.

By diviſion, and extracting the ſquare root on both ſides of the propoſed equation, we have $\sqrt{\frac{q}{n}} \times \frac{\dot{x}}{x\sqrt{\frac{mx}{n}-1}} = \frac{1}{p} \times \frac{\dot{z}}{1+\frac{zz}{p}}$; which, by ſubſtituting $u = \frac{z}{\sqrt{p}}$, and $y = \sqrt{\frac{mx}{n}-1}$, is further transformed to $\frac{\dot{y}}{1+yy} = \frac{1}{2}\sqrt{\frac{n}{pq}} \times \frac{\dot{u}}{1+uu}$: whence, ſuppoſing *A* and *B* to denote the two circular arcs whoſe tangents are $y\,(\sqrt{\frac{mx}{n}-1})$ and $u\,(\frac{z}{\sqrt{p}})$, we ſhall, by taking the fluent, have $A + C = \frac{1}{2}B\sqrt{\frac{n}{pq}}$; *C* being a conſtant arch, ſerving to correct the fluent. From this equation, when either of the quantities *z* or *x* is given, the other may be determined (from a table of tangents) in all caſes, except when imaginary quantities enter into the conſideration: Thus, *z* being ſuppoſed given, the tangent $u\,(\frac{z}{\sqrt{p}})$ will alſo be given; and from thence (by the table)

table) the arch B correſponding; whereby the arch A $(=\frac{1}{2}B\sqrt{\frac{n}{pq}}-C)$, and its tangent y, will be known, and conſequently x $(=\frac{n}{m}\times\overline{1+yy})$.

When n, p, and q are ſuch, that $\frac{1}{2}\sqrt{\frac{n}{pq}}$ is a rational quantity, then may the relation of x and z be algebraically expreſſed by a finite equation: For, here the fraction $\frac{r}{s}$, where r and s are integers, may be aſſumed $=\frac{1}{2}\sqrt{\frac{n}{pq}}$; and we ſhall have $s\times\overline{A+C}=rB$: But, if c be taken to denote the tangent of the arch C, that of $A+C$ will be $=\frac{c+y}{1-cy}$. Whence (by the theorem in the ſolution of Q. 388) the tangent of $s\times\overline{A+C}$ will be had $=$ $s\cdot\frac{c+y}{1-cy}-\frac{s}{1}\cdot\frac{s-1}{2}\cdot\frac{s-2}{3}\cdot\overline{\frac{c+y}{1-cy}}\big|^{3}+\&c.\div$ $1-\frac{s}{1}\cdot\frac{s-1}{2}\cdot\overline{\frac{c+y}{1-cy}}\big|^{2}+\&c.$ which muſt neceſſarily be equal to the tangent of $rB=ru-\frac{r}{1}\cdot\frac{r-1}{2}\cdot\frac{r-2}{3}\cdot u^{3}\ \&c.$ $\div 1-\frac{r}{1}\cdot\frac{r-1}{2}\cdot u^{2}\ \&c.$ where r and s being integers, the querieſes will terminate, and the relation of y $(=\sqrt{\frac{mx}{n}-1})$ and u $(\frac{z}{\sqrt{p}})$ will therefore be expreſſed in finite terms.

Mr. *Rollinſon* (the propoſer) anſwers this queſtion, in almoſt the very ſame manner. *Peter Walton* has brought out an elegant ſolution to it, from p. 66 of Landen's Mathematical Lucubrations; and, in a way not greatly different, the ſolution is alſo given by Mr. *R. Butler*, and Mr. *L. Charlton*.

XIII. QUESTION 433 *anſwered by* Peter Walton.

Let A denote the force which, acting at the end n, at right angles to the rod, would accelerate the velocity of that end about (G) the center of gravity of the rod, at the ſame rate as the ſaid velocity is accelerated by the action of the rod againſt AB; and let A^{a} denote the force which, acting at the end m, at right angles to the rod, would retard the velocity of that end about G, at the ſame rate as the ſaid velocity is retarded by the action of the rod againſt AC: Alſo let u be the velocity of m, or n, about G; v, the velocity of G towards the horizon; w, its velocity in a direction parallel to the horizon; t, the time the rod has been in motion, while it touches AC; T, the time from the rod's ceaſing to touch AC; d, the diſtance of G from AB, at the commencement of the motion; x, the diſtance of G from AB, at any time after the motion has commenced; $2a$, the length of the rod; $s = 16\frac{1}{12}$ feet; and $y = \sqrt{aa - xx}$.

Then will the abſolute weight of the rod, the preſſure againſt AB, and the preſſure againſt AC, be as $2s$, $\frac{aA'}{3y}$, and $\frac{aA''}{3x}$ reſpectively: And $\frac{aA''}{3x} \times \frac{-\dot{x}}{v}$ will be $= \dot{w}$, $\overline{A' - A''} \times \frac{-\dot{x}}{v} = \dot{u}$, $\overline{2s - \frac{aA'}{3y}} \times \frac{-\dot{x}}{v} = \dot{v}$, $av = uy$, and $aw = ux$.

By means of which equations, we find $3w\dot{w} + u\dot{u} = -6s\dot{x} - 3v\dot{v}$, while the rod continues to touch AC.

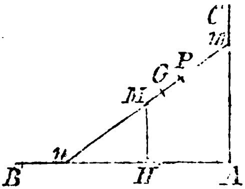
Hence, by taking the correct fluents and expunging u and w, we have $v = \frac{\sqrt{\overline{3ds - 3sx}} \times y}{a} = \frac{\sqrt{\overline{3ds - 3sx}} \times \sqrt{\overline{aa - xx}}}{a}$.

Therefore w is $= \frac{\sqrt{\overline{3ds - 3sx}} \times x}{a}$; whoſe fluxion is $= 0$, and conſequently $x = \frac{2d}{3}$, when $A'' = 0$, i. e. when the rod ceaſes to touch AC. Moreover $\dot{t}$ is $= \frac{-a\dot{x}}{\sqrt{\overline{3ds - 3sx}} \times \sqrt{\overline{aa - xx}}}$; whoſe correct fluent, when x is $\frac{2d}{3}$, is the time required in the queſtion.

After the rod has ceaſed to touch AC, (A'' being $= 0$, and w invariable) we ſhall have $A' \times \frac{-\dot{x}}{v} = \dot{u}$, $\overline{2s - \frac{a}{3y}} \times \frac{-\dot{x}}{v} = \dot{v}$, and $av = uy$. From which equation we get $u\dot{u} = -6s\dot{x} - 3v\dot{v}$: Hence, by taking the correct fluents expunging u, and putting $d - \frac{d^3}{9aa} = h$, we find $v =$

$\frac{2y\sqrt{3sb-3sx}}{\sqrt{aa+3yy}} = \frac{2\sqrt{aa-xx}\times\sqrt{3sb-3sx}}{\sqrt{4aa-3xx}}$. Conſequently $\dot{T}$ is $= \frac{\dot{x}\sqrt{4aa-3xx}}{2\sqrt{aa-xx}\times\sqrt{3sb-3sx}}$; whereby the time from the rod's ceaſing to touch *AC*, to its coincidence with *AB*, will be found; from whence and the given (invariable) velocity $\frac{2d\sqrt{ds}}{3a}$, of the center of gravity from the plane *AC*, during that time, the required diſtance of the end of the rod from *AC* will be obtained.

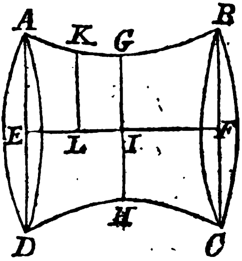
Mr. *O'Cavanah* has alſo obliged us with a ſolution to this very difficult problem, which agrees in every particular with that exhibited above. This gentleman, at the end of his ſolution, ſubjoins the following remark: ‘If the figure of the rod ſhould be ſuch that the center of gravity (G) is not in the middle-point M; then, putting $h = nG$, and k $(= nP)$ = the diſtance of the center of oſcillation (P) from the end n (conſidered as the point of ſuſpenſion), it will appear, in like manner, that the celerity with which the ſaid end recedes from the plane *AC*, will be $2y\sqrt{\frac{hf\times\overline{c-y}}{\frac{1}{4}dhk-\overline{h-d}\times yy}}$.* When this quantity is a maximum, the end (m) will quit the plane *AC*; after which the relative celerity wherewith n recedes from *MH*, will be $= 2y\times\sqrt{\frac{df\times\overline{r-y}}{ddk-hyy}}$: wherein r is a conſtant quantity, to be determined (like m above) from the value of y, when the end m ceaſes to to touch the plane *AC*.’



* Note, That, in this author's ſolution, $d = Mn$, $y = MH$, $c =$ the firſt value of y, and $f = 16\frac{1}{12}$ feet.

PRIZE

PRIZE QUESTION *anſwered by Mr.* E. Rollinſon.



Let $ABCD$ repreſent the caſk or veſſel; put the given ſemi-length $EI = c$, and the given ſemi-diameter (AE) at the end $= b$; put alſo $IL = x$, and $LK = y$, ſuppoſing LK to be variable, and $\parallel IG$: Then, the fluxion of the curve ſurface being $3{\cdot}1416 \times 2y\sqrt{\dot{y}\dot{y} + \dot{x}\dot{x}}$, it follows, from the conditions of the problem, that the fluent of $\sqrt{\dot{y}\dot{y} + \dot{x}\dot{x}}$ ought to be a minimum, when that of $\dot{x}$ becomes equal to the given quantity IE. Hence, by the general rule for the reſolution of iſoperimetrical problems (vid. p. 101 of Simpſon's Miſcellaneous Tracts) it appears that $\frac{y\dot{x}}{\sqrt{\dot{y}\dot{y} + \dot{x}\dot{x}}} - a = 0$; whence $\dot{x}$ is found $= \frac{a\dot{y}}{\sqrt{yy - aa}}$; and conſequently $x = a \times$ hyp. log. $\frac{y + \sqrt{yy - aa}}{a}$, anſwering to the catenaria; in which $a = IG$ (becauſe when $y = a$, x will be $= 0$).

The equation of the curve being thus known, the fluxion of the generated ſolid, $3{\cdot}1416 \times yy\dot{x}$, will be found $= 3{\cdot}1416a \times \frac{y^2\dot{y}}{\sqrt{yy - aa}}$, and conſequently the ſolid itſelf $= 3{\cdot}1416a \times \frac{1}{2}y\sqrt{yy - aa} + \frac{1}{2}ax$; the double of which, when $x = c$, and $y = b$, will be the whole content of the veſſel.

But now to find a (which is yet unknown), the general equation, when $y = b$, and $x = c$, will give $c = a \times$ hyp. log. $\frac{b + \sqrt{bb - aa}}{a}$, or $\frac{c}{a} =$ H.L. $\frac{b}{a} + \sqrt{\frac{bb}{aa} - 1}$; which, by putting $\frac{b}{a} = v$, will become hyp. log. $v + \sqrt{vv - 1} = \frac{cv}{b} = {\cdot}5v$ (in the preſent caſe); whence v is found $= 1{\cdot}1788$; and from thence $a = 31{\cdot}388$. Hence the content will come out 16121 cubic inches, or 69·79 wine gallons. And the curve ſurface will be leſs than that of the circumſcribing cylinder, in the proportion of 95·54 to 100.

The ſolution by Mr. *Rollinſon*, here given, being the only true one that came to hand before Candlemas-day, that gentleman is therefore intitled to a prize of 12 diaries, without the chance of drawing lots.

Of the Eclipſes in the Year 1758.

There will this year be ſix eclipſes, four of the ſun, and two of the moon; whereof one of the moon only will be viſible to the inhabitants of Great Britain.

The 1ſt is of the ſun, January 9th, at 6 in the evening, inviſible.

The 2d is a viſible, and total eclipſe of the moon, on January 24th, ſome calculations whereof are as follow:

Calculated by	For	Beg. of Eclip.	Beg. total Dark.	Mid. of Eclip.	End total Dark.	End of Eclip.	Digits eclipſed
		h. m.	h. m.	h. m.	h. m.	h. m.	° ′
Mr. S. Towndrow,	London	4 22	5 28	6 21	7 14	8 20	21 25
	Cheſterfield Derbyſhire	4 13	5 19	6 12	7 05	8 11	
Mr. W. Terrill,	London	4 21	5 27	6 20	7 13	8 19	21 30
	Rudruth, in Cornwall	4 1	5 7	6 0	6 53	7 59	
Mr. J. Gutteridge,	London	4 24	5 30	6 23	7 16	8 22	21 32
	Loughboro'	4 19	5 25	6 18	7 11	8 17	

The 3d is of the ſun, February 8th, between 4 and 5 in the morning, inviſible.

The 4th is of the ſun, July 4th, at 9 in the morning, inviſible.

The 5th is of the moon, July 20th, at 5 in the afternoon, inviſible.

The ſixth is of the ſun, December 30th, between 7 and 8 in the morning, inviſible.

⁂ Mr. *Charles Brent* obſerves (very juſtly) that thoſe authors are miſtaken who affirm, That there can never happen more than ſix eclipſes in one year. Beſides thoſe mentioned above, he takes notice of a ſmall ſolar defect of about half a digit, on Auguſt 3d, between 10 and 11 at night; which no other author has adverted to.—But whether the errors of the beſt tables extant, may not be ſufficient to render the happening of *ſuch* an eclipſe doubtful, we do not undertake to decide.

New Queſtions.

I. QUESTION 434, *by Miſs* T. S—e.

Addreſſed to Mr. U. T——r, *who took the Liberty to aſk her the following Queſtions, viz. What age? What fortune? And what height ſhe was?*

My height, ſir, in inches, is three times my years;
My fortune their ſquares will both ſhew;
Put all theſe together, there then, ſir, appears
The number expos'd to your view.* (* 4494.
From which, ſir, determine the things you requir'd;
And then, if more favours you want,
As lovers of ſcience I always admir'd,
Thoſe favours, perhaps, I may grant.

II. QUESTION 435, *by Mr.* P. O'Cavanah.

A tortoiſe once (or Æſop lies)
Run with a hare, and won the prize;
The hare, in the firſt minute's ſpace,
Four furlongs run o'th' deſtin'd race,
While far behind her foe crept on,
And only crawl'd the fourth of one:
Puſs, looking back, obſerves the caſe,
Diſdains her foe, and bates her pace:
In the next minute ſhe run o'er
But half the ground ſhe run before;
And in the third was only reckon'd,
One half the ſpace ſhe went the ſecond;
Decreaſing ſtill (as artiſts call)
In ratio geometrical:
Mean time the tortoiſe ſtill crept on,
At the ſame rate as he began:
Now deign to ſhew, ye learned fair,
In what time he o'ertook the hare.

III. QUESTION 436, *by Mr.* John Brickland, *Teacher of the Mathematics in* Oxford.

What is the ſide of that equilateral triangle, whoſe area coſt as much paving at 8 d. a foot, as the palliſadoing the three ſides did at a guinea a yard?

IV. QUESTION 437, *by Mr.* Thomas Moſs.

From the vertical angle of a triangle (whoſe baſe is 70, and its two ſides 40 and 50) to draw a line, terminating in the baſe, ſo as to be a mean proportional between the two ſegments of the baſe made thereby.

V. QUESTION 438, *by Mr.* Peter Walton.

To place three circles (whoſe radii are 10, 15, and 20) ſo that three right lines may be drawn each, to touch all the three circles.

VI. QUESTION 439, *by Mr.* W. Bevil.

The baſe of an iſoſceles triangle being given = 100, and each of the equal ſides = 60; ſo to draw a right line from the vertex to the baſe, that the ſolid (or continual product) under it and the two ſegments of the baſe ſhall be the greateſt poſſible.

VII. QUESTION 440, *by Mr.* J. Fellows, *of his Majeſty's Ship* Captain.

Under the line (or very near)
NE, SW, two ports do bear,
Whoſe diſtance, if I rightly gueſs,
Is fifty leagues, no more nor leſs;
Between which ports a current goes,
SE, three knots, as I ſuppoſe:
The quere is, how muſt I ſail
From northern port, in five-knot gale,
That I the ſouthern port may make,
In the leaſt time the thing can take?
And what that time will be, declare:
I'll do as much for you next year.

VIII. QUESTION 441, *by Mr.* Lionel Charlton.

It being a common practice among ſhip-chandlers to make their log-lines 42 feet long, and their half-minute glaſſes 28 vibrations of a pendulum, whoſe length is 38 inches and an half: Quere, how far a ſhip's reckoning will vary from the truth, that uſes only theſe, and ſails by mercator on one rhumb from the Lizard to the Capes of Virginia?

IX. QUESTION 442, *by Mr.* J. Wilſon.

$$\text{Given}\left\{\begin{array}{l} x^2z+z^2x=a \\ x^2z^7+z^2z^7=b \end{array}\right\}$$
To find x and z, without having the root of any adfected equation to extract higher than a quadratic.

X. QUES-

X. Question 443, *by Mr.* H. Watſon.

From the equation $44000xx+1=zz$, to find both x and z in whole numbers.

XI. Question 444, *by* Fr. Bell.

A gentleman having in his garden an elliptical fountain, whoſe greater diameter is 30, and the leſſer 24 feet, orders a free-ſtone walk to be made round it, to be every where of an equal breadth, and to take up juſt the ſame quantity of ground as the fountain itſelf: Quere, what muſt the breadth of the walk be?

XII. Question 445, *by* Plus-Minos.

One morning laſt ſummer, being in a gentleman's garden, I ſaw a very good horizontal ſun-dial, which was not fixed down, but neatly let into the poſt on which it ſtood: Taking hold of the gnomon, I pulled it towards me, thereby elevating the ſouth ſide of the dial, ſtill keeping the gnomon in the plane of the meridian: Upon this, the ſhadow came forward from the hour-line of 7, till it marked 7h. 40m. and no nearer to the meridian, or 12 o'clock, would it come; for, if the dial was farther elevated, it turned back again: Now the day of the month (which I have forgot, and for private reaſons ſhould be glad to recollect) is here required.

XIII. Question 446, *by Mr.* P. O'Cavanah, *of* Dublin.

How muſt I incloſe an acre of land into a garden, with a fence of 80 poles in circumference, ſo as to be able to form therein the longeſt (ſtraight) walk poſſible, and what will that length be, ſuppoſing the breadth of the walk to be 10 feet?

XIV. Quest. 447, *by* Peter Walton, Diſcip. Landenii.

A chain, 10 yards long, conſiſting of indefinitely ſmall equal links, being laid ſtraight on an horizontal (perfectly poliſhed) plane, except one part, a yard in length, which hangs down perpendicularly below the plane: In what time will the ſaid chain (drawn by the gravity of the deſcending part) entirely quit the plane?

Prize Question, *by Mr.* G. Witchell.

To determine the neareſt diſtance of the orbit of the expected comet from that of the earth, together with the longitude of the earth and comet in that ſituation, ſuppoſing, 1. That the orbit of the earth is a circle. 2. That the trajectory of the comet is a parabola, whoſe focus is the center of the earth's orbit. 3. That the perihelion diſtance of the comet is to the radius of the circle, as 0'5868 to 1. 4. That the inclination of the planes is 17° 2′, the place of the perihelion ♒ 3° 30′, and that of the deſcending node ♍ 22° 13′.

1759.

Queſtions anſwered.

I. QUESTION 434 *anſwered by Mr.* Tho. Baker, *to Miſs* T. S——e, *the Propoſer.*

YOUR age, dear Miſs, is twenty-one, your height is five
feet three;
Forty-four hundred pounds and ten, will juſt your fortune be.

Algebraic Solution.

Let x repreſent the lady's age, then her height (in inches) will be $3x$, and her fortune (in pounds) $= 10xx$, by the conditions of the queſtion; from whence we have alſo given $10xx + 3x + x = 4494$; therefore $xx + 0'4x = 449'4$, and conſequently $x = \surd 449'44 - 2 = 21$. Hence the lady's age appears to be 21, her height five feet three inches, and her fortune 4410 pounds.

In this manner the anſwer is given by Mr. *W. Aſhton*, Mr *G.* and *J. Atkinſon*, Mr. *Bamfield*, Mr. *Barras*, Mr. *J. Brickland*, Mr. *Cha. Cave*, Mr. *J.* and *W. Chapman*, Mr. *T. Corbett*, Mr. *N. Cory*, Mr. *T. French*, Mr. *E. Hare*, Mr. *D. Haſtings*, Mr. *R. Hodges*, Mr. *T. Jeffery*, Mr. *Sam. Koit*, Mr. *G. Langley*, Mr. *J. Lewin*, Mr. *B. Lydal*, *Little Conjurer*, Mr. *R. Marſh*, *Mathematicus*, Mr. *G. Nokes*, Mr. *R. Quant*, Mr. *Pet. Rogers*, Mr. *W. Swift*, Mr. *J. Thorne*, Mr. *Joſ. Waſtel*, Mr. *J. Writer*, and by a multitude of others.

II. QUESTION 435 *anſwered by Mr.* B. Lydal.

Since the ſpaces run by the hare, in the ſeveral ſucceeding minutes, are expreſſed by 4, 2, 1, ½, ¼, &c. reſpectively, it is plain, from addition only, that the whole ſpace gone over from the beginning, will be always leſs than 8 furlongs by a ſpace equal to that paſſed over in the laſt minute. Therefore ſince the ſpace paſſed over in the laſt minute (after a continual decreaſe of one half every minute) muſt be extremely ſmall, the whole ſpace gone over by the hare, when the tortoiſe overtakes her, will amount to 8 furlongs, extremely near; which, at the rate of $\frac{1}{4}$ of a furlong in a minute, will take the tortoiſe 32 minutes to crawl over, in order to come up with the hare.

The ſame anſwered by Mr. W. Kingſton, *of* Bath.

If the number of minutes be denoted by x, the ſpace gone over by the tortoiſe will be $= \frac{x}{4}$, and that by the hare $= 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4}$ &c. $= 8 \times \overline{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}}$ &c. continued to x terms, whereof the laſt is evidently $= 8 \times \frac{1}{2^x} \left(= \frac{8}{2^x}\right)$. Whence the ſum of the whole progreſſion (being = the double of the firſt term *minus* the laſt, in all caſes where the ratio is $\frac{1}{2}$) is had $= 8 - \frac{8}{2^x}$. Hence we have $8 - \frac{8}{2^x} = \frac{x}{4}$, or $32 - \frac{1}{2^{x-5}} = x$; where, if $\frac{1}{2^{x-5}}$ be rejected on account of its extreme ſmallneſs, we ſhall have $x = 32$. To correct this value, let $\frac{1}{2^{x-5}} = \frac{1}{134217728}$, be now ſubtracted from 32; whence $x = 31\frac{134217727}{134217728}$.

The ſame anſwered by Penovius.

Suppoſing that the hare moved uniformly in each minute, the diſtance run by her was $= 16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{2})^2 + \frac{1}{2})^3 + \&c. \ldots + \frac{1}{2})^{27} \times x = 31 + \frac{67108863}{67108864} + \frac{x}{134217728}$, and by the tortoiſe $= 1 + 1 + 1 + \&c. \ldots + x = 31 + x$: Therefore $x = \frac{67108863}{67108864} + \frac{x}{134217728}$; whence $x = \frac{134217726}{134217727}$, and therefore $31\frac{134217726}{134217727}$ minutes is the true time required.

According to ſome one or other of the three methods above given, the problem is alſo anſwered by Meſſ. *T.* and *W. Allen*, *W. Aſhton*, *W. Bamfield*, *T. Barker*, *E. Barras*, *J. Beresford*, *T. Boſworth*, *Turner Boſton*, *J. Buddles*, *T. Corbett*, *T. Crabtree*, *E. Ellis*, *R. Gibbons*, *H. Green*, *J. Hampſon*, *T.* and *W. Harriſon*, *Joſ. Harrington*, *D. Haſtings*, *G. Hicks*, *Malachi Hitchins*, *J. Honey*, *Abr. Horsfall*, *T. Hopkinſon*, *T. Howe*, *Steph. King*, *R. Langley*, *W. Matthewſon*, *Chr. Meſban*, *S. Pedley*, *R. Pitches*, *W. Fenn*, *Alex. Rowe*, *J. Scott*, *W. Spicer*, *G. Stapley*, *W. Swift*, *J. Thompſon*, *J. Thorne*, *Ja. Vicary*, *Matt. Ward*, *Joſ. Waſtel*, *R. Walton*, *T. Wilkin*, *J. Wilſon*, and a great many others.

III. QUESTION 436 *anſwered*.

If the length, in feet, of each ſide of the triangular garden be denoted by x, then the palliſadoing of the three ſides, at 21 s. per yard, will amount to 21 x ſhillings: Moreover, the perpendicular being $= \sqrt{xx - \frac{xx}{4}} = \frac{x\sqrt{3}}{2}$, the area of the garden, in ſquare feet, will be $= \frac{xx\sqrt{3}}{4}$; whereof the paving, at $\frac{2}{3}$ of a ſhilling per foot, will come to $\frac{xx\sqrt{3}}{6}$ ſhillings. Hence, by the queſtion, we have $\frac{xx\sqrt{3}}{6} = 21x$; and therefore $x = \frac{126}{\sqrt{3}} = 42\sqrt{3} = 72{\cdot}74615$, the required length of each ſide.*

Thus the problem is reſolved by Mr. *J. Brickland* (the propoſer), Mr. *E. Barras*, Mr. *Turner Boſton*, Mr. *J. Buddles*, Mr. *F. Butcher*, Mr. *C. Cave*, Mr. *W. Cooke*, Mr. *T. Crabtree*, Mr. *W. Harriſon*, Mr. *Joſ. Harrington*, Mr. *Malachi Hitchins*, Mr. *E. Houlſton*, Mr. *T. Jeffery*, *Little Conjurer*, Mr. *W. Matthewſon*, Mr. *Chr. Meſban*, Mr. *S. Pedley*, Mr. *W. Penn*, Mr. *R. Quant*, Mr. *Alex. Rowe*, Mr. *Oliver Shakeſpear*, Mr. *G. Stapley*, Mr. *R. Walton*, Mr. *Matt. Ward*, Mr. *Joſ. Waſtel*, Mr. *T. Wilkin*, and by upwards of 40 other perſons.

IV. QUES-

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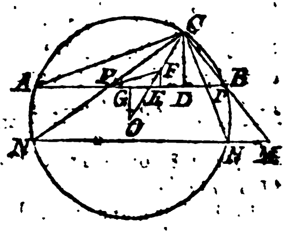
* See this Queſtion ſolved without Algebra p. 118 Hutton's Menſuration.

IV. QUESTION 437 *anſwered by* Mr. Rob. Shirtcliffe; *Author of the* Theory and Practice of Gauging.



About the given triangle *ABC* let a circle be circumſcribed, and in the ſide *CB*, produced, take *BM* = *CB*, drawing *MNN* parallel to *AB*, interſecting the circle in *N*, *N*; then draw *CN*, cutting *AB* in *P*; ſo ſhall *CP* be a mean proportional between *AP* and *BP*. For, *MN* being parallel to *AB* (by conſtruct.) and *BM* = *BC*, it is evident that *PN* is alſo = *PC* (Euc. 2. 6.); and conſequently $CP^2 = CP \times PN = AP \times BP$ (Euc. 35. 3.) or $AP : CP :: CP : BP$ (Euc. 16. 6.) *Q.E.D.*

Thus the problem is alſo conſtructed by Mr. *Moſs* (the propoſer), and Mr. *W. Spicer*.

The ſame conſtructed, and anſwered otherwiſe, by Mr. W. Davies.

Let a circle be deſcribed about the given $\triangle\, ABC$; and, having biſected the radius *OC* in *F*, from this point to *AB* apply *FP* = *FC* (or *FO*); then will *CP*, when drawn, be the required mean proportional between *AP* and *BP*.

CALCULATION. There being given $BC = 40$, $AC = 50$, and $AG = BG = 35$, the perpendicular *CD* will be found $= \frac{80\sqrt{6}}{7}$, $GD = \frac{45}{7}$, and $GE = ED = \frac{45}{14}$: Then $\frac{AC \times BC}{2CD} = OC = \frac{175}{2\sqrt{6}}$, $\frac{1}{2}CD - \frac{1}{2}OG = FE = \frac{40\sqrt{6}}{7} - \frac{35}{4\sqrt{6}}$, and $\overline{FP^2 - FE^2}\big|^{\frac{1}{2}} = EP = \frac{5\sqrt{1649}}{14} = 14{\cdot}5$: Whence $AP = 23{\cdot}712$, and $BP = 46{\cdot}288$.

In this manner the ſolution is alſo given by Mr. *Rob. Butler*, and Mr. *Henry Watſon*.

An

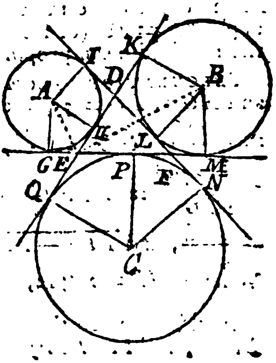
An Algebraical Solution to the ſame by Mr. Ed. Johnſon.

All the ſides of the triangle being given, the ſegments of the baſe are readily obtained, viz. $AD = 41\frac{3}{7}$, and $BD = 28\frac{4}{7}$. Putting therefore $AC = a$, $AD = b$, $BD = c$, and $DP = x$, we ſhall have $AP = b - x$, and $BP = c + x$; whence CP^2 (= $AP \times BP$ per queſt.) $= \overline{b - x} \times \overline{c + x}$: But CP^2 is alſo $= CD^2 + DP^2 = aa - bb + xx$; conſequently $aa - bb + xx = \overline{b - x} \times \overline{c + x}$; whence $xx - \frac{b - c}{2} \times x = \frac{bb + bc - aa}{2}$, and $x = \frac{b - c \pm \sqrt{\overline{3b + c}|^2 - 8aa}}{4}$ $= 17·71708$, or $- 11·28851$. From which $AP = 23·71148$, or $52·71708$.

In this laſt manner it is likewiſe anſwered by Meſſ. *T.* and *W. Allen*, *W. Aſhton*, *T. Barker*, *Ja. Beresford*, *J. Buddles*, *F. Butcher*, *Tim. Drury*, *Edward Ellis*, *G. Godhelp*, *J. Goodhead*, *J. Hampſon*, *D. Haſtings*, *G. Hicks*, *J. Honey*, *Abr. Horsfall*, *T. Hopkinſon*, *T. Howe*, *J.* and *R. Hudſon*, *Stephen King*, *R. Langley*, *J. Lightfoot*, *B. Lydal*, *R. Mallock*, *J. Milbourn*, *J. Pearce*, *R. Pitches*, *Joſ. Rennard*, *W. Spalton*, *J. Scott*, *Paul Sharp*, *J. Taylor*, *J. Vicary*, *Stephen Weſt*, *J. Wilſon*, *Geo. Witchell*, and ſome others.

V. QUESTION 438 *anſwered by Mr.* Henry Watſon.

Let GPM, QHK, and NLI be the three right lines which each circle is to touch, (forming by their interſections the $\triangle EDF$); and from the centers A, B, C, to the points of contact, let radii be drawn. Becauſe $EG = EH$, and $DI = DH$, it is evident that the ſum of the two (equal) tangents FG and FI is equal to the ſum of all the three ſides of the triangle EDF. And, by the ſame argument, the ſum of the tangents, $EM + EK$, or $DQ + DN$, muſt alſo be equal to the very ſame quantity, and conſequently all theſe tangents equal among themſelves. Now from $FG = EM$, let EF (common) to be taken away, and



there remains $EG = FM$; but $EH = EG$, and $FL = FM$; conſequently $EH = FL$. In the very ſame manner, $DH = PF$, and $DL = EP$.

Now, per ſim. fig. $\begin{cases} AH : EH :: CP : EP = \dfrac{EH \times CP}{AH} \\ BL : FL(EH) :: CP : FP = \dfrac{EH \times CP}{BL} \end{cases}$

Hence it appears, that the three ſides EF, ED, DF, of the triangle DEF, are equal to $\frac{EH \times CP}{AH} + \frac{EH \times CP}{BL}$, $EH + \frac{EH \times CP}{BL}$, and $EH + \frac{EH \times CP}{AH}$, reſpectively; which, therefore, are to each other in the given ratio of $CP + \frac{AH \times CP}{BL}$, $AH + \frac{AH \times CP}{BL}$, and $AH + CP$: From whence the problem may be very eaſily conſtructed, and the angles, &c. all found by common trigonometry.

The ſame anſwered algebraically by Mr. O'Cavanah.

This gentleman, after proving EG and FM to be equal, (as in the ſolution above given by Mr. *Watſon*) puts $AG = a$, $BM = b$, $CP = c$, and $EM = x$: Then (from the ſimilarity of the triangles EBN, AEG) it will be $EM\ (x) : BM\ (b) :: AG\ (a) : EG = \frac{ab}{x}$. Again (by ſim. fig.) $AG\ (a) : GE\ (\frac{ab}{x}) :: CP\ (c) : EP = \frac{bc}{x}$; and $BM\ (b) : FM\ (= EG = \frac{ab}{x}) :: CP\ (c) : FP = \frac{ac}{x}$. Therefore $\frac{bc}{x} + \frac{ac}{x} + \frac{ab}{x}$ ($= EP + FP + FM = EM$) $= x$; and conſequently $x = \sqrt{bc + ac + ab} = 25{\cdot}495098$: Whence every thing elſe is eaſily determined.

Meſſ. *L. Charlton*, *R. Pitches*, and *Penovius* have alſo given new and very neat algebraical ſolutions to this problem. The two latter have determined the diſtances of the centers A, B, C, in numbers, viz. $AB = 31{\cdot}774$, $AC = 34{\cdot}807$, and $BC = 27{\cdot}596$.

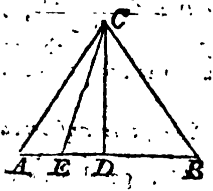
Penovius, making $m = \sqrt{ab + ac + bc}$ ($= EM$), proves alſo, that the ſides (DE, DF, EF) of the $\triangle\, DEF$ will be

$= a \times \frac{b+c}{m}$, $b \times \frac{a+c}{m}$, and $c \times \frac{a+b}{m}$, reſpectively; and the radius of the circle inſcribed therein $= \frac{abc}{ab+ac+bc}$.

Meſſ. *T. Allen*, *J. Hudſon*, and *E. Johnſon*, have likewiſe obliged us with the ſame elegant concluſions, from Ex. 19. p. 23. of Mr. Landen's Mathematical Lucubrations, to which they ſeverally refer.

VI. QUESTION 439 *anſwered by Meſſ.* J. *and* R. Hudſon.

Let CD be perpendicular to AB, and put AC $(= BC = 60) = a$, AD $(= BD = 50) = b$, and $x = CE$, the line required: Then $aa - bb = CD^2$, and $\sqrt{xx+bb-aa} = DE$ (Euc. 47. 1.): whence $AE = b - \sqrt{xx+bb-aa}$, and $BE = b + \sqrt{xx+bb-aa}$: Therefore $AE \times BE \times CE = \overline{aa - xx} \times x = a^2x - x^3$; which being a maximum, we have, in fluxions, $a^2\dot{x} - 3x^2\dot{x} = 0$; whence $x = \frac{a}{\sqrt{3}}$: Conſequently $AE = 40$, and $BE = 60$.

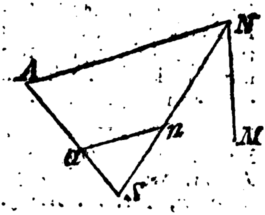


The ſame anſwered by Mr. T. Hopkinſon.

Making $CD \perp AB$, and putting $AC = a$, and $EC = x$, we have (by Euc. 5. 2.) $AE \times BE = AD^2 - ED^2 = AC^2 - EC^2 = aa - xx$; and conſequently $AE \times BE \times CE = aax - x^3$: Which, in fluxions, gives $aa\dot{x} - 3x^2\dot{x} = 0$; whence $x = \sqrt{\frac{aa}{3}}$; therefore DE $(= \sqrt{AD^2 - \frac{2}{3}AC^2})$ $= 10$, $AE = 40$, and $BE = 60$.

From this ſame ingenious method and ſubſtitution, the anſwer is likewiſe brought out by Mr. *T. Barker*, Mr. *E. Lydal*, Mr. *J. Milbourn*, Mr. *Paul Sharp*, Mr. *J. Wilſon*; Mr. *G. Witchell*, and a few others. Mr. *O'Cavanah* has given the conſtruction and demonſtration from geometrical principles independent of fluxions. But the ſcantineſs of our room will not allow us to inſert all that our moſt able and experienced correſpondents favour us with.

VII. QUESTION 440 *anſwered by Mr.* J. Milbourn.



Let *AS* be the given direction of the current at the ſouthermoſt of the two ports *N*, *S*: Take *Sa* = 3, (expreſſing the rate of the current's motion) and, from *a* to *SN*, apply *an* = 5, the rate of the ſhip's motion. Then draw *NA* parallel to *na*, which will give the courſe and diſtance required; becauſe, by the compoſition of motions, the ſhip will then really move in the right line *NS*, directly towards her port.

CALCULATION. In the right-angled triangle *Sna*, we have given $\frac{Sa}{na} = 0·6$ = nat. ſin. of *Sna* (or *SNA*) = 36° 52′, whence the angle of the courſe *ANM* = 81° 52′; alſo *NA* (the diſt. run by the log) = $NS \times \frac{5}{4}$ = 62·5 l. and conſequently the time = 37½ hours.

Meſſ. *L. Charlton*, *W. Davies*, *T. Drury*, *T. Hopkinſon*, and *G. Witchell*, ſolve it in the ſame manner.

The ſame anſwered by Mr. Richard Terry.

Let *NM* repreſent the meridian of the place ſailed from, *N* and *S* the two points, *AS* the direction of the current, and *NA* the courſe ſteered by the compaſs; then, by the compoſition of motions, the ſhip's real direction will be in the line *NS*. Put now *NS* = *x*, *SA* = 3*x*, and *NA* = 5*x*. Then (by Euc. 47. 1.) $25xx - 9xx$ ($= NA^2 - SA^2 = NS^2$) $= aa$; whence $x = \frac{1}{4}a = 37·5$ miles. From which the courſe is found to be W. by S. 3° 7′. S. and the time required 37h: 30m.

Thus it is alſo anſwered by Meſſ. *T. Allen*, *Birchoverenſis*, *D. Haſtings*, *T. Howe*, *J. Goodhead*, *J.* and *R. Hudſon*, *J. Pearce*, *Joſ. Rennard*, and *W. Spicer*.

VIII. QUESTION 441 *anſwered by Mr.* Rich. Mallock, *of* Lyme Regis.

By the known rules for pendulums, $\sqrt{39·2} : \sqrt{38·5} :: 28 : 27·75$ = the number of ſeconds that the glaſs is running, which let be denoted by n; then it will be as 3600 (the number of ſeconds in an hour) is to n, ſo is 6120 (the feet in a nautical mile) to $1{\cdot}7n =$ the diſtance which the knots ought to be aſunder, in order to meaſure truly the ſhip's way; which diſtance, in the preſent caſe, coming out 47·175 feet, (inſtead of 42 feet) we therefore have, as 42 to 47·175, ſo is 3100 miles (the whole given diſtance of the Lizard from the capes of Virginia) to 3482 miles, the diſtance run, by the ſhip's reckoning, when ſhe arrives at her port; exceeding the truth by 382 miles.

Almoſt in the ſame manner the ſolution is alſo given by Mr. *Lionel Charlton* (the propoſer), *Birchoverenſis*. Mr. *Tim. Drury*, Mr. *T. Hopkinſon*, Mr. *J. Hudſon*, Mr. *Edward Johnſon*, Mr. *W. Kingſton*, Mr. *R. Terrey*, and Mr. *G. Witchell*.

IX. QUESTION 442 *anſwered*.

In the given equations ($xz \times \overline{x+z} = a$, and $xxzz \times \overline{z^5 + x^5} = b$) let there be aſſumed $x + z = s$, and $xz = p$; then will $z^5 + x^5 = s^5 - 5s^3p + 5sp^2$; and ſo, by ſubſtitution, there will be had $ps = a$, and $p^2 \times \overline{s^5 - 5s^3p + 5sp^2} = b$: In the latter of which equations let $\frac{a}{s}$, the value of p (as given by the former) be ſubſtituted; then will $\frac{aa}{ss} \times \overline{s^5 - 5s^3 \times \frac{a}{s} + 5s \times \frac{aa}{ss}} = b$; whence $s^6 - \overline{5a + \frac{b}{aa}} \times s^3 = -5aa$; and conſequently $s =$

$$\sqrt[3]{\frac{5a}{2} + \frac{b}{2aa} \pm \sqrt{\left[\frac{5a}{2} + \frac{b}{2aa}\right]^2 - 5aa}}.$$

From which p $(= \frac{a}{s})$ will alſo be known; whence x and z are eaſily obtained.

Thus the anſwer is given by Mr. *L. Charlton*, Mr. *Rich. Harvey*, Mr. *J. Honey*, Meſſ. *J.* and *R. Hudſon*, Mr. *E. Johnſon*, Mr. *W. Kingſton*, Mr. *R. Langley*, Mr. *W. Spalton*, Mr. *H. Watſon*, Mr. *J. Wilſon* (the propoſer), and ſome others.

The ſame anſwered otherwiſe.

Put $y+v=x$, and $y-v=z$; then the two given equations will become $\overline{yy-vv}\times 2y=a$, and $\overline{yy-vv}^2\times\overline{2y^5+20y^3v^2+10yv^4}=b$; from the former whereof $vv=yy-\frac{a}{2y}$; which value ſubſtituted in the latter, gives (after proper reduction) $y^6-\frac{5a^3+b}{8aa}\times y^3=-\frac{5aa}{64}$; whence $y^3=\frac{5a^3+b\pm\sqrt{5a^6+10a^3b+bb}}{16aa}$.

In this manner the ſolution is given by Mr. *R. Butler*, *Birchoverenſis*, Mr. *Jos. Harrington*, Mr. *Steph. King*, *Ponovius*, Mr. *R. Pitches*, Mr. *W. Spicer*, and Mr. *J. Thompſon*.

Mr. W. M. *of* Plymouth, *anſwers it thus.*

Let $c=\frac{1}{2}a$, $c-y=xxz$, and $c+y=zzx$; which values being wrote in the ſecond equation, it becomes $\frac{\overline{c-y}^4}{c+y}+\frac{\overline{c+y}^4}{c-y}=b$; whence, by reduction, $y^4+2ccyy+\frac{byy}{10c}=\frac{bc-2c^4}{10}$; and conſequently (by putting $d=2cc+\frac{b}{10c}$) $yy=\overline{\frac{dd}{4}+\frac{bc-2c^4}{10}}\Big|^{\frac{1}{2}}-\frac{d}{2}$: But $\frac{\overline{c-y}^2}{c+y}=x^3$, and $\frac{\overline{c+y}^2}{c-y}=z^3$; whence x and z are alſo known.

X. QUESTION 443 *anſwered by* Birchoverenſis.

In the given equation $44100xx+1=zz$, let z be aſſumed $=210x+b$; then will $100xx-1=420xb+bb$: Here x being greater than $4b$, and leſs than $5b$, let $4b+c=x$, then will $79bb+1=380bc+100cc$. Here $b\sqsubset 5c$, and $\sqsupset 6c$; let therefore $5c+d=b$; then will $25cc-1=410cd+79dd$. Here c being $\sqsubset 16d$, and $\sqsupset 17d$, make $17d-e=c$; then will $176dd-1=440de-25cc$. From whence, by proceeding on in this manner, (making $2e+f=d$, $2f+g=e$, $4g-h=f$, $3h-i=g$, $17i-k=h$, $105k-l=i$, $37l-m=k$, $5m-n=t$, $4n+o=m$, 20

$+p$

$+ p = n$, $2p - q = o$, $17q + r = p$, and $5r + s = q$) you will, at laſt, come to the equation $100rr - 1 = 380sr + 79ss$; where $4s = r$, and $s = 1 = ss$; whence $r = 4$, $q = 21$, $p = 361$, $o = 701$, $n = 1763$, &c. x coming out $= 40482981221781$, and $z = 8491781781142001$; From which two numbers, anſwering the conditions of the problem, an infinity of others may be found.

Mr. *H. Watſon* (the propoſer), Mr. *Edward Johnſon*, Mr. *Stephen King*, and Mr. *R. Pitches*, ſolve it exactly the ſame way, and have brought out the very ſame numbers.

The ſame otherwiſe anſwered by Peter Walton, *In a new Method.*

Firſt let $\frac{y}{20}$ be ſubſtituted, in the propoſed equation ($44000xx + 1 = zz$), inſtead of x; then will $110yy + 1$ be $= zz$. Now, z being greater than $10y$, ſuppoſe $z = 10y + b$, and you will, by ſubſtitution and reduction, have $10yy - 20by = bb - 1$. In this laſt equation y is greater than $2b$; therefore ſuppoſe $y = 2b + c$, and you will find $bb - 20cb = 10cc + 1$; from whence is found $b = 10c + \sqrt{110cc + 1}$.

Suppoſe now $c = 0$; then will you have $b = 1$, $y = 2$, $z = 21$, and $x = \frac{2}{20}$; which laſt not being an integer, we muſt ſeek farther. To that end, take $c = 2$ (the value of y juſt now found); then will $\sqrt{110cc + 1}$ be $= 21$ (the above-found value of z), and b will be $= 41$, $y = 84$, $z = 881$, and $x = \frac{84}{20}$; which not being an integer, we muſt proceed farther, by taking $c = 84$ (the laſt value of y); then will $\sqrt{110cc + 1}$ be $= 881$ (the laſt value of z); and b will be $= 1721$, $y = 3526$, $z = 36981$, and $x = \frac{3526}{20}$; which not being an integer, we muſt proceed yet farther, by taking $c = 3526$, &c. At length, by proceeding in that manner, we find $x = 40482981221781$, and $z = 8491781781142001$.

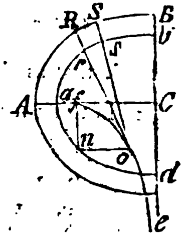
A third Method of Solution by Mr. Patrick O'Cavanah, *of* Dublin.

From the given equation ($44000xx + 1 = zz$) it is evident that z is nearly $= 210x$. Putting therefore $\overline{210 - y} \times x = z$, we thence get $420y - yy - 100 + \frac{1}{xx} = 0$; which, by making $v = \frac{y}{10}$ (to reduce the coefficients ſtill lower) will become $42v - vv - 1 + \frac{1}{100xx} = 0$. Now, it may be obſerved, that as the term $\frac{1}{100xx}$ is very ſmall, the true value of v, in this equation, muſt be an approximate value of v, in the equation $42v - vv - 1 = 0$ (where the ſaid term $\frac{1}{100xx}$ is omitted). In order therefore to find ſuch an approximate value of v, let a ſeries of numbers, 1, 42, 1763, 74004, 3106405, &c. be ſo formed, that each new term may be equal to 42 times the laſt, *minus* the laſt but one (42 and -1 being the coefficients of v and vv in the equation here to be reſolved): Then (by what is demonſtrated at page 174 of Simpſon's Algebra, 2d edit.) it will appear, that $\frac{1}{42}$, $\frac{42}{1763}$, $\frac{1763}{74004}$, $\frac{74004}{3106405}$, &c. are ſo many ſucceſſive approximations to the value of v, each more exact than the preceding one. It will alſo appear (and might be eaſily demonſtrated in a general manner, if room would permit) that the error, or quantity ariſing by ſubſtituting any one of theſe fractions for v, in the given equation $42v - vv - 1 + \frac{1}{100xx} = 0$, will be always expreſſed by $-\frac{1}{D^2} + \frac{1}{100xx}$, D denoting the denominator of the fraction ſo ſubſtituted. Therefore, in order that the error may entirely vaniſh, and a ſolution in integers be obtained, we have nothing to do but to continue the preceding ſeries of numbers to five terms farther, ſo that the laſt of theſe (D) may be a multiple of 10, or ſo that x ($= \frac{D}{10}$) may be an integer; by which means D is found $= 404829812217810$, $x = 40482981221781$, and $z = 8491781781142001$. *W. W. R.*

XI. QUES-

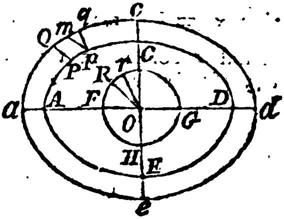
XI: QUESTION 444 *answered by Mr.* C. Wildbore.

Let dab repreſent half the elliptic fountain, foe its evolute, ro and so two radii of curvature indefinitely near each other, and let $Aa = Rr = Ss = Bb$ be every where perpendicular both to the ellipſis and the curve $ARSB$, and then it is evident that this will repreſent the breadth of the required walk.



Put $ar = z$, $AR = v$, $Rr = b$, and $ro = r$; then $ro : \dot{z} :: Ro : \dot{v} = \frac{\overline{r+b}\times\dot{z}}{r}$, which multiplied by $\frac{1}{2}Ro$, gives $\frac{\overline{r+b}|^2\times\dot{z}}{2r}$ for the area $R\dot{o}S$; and in the ſame manner the area ros is found to be $\frac{r\dot{z}}{2}$; the difference of which area is $b\dot{z}+\frac{b^2}{2}\times\frac{\dot{z}}{r}$ the fluxion of the walk $AaRr$; the fluent of which is $bz+\frac{b^2}{2}\times\angle ron$ (to rad. unity): This, when AR becomes $= AB$, is $bz+\frac{b^2}{2}\times 1\cdot5708$; which expreſſion is general, let the curve arb be what it will, and, in the preſent caſe, muſt be equal to the area of the quarter of the ellipſis $abC = a$; this reduced (putting $1\cdot5708 = c$) gives $b = \frac{\sqrt{2ac+zz}-z}{c} = \frac{\sqrt{282\cdot744\times1\cdot5708+21\cdot25\times21\cdot25}-21\cdot25}{1\cdot5708} = 5\cdot5247$ feet, the breadth of the walk required.

The ſame anſwered by Mr. E. Rollinſon.



Let $ACDE$ be the fountain, and PQ (pq, Aa, Cc, or Dd) the breadth of the walk ſurrounding it: Let pq be ſuppoſed indefinitely near to PQ, and pm parallel to PQ; moreover, ſuppoſing $OFRG$ to be a circle whoſe radius is $= PQ$, let OR be conceived parallel to PQ, and Or to pq. Then, the area mpq being $= ROr$, and $QPpm = PQ \times Pp = Aa \times Pp$, it is manifeſt that ($QPpq$) the whole increment of the area $APQa$, is every where equal to a rectangle under Aa, and the correſponding increment (Pp) of the arch AP, together with the increment (ROr) of the circular ſector ROF; and, conſequently, that the area $APQa = Aa \times AP + RO \times \frac{1}{2}FR = Aa \times \overline{AP + \frac{1}{2}FR}$. Hence the area of the whole walk is evidently $= Aa \times \overline{ACDEA + \frac{1}{2}FRGHF}$; which is general, let the curve $ACDEA$ be what it will. But, in the preſent caſe, $ACDEA$ being an ellipſe (whoſe two axes are 30 and 24) the length thereof, by the known ſeries for the periphery of an ellipſis, will be found $= 27{\cdot}0834 \times p$ (p being $= 3{\cdot}14159$ &c.) Whence, if Aa be denoted by x, the area of the walk will here come out $x \times \overline{27{\cdot}0834p + px} = 30 \times 24 \times \frac{1}{2}p$ (by the queſt.) From which $xx + 27{\cdot}0834x = 180$; and conſequently $x = 5{\cdot}520766$, the required breadth of the walk.

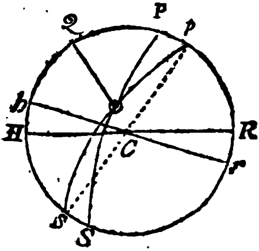
The ſame anſwered by Mr. T. Barker, *of* Weſthall, *in* Suffolk.

Although the external boundary of the walk is not accurately an ellipſis, yet the deviation therefrom is ſo very little, that the conſidering of it as ſuch, will be ſufficient to give the ſolution very near the truth. Let therefore AO $(= 15) = a$, OC $(= 12) = b$, and Aa $(= Cc) = x$; then we ſhall have $\overline{a + x} \times \overline{b + x}$ $(Oa \times Oc) = 2ab$ $(2OA \times OC)$ by the queſtion; whence $xx + \overline{a + b} \times x = ab$, and conſequently $x = \sqrt{ab + \frac{\overline{a+b}|^2}{4}} - \frac{a+b}{2} = 5{\cdot}5328 =$ the required breadth of the walk, very near.

In this laſt manner it was alſo anſwered by Meſſ. *W. Allen, W. Aſhton, Edward Barras, Ja. Beresford, T. Boſworth, Turner Boſton, J. Buddles, J. Chapman, Charles Cave, T. Crabtree, Edward Ellis, G. Godhelp, J. Goodhead, H. Green, Joſ. Harrington, W. Harriſon, G. Hicks, R. Hodges, J. Honey, Abr. Horsfall, E. Houlſton, B. Lydal, W. Matthewſon, Chr. Meſhan, J. Milbourn, J. Pearce, S. Pedley, W. Penn, Joſ. Rennard, Alex. Rowe, J. Scott, G. Stapley, H. Stephens, J. Taylor, J. Thompſon, T. Wilkin,* and ſome others.

XII. QUESTION 445 *anſwered by Mr.* O'Cavanah.

Let *HPRS* be the meridian (or plane of the ſtyle), *P* the north pole, *HCR* the horizon, ⊙*PH* the given hour angle from noon, *P*⊙ the ſun's diſtance from the north pole, and ⊙*Q* a great circle perpendicular to the meridian: Moreover, let *hCr* be ſuppoſed to repreſent the poſition of the plane of the dial, and *Cp* that of the gnomon, when the ſouth ſide of the dial *hC* is elevated (as by the queſtion) above its proper horizontal poſition *HC*.



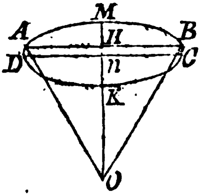
It is evident, from the general conſtruction of all dials, that the hour from 12 marked out by the ſhadow (converted into degrees) will, in every poſſible poſition of any dial, be the true meaſure of the angle made by the plane of the ſtyle and another (imaginary) plane paſſing by the edge of the ſtyle thro' the ſun's center: Whence it appears that the ∠*Qp*⊙, made (in the preſent caſe) by the plane of the ſtyle *HPRS* and the plane *p*⊙*sC* paſſing by the edge of the ſtyle in the poſition *pC*, will be truly expreſſed by 65°, anſwering to 4h. 20m. the time from 12, given by the queſtion. Now, the ∠*Qp*⊙ (here given) being (by the queſt.) leſs than it can, otherwiſe, be in any other poſition of *p* (or *Cp*) it is plain that, let ⊙*Q* be what it will, *Qp* and ⊙*p* muſt be archs of 90 degrees each; ſince, in this caſe, the meaſure of the ∠*Qp*⊙, or *Qs*⊙, (which in all other poſitions of *p* is greater) will become = ⊙*Q*. Hence we have (per ſpherics) as ſin. ⊙*PQ* (75°) : ſin. ⊙*Q* (65°) :: rad. : ſin. *P*⊙ = 69° 45′ 52″; whoſe complement 20° 14′ 8″, is the declination ſought: The time correſponding being May the 21ſt, or July the 22d.

COROLLARY. In all latitudes, the ſhadow, while the plane of the dial is elevating, will proceed to the ſame hour before it returns back: Becauſe the true time (or hour) being ſuppoſed the ſame in all latitudes, ⊙*Q* as well as *PQ* will remain invariable, and therefore *Pp* (the complement of *PQ*) invariable likewiſe.

In this manner the problem was alſo determined by Mr. *L. Charlton*, Mr. *W. Davies*, Mr. *E. Rollinſon*, and Mr. *C. Wildbore*.

XIII. QUESTION 446 *anſwered by Mr.* L. Charlton.

Suppoſing *AMBCKD* to repreſent the garden, and *ABCD* the walk; it is evident, ſeeing the length *AMB* is given, that *AB* will be a maximum, when the exceſs of *AMB* above *AB* is the leaſt quantity poſſible (the area being given, or ſuppoſed to remain the ſame; which will therefore be when *AMB* is an arch of a circle; becauſe, the circle being the moſt capacious figure, any other curve drawn from *A* to *B*, to contain with *AB* an area equal to *AMBHA*, muſt neceſſarily differ more from *AB* than *AMB* does, in this ſuppoſition. To determine, now, the radius of the circle (and from thence the length of the walk) put *AM* (= 325 feet) $= a$, *AD* (= 10) $= b$, the area *AMBCKD* (= 43560) $= c$, and $AO\ (= MO = BO) = r$. Then, the area *AMH* being $= \frac{1}{2} AM \times OM - \frac{1}{2} AH \times OH = \frac{1}{2} ar - \frac{1}{2} AH \times OH$, and the area $AHnD = b \times AH$, we thence get $2ar - 2AH \times OH + 2b \times AH = c$. From whence, by ſeries, or any of the known methods of approximation, the required length *AB* of the walk may be determined, and will come out = 642·312 feet, or 38·928 furlongs.



Meſſ. *Goodhead*, *Hudſon*, *Rollinſon*, and *Wildbore*, have alſo given ſolutions to this problem, in a manner very little different.

XIV. QUESTION 447 *anſwered by Mr.* T. Allen, *of* Spalding.

Let the given length of the chain (= 30 feet) $= a$; the part hanging down at the commencement of motion (= 3 feet) $= c$; $s = 32\frac{1}{6}$ feet, the velocity generated at the earth's ſurface in one ſecond; x the ſpace moved over by a particle in any variable time t; and $v =$ the correſponding velocity of the chain at that inſtant: Then it is evident, that $c + x$ is as the motive force acting on the chain; and therefore, $\frac{\dot{x}}{v}$ being $= \dot{t}$, $s \times \overline{c + x} \times \frac{\dot{x}}{v}$ will be $= a\dot{v}$, the fluxion of the quantity of motion generated in the time t; whence $s \times \overline{c\dot{x} + x\dot{x}} = av\dot{v}$; and, taking the fluents, &c. $v =$

$\sqrt{\frac{s}{a}} \times \overline{2cx+xx}$, which ſubſtituted in the above value of $\dot{t}$, gives $\dot{t} = \frac{\dot{x}}{\sqrt{\frac{s}{a}} \times \overline{2cx+xx}}$, whoſe correct fluent is $t = \sqrt{\frac{a}{s}} \times$ hyp. log. $\frac{c+x+\sqrt{2cx+xx}}{c}$, which, when x becomes $= a - c$, is $= 2{\cdot}890663$ ſeconds, the time required.

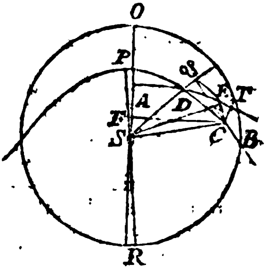
The ſame anſwered by Mr. Robert Butler.

Let $a =$ the chain's length $(= 30$ feet$)$, $d = 32\frac{1}{6}$ feet, the velocity generated by the uniform force of gravity in one ſecond, $x =$ the length of the part of the chain diſengaged from the plane at the end of any time t, and $v =$ the velocity acquired in that time. Then $a : x :: d : \frac{dx}{a}$ the velocity that would be acquired in one ſecond by the uniform force of x length: Therefore, as 1 (ſecond) is to $\frac{\dot{x}}{v}$ $(= \dot{t}) :: \frac{dx}{a} : \frac{dx\dot{x}}{av} = \dot{v}$ (the uniform increaſe of the velocity in the time $\dot{t}$). Hence $\frac{dx\dot{x}}{a} = v\dot{v}$; and therefore, by taking the correct fluents, $\frac{dxx}{a} - \frac{dbb}{a} = vv$, b being (= 3 feet) = the firſt or given value of x, when the motion commences. Now $\dot{t}$ $(= \frac{\dot{x}}{v})$ being given from hence $= \sqrt{\frac{a}{d}} \times \frac{\dot{x}}{\sqrt{xx-bb}}$, by taking again the correct fluents, we get $t = \sqrt{\frac{a}{d}} \times$ hyp. log. $\frac{x+\sqrt{xx-bb}}{b} = \sqrt{\frac{a}{d}} \times$ hyp. log. $\overline{10+\sqrt{99}}$ (when $x = a = 30$) $= 2{\cdot}89066$ ſeconds, the time required.

Peter Walton (the propoſer), Mr. *O'Cavanah*, Mr. *Rollinſon*, Mr. *Watſon*, and Mr. *C. Wildbore*, anſwer this queſtion in the ſame manner.——*Tyro* has given a very pretty ſolution to it upon different principles.

PRIZE

PRIZE QUESTION *anſwered by Mr.* G. Witchell, *the Propoſer.*



Let OTR be the earth's orbit, PDC the comet's trajectory, ADB the orthographic projection of it on the plane of the ecliptic, D the deſcending node, and P the place of the perihelion. From any point C of the parabola let fall CE perpendicular to the plane of the ecliptic; through S and E draw ST and join C, T, it is evident that T is the neareſt point in the circle to C. Now put $SC = 1 - x$, $PS = \frac{a}{4}$, $n = \frac{a}{2} - 1$, $m = a - \frac{aa}{4}$, $t =$ ſin. of E☋C (the greateſt inclin.) $b =$ the ſine and $d =$ the coſine of PS☋; then, by the nature of the parabola, we ſhall have $SF = n + x$ and $FC = \sqrt{m - ax}$; therefore $1 - x : 1 :: \sqrt{m - ax} : \frac{\sqrt{m - ax}}{1 - x} :: n + x : \frac{n + x}{1 - x} =$ ſin. and coſ. of PSC; but ſin. $PSC - PS$☋ $=$ ſin. $PSC \times d -$ coſ. $PSC \times b = \frac{d\sqrt{m - ax} - b \times \overline{n + x}}{1 - x} =$ ſin. ☋SC; then $1 : 1 - x :: \frac{d\sqrt{m - ax} - b \times \overline{n + x}}{1 - x} : d\sqrt{m - ax} - b \times \overline{n + x} =$ ☋C, and $1 : d\sqrt{m - ax} - b \times \overline{n + x} :: t : td\sqrt{m - ax} - tb \times \overline{n + x} = EC$; which being ſquared and ſubtracted from $\overline{1 - x}|^2 = SC^2$, we have $\overline{1 - x}|^2 - ttdd \times \overline{m - ax} - ttbb \times \overline{n + x}|^2 + 2ttdb \times \overline{n + x} \times \sqrt{m - ax} = SE^2$: This being reſolved into a ſeries (and A, B, C, &c. ſubſt. for the known coefficients) we ſhall get $A^{\frac{1}{2}} \times \overline{1 - \frac{B}{2A} \times x - \frac{B^2 + 4AC}{8A^2} \times xx + \frac{B^3 - 4ABC + 8A^2D}{16A^3} \times x^3 \text{ \&c.}} = SE$: But $1 + SC^2 - 2SE = CT^2$; whence we have $2 - 2A^{\frac{1}{2}} - \overline{2 + \frac{B}{A^{\frac{1}{2}}}} \times x + \overline{1 + \frac{B^2 - 4AC}{4A^{\frac{3}{2}}}} \times xx -$

$\frac{B^3-4ABC+8A^2D}{8A^{\frac{5}{2}}} \times x^3$ &c. $= CT^2$; which, when CT is a minimum, will give $2+\frac{B}{A^{\frac{1}{2}}} = \overline{\frac{B^2-4AC}{2A^{\frac{3}{2}}}+2}$ $\times x - \frac{B^3-4ABC+8A^2D}{8A^{\frac{5}{2}}} \times 3xx$ &c. Whence we get $x = 0{\cdot}0146$, and $CT = 0{\cdot}041974$; which (taking $ST =$ 8000000) is = 335792o miles. Moreover, the coſ. of PSC being $= \frac{n+x}{1-x} =$ coſ. 78° 59′, we ſhall have the long. of the comet in its orbit ♏ 14° 31′, and its arg. of latitude 7° 42′; from which, and the given inclination, its longitude in the ecliptic will be found ♏ 14° 51′; which place the earth tranſits, May 4th.

Hence it appears, that we need not be under any apprehenſions from the return of this comet, notwithſtanding the prediction of a certain experimental philoſopher; for if it ſhould return at the time he mentioned, it would be diſtant from the earth (by his own ſcheme) 9000000 miles.

In a manner equally conciſe and ingenious, the ſolution to this problem is alſo given by Mr. *O'Cavanah*, and Mr. *Wildbore*; but the prize belongs to Mr. *Witchell*, his ſolution being the only true one that came to hand before Candlemas-day.

The Eclipſes calculated for 1759.

There will this year be three eclipſes, one of the moon, and two of the ſun; whereof that of the moon, only, will be viſible to the inhabitants of Great Britain :. Some calculations of which are as follow.

☽ Ecl. Jan. 13, in the morning.

Calculated by		Beg.	Mid.	End	Dur.	Dig.
		h. m.	h. m.	h. m.	h. m.	
Mr. G. Witchell, from Mayer's Tables	for London	6 37	7 57	9 18	2 41	6 29
Mr. Gutteridge,	for London	6 38	7 59	9 21	2 43	6 39
	Loughbor.	6 33	7 54	9 16		
Mr. E. Greenſtead,	for London	6 38	8 0	9 21	2 43	6 39
Mr. T. Hopkinſon,		6 30	7 55	9 18	2 46	6 41
Mr. R. Langley,		6 31	7 53	9 15	2 44	6 35
Mr. T. Allen, from Halley's Tables	Spalding	6 43	8 1	9 19	2 46	6 24

The ſecond eclipſe is of the ſun, June 24th, about 5 in the afternoon.

The third is alſo of the ſun, Dec. 19th, about 2 in the afternoon.——Theſe two eclipſes are both inviſible in England, but very large in the Weſt Indies.

New-Queſtions.

I. QUESTION 448, *by* Richard *of the* Vale.

Teach me, fair artiſts, how to find,
When my dear Nancy will be kind;
The charming maid my love allows,
Yet is averſe to crown my vows;
In myſtic terms * preſcribes her will,
Which to diſcloſe, exceeds my ſkill.
T' unfold the knot, * ye fair, deſcend,
And ſerve, for once, an unlearn'd friend.

* $\left\{\begin{array}{l} xx+yy-5x+9y=968 \\ 8yy+xy-30x+54y=5808 \end{array}\right\}$; *x* being (ſhe ſays) the months, and *y* the days, before ſhe can conſent to compleat my happineſs.

II. QUESTION 449 *by Mr.* Paul Sharp.

The difference of the two legs of a right-angled triangle, whoſe area is double to that of its inſcribed circle, being given (=6); to find all the ſides, and the radius of the circle.

III. QUESTION 450, *by Mr.* Richard Gibbons.

Somewhere, on fam'd Europa's ſtrand,
A lofty cone aſpiring ſtands,
O'erlooking far the watry plain,
A guide to thoſe who plow the main;
Whoſe ample bulk contains compleat
One fifteenth of two million feet.
It chanc'd, that from the top, a ball
Down by its ſide to th' ground did fall,
Which reach'd, in five half-ſecond's ſpace,
The utmoſt limit of the baſe.
From hence, fam'd artiſts, let be ſhown
The heigh of this ſtupendous cone.

IV. QUESTION 451, *by Mr.* T. Barker, *of* Weſthall, *in* Suffolk.

Two lines drawn to biſect and terminate in the oppoſite ſides of any trapezium, will alſo biſect each other: A demonſtration of this is required.

V. QUESTION 452, *by Mr.* Chriſtopher Maſon.

A farmer propoſes to have a buſhel to contain three quarts above ſtatute (being the cuſtomary meaſure of the neighbourhood) to be 1-5th of an inch in thickneſs, of hammered braſs; the depth and diameter to be ſuch as will require the leaſt metal; he will likewiſe allow 16 d. per lb. and deſires to know the expence at that rate.

VI. QUESTION 453, *by Mr.* J. Hudſon, *of* Louth.

Having given the lengths (14 and 20) of two right lines drawn from the ſame point to make a given angle (60°) with each other; ſo to draw another right line through the point of concourſe, that two perpendiculars being let fall thereon from the extremes of the firſt lines, the ſum of the two triangles, thereby formed, ſhall be the greateſt poſſible.

VII. QUESTION 454, *by Mr.* William Toft.

A ſhip, plying to windward, then at N. N. E. ſails, with her ſtarboard tacks on board, 60 miles; ſhe then tacks, and, after having run 45 miles farther, finds, by an obſervation, that her whole difference of latitude, on both tacks, is juſt 40 miles: Hence you are deſired to determine how near to the wind the ſhip made good her way.

VIII. QUESTION 455, *by Mr.* Tho. Moſs.

To divide a given trapezium into two equal parts (geometrically) by a right line cutting off from oppoſite ſides two ſegments, adjacent to the baſe, which ſhall alſo be equal to each other.

IX. QUESTION 456, *by Mr.* Mor. McRoy.

In a right-angled triangular field there are three trees, viz. one in each fence: The diſtance of the tree in the baſe from that in the hypothenuſe, and from the acute angle adjacent, and of the tree in the perpendicular from the right angle, are all equal, and given; and if lines be drawn from the tree in the baſe to the other two, thoſe lines will form a right angle: The perpendicular of the propoſed triangle is known to be the leaſt of its kind (or that the data will admit of): Hence you are deſired to find the ſides of the triangle, and to conſtruct the ſame geometrically.

X. QUESTION 457, *by Mr.* G. Witchell.

The poet Heſiod tells us, That, in his time Arcturus roſe at the ſetting of the ſun 60 days after the winter ſolſtice. It is required to find from hence, and the ſubjoined data*, collected from aſtronomical obſervations, at what time that author flouriſhed.

* Lat. of the place 38° 54′ N. long. of the ſtar, at the beg. 1690, ♎ 10° 53′ 52″, its lat. 30° 57′ N. annual preceſſion of the equinox 50·3″.

XI. QUESTION 458, *by Mr.* Edw. Johnſon.

On May 12th, 1758, in latitude 53° 40′ N. a perſon ſets out at ſix in the morning, and walks uniformly in the direction of his own ſhadow, till 50 minutes paſt ten; when he arrives at a place known to lie 12 miles weſt of the meridian of that from whence he ſet out: The queſtion is to find how far, and at what rate he travelled in going from one place to the other.

XII. QUESTION 459, *by Mr.* Hugh Brown.

Suppoſe a ciſtern, whoſe length, breadth, and depth are 60, 28, and 32 inches, to be ſupplied with water by a cock, running uniformly into it, at the rate of 2 gallons per minute; and that the diſcharge of water by a cock at the bottom, when the ciſtern is full, is at the rate of 5 gallons per minute. Now I deſire to know, ſuppoſing the veſſel to be full, and both cocks opened together, in what time 150 gallons may be drawn off, at the evacuating cock.

XIII. QUESTION 460, *by Mr.* Edw. Rollinſon.

To determine the curve in which a body muſt move, ſo as to continue always at the ſame invariable diſtance from another body moving uniformly in a right line; the velocity of the former body being alſo uniform, and exceeding that of the latter, in any given ratio.

XIV. QUEST.

XIV. QUEST. 361, *by* Peter Walton, *Diſcip. Landenii.*

To aſſign the ſum of the ſeries $1 - \frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} - \frac{1^2.3^2.5^2}{2^2.4^2.6^2}$ + &c. by means of circular and elliptic arcs.

The PRIZE QUESTION, *by Mr.* O'Cavanah *of Dublin.*

To determine on what day of the year, in the lat. of London, the length of the afternoon exceeds that of the forenoon by the greateſt difference poſſible, reckoning the day to begin at ſun-riſing, and to end at ſun-ſetting.

1760.

Queſtions anſwered.

I. QUESTION 448 *anſwered by Mr.* G. Witchell.

AH! Richard, take a friend's advice,
Or you'll be ruin'd in a trice,
Don't wed the artful fair;
When ſhe ſays Aye, do you ſay No:
Believe me, ev'ry ſcene of woe
Attend the ill-match'd pair.
Her head is turn'd with algebra;
She will not riſe before 'tis day,
Nor trudge to milk your kine:
But, if no reaſon can perſuade
You to refuſe the yielding maid,
Behold your quarentine.*

* The firſt equation being multiplied by 6, and compared with the ſecond, there reſults $yy + \frac{xy}{2} = 3xx$; whence, by compleating the the ſquare, $y = \sqrt{3xx + \frac{xx}{16}} - \frac{x}{4} = \frac{3x}{2}$; which value ſubſtituted in the firſt equation, gives $\frac{13xx}{4} + \frac{17x}{2} = 968$, whence $x = 16$ months, and $y = 24$ days.

In this manner the ſolution is alſo given by Mr. *T. Barker*, Mr. *E. Batten*, Mr. *J. Bereſford*, *Birchoverenſis*, Mr. *Turner Boſton*, Mr. *J. Buddles*, Mr. *R. Butler*, Mr. *W. Fowler*, Mr. *P. George*, Mr. *Nath. Gerrad*, Mr. *R. Gibbons*, Mr. *J. Hampſon*, Mr. *Joſ. Harrington*, Mr. *G. Hicks*, Mr. *Abr. Horsfall*, Mr. *E. Houlſton*, Mr. *Steph. King*, Mr. *W. Kingſton*, Mr. *W. Litſon*, Mr. *R. Mallock*, Mr. *Chr. Meſban*, Mr. *J. Milbourn*, Mr. *Alex. Rowe*, Mr. *Joſ. Ruſton*, Mr. *W. Spicer*, Mr. *W. Stoker*, Mr. *W. Toms*, Mr. *Joſ. Waſtel*, and ſeveral others.

II. QUESTION 449 *anſwered by Mr.* T. Barker, *of* Weſthall, Suffolk.

Put $\frac{1}{2}$ the ſum of the two legs $= x$, and $\frac{1}{2}$ their difference $= a$; then $x - a =$ the leſs leg, and $x + a =$ the greater; and conſequently $\sqrt{2xx + 2aa} =$ the hypothenuſe. Alſo, by a well-known theorem, $2x - \sqrt{2xx + 2aa}$ ($=$ the ſum of the legs minus the hyp.) $=$ the circle's diameter; whence its periphery (putting $n = 3{\cdot}1416$) will be $= 2nx - n\sqrt{2xx + 2aa}$; the double of which (by another well-known theorem, and the conditions of the queſtion) muſt be equal to the perimeter of the triangle; that is, $4nx - 2n\sqrt{2xx + 2aa} = 2x + \sqrt{2xx + 2aa}$; whence x is found $= a\sqrt{\dfrac{\overline{2n+1}|^2}{2 \times \overline{2n-1}|^2 - \overline{2n+1}|^2}} = 13{\cdot}1$; therefore the ſides are $10{\cdot}1$, $16{\cdot}1$, 19, and the diameter $= 7{\cdot}19$.

Thus (or in a manner very little different) the anſwer is alſo given by Mr. *T. Allen*, Mr. *E. Batten*, Mr. *T. Baxtonden*, *Birchoverenſis*, Mr. *R. Butler*, Mr. *G. Brown*, Mr. *W. Chapman*, Mr. *Chr. Cave*, Mr. *Tim. Drury*, Mr. *P. George*, Mr. *Nath. Gerrad*, Mr. *R. Gibbons*, Mr. *J. Goodhead*, Mr. *T. Harris*, Mr. *J. Hampſon*, Mr. *Abr. Horsfall*, Mr. *R. Mallock*, Mr. *Chr. Meſban*, *Penovius*, Mr. *W. Spicer*, Mr. *Abr. Stone*, Mr. *W. Stoker*, Mr. *Joſ. Waſtell*, Mr. *G. Witchell*, and ſome others.

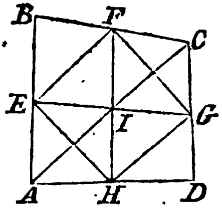
III. QUESTION 450 *anſwered.*

Put the content of the cone ($= 133333\frac{1}{3}$) $= c$, and its altitude $= x$, alſo put $p = 3{\cdot}1416$, and let $a =$ diameter of a ſphere deſcribed about the cone; which (by the laws of deſcent of heavy bodies) is given $= 100\frac{25}{48} =$ the perp. deſcent in the given time in which the chord or ſide of the cone is deſcribed. Now, by the property of the circle, $x \times \overline{a-x}$ is = the ſquare of the rad. of the cone's diam. at the baſe; and conſequently $\frac{1}{3}\ pxx \times \overline{a-x} = c$ (= the cone's content): From which equation $x = 81{\cdot}219$, or $x = 50{\cdot}408$.

Thus the problem is reſolved by Mr. *W. Bamfield*, Mr. *T. Barker*, Mr. *T. Baxtonden*, *Birchoverenſis*, Mr. *Turner Boſton*, Mr. *G. Brown*, Mr. *J. Buddles*, Mr. *R. Butler*, Mr. *W. Chapman*, Mr. *Chriſ. Cave*, Mr. *W. Davies*, Mr. *Tim. Drury*, Mr. *P. George*, Mr. *Ja. Giles*, Mr. *R. Gibbons*, Mr. *J. Goodhead*, Mr. *T. Harris*, Mr. *Abr. Horsfall*, Mr. *Steph. King*, Mr. *W. Kingſton*, Mr. *W. Litſon*, Mr. *R. Mallock*, Mr. *J. Milbourn*, *Penovius*, Mr. *Alex. Rowe*, Mr. *Joſ. Ruſton*, Mr. *W. Spicer*, Mr. *Abr. Stone*, Mr. *W. Toms*, and Mr. *G. Witchell*.

IV. QUESTION 451 *anſwered.*

Let the biſecting points be joined by the lines *EF*, *FG*, *GH*, and *HE*, and let *AC* be drawn: Then, becauſe the ſides of the triangles *ABC* and *ADC* are divided proportionally, both *EF* and *HG* will be parallel to *AC* (Euc. 2. 6.); and, therefore, parallel to each other; and in the very ſame manner is *EH* parallel to *FG*; and ſo, the triangles *EIF* and *HIG* being equiangular (Euc. 29. 1.) and *EF* being = *HG*, thence will *EI* = *GI*, and *FI* = *HI*. *Q.E.D.*



In this manner the demonſtration is given by Meſſ. *Allen*, *Barker*, *Baxtonden*, *Butler*, *Brown*, *Davies*, *George*, *Goodhead*, *Mallock*, *Milbourn*, *Penovius*, *Spicer*, *Wildbore*, and *Witchell*; to none of whom, in particular, we have a right to aſcribe the ſolution above exhibited in preference to the others

V. QUESTION 452 *anſwered.*

Put $\cdot 7854 = p$, the content of the buſhel in cubic inches $= a$, and the internal diameter $= x$; then will the area of the baſe $= pxx$, the circumference of the baſe $= 4px$, the depth of the buſhel $= \frac{a}{pxx}$, the concave ſurface $= 4px \times \frac{a}{pxx} = \frac{4a}{x}$, and the whole internal ſurface (including the baſe) $= \frac{4a}{x} + pxx$: Which being a minimum (by the queſt.) we thence have $- \frac{4ax}{xx} + 2px\dot{x} = 0$; whence $x = \sqrt[3]{\frac{2a}{p}} = 18{\cdot}1604$; and the depth $(\frac{a}{pxx} = \frac{x}{2}) = 9{\cdot}0802$: Hence 158·85 = the quantity of the metal; which, at 16d. per pound (ſuppoſing a cubic foot of hammered braſs to weigh 8349 ounces, as by the tab. in Miſ. Cur.) comes to 3l. 3s. 11d.

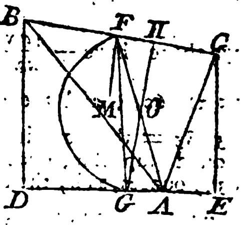
Thus the ſolution is given by Meſſ. *Barker*, *Bereſford*, *Birchoverenſis*, *Boſton*, *Buddles*, *Chapman*, *Drury*, *Haſtings*, *Gibbons*, *Giles*, *Goodhead*, *Harris*, *Hicks*, *Horsfall*, *Houlſton*, *King*, *Kingſton*, *Rowe*, *Toms*, and ſome others.

The ſame anſwered otherwiſe.

Let x and y denote the internal diameter and depth, and a the thickneſs of the metal; then the external diameter and depth being $x + 2a$ and $y + a$, we ſhall have $pyxx = c$ (= the given content) and $p \times \overline{y + a} \times \overline{x + 2a}^2 - c$ a minimum (by the queſtion): Whence, in fluxions, $\dot{y}xx + 2yx\dot{x} = 0$, and $\dot{y} \times \overline{x + 2a}^2 + \overline{y + a} \times \overline{x + 2a} \times 2\dot{x} = 0$; hence $\dot{y} = -\frac{2y\dot{x}}{x} = -\frac{\overline{y + a} \times 2\dot{x}}{x + 2a}$, $\frac{y}{x} = \frac{y + a}{x + 2a}$, and $x = 2y$: Whence alſo $\frac{1}{2}px^3 \; (= px^2y) = c$; from which x is given $= \sqrt[3]{\frac{2c}{p}} = 18{\cdot}1604$, and $y = 9{\cdot}0802$; hence the quantity of metal $(= p \times \overline{y + a} \times \overline{x + 2a}^2 - p \times yx^2 = 4pa \times 3yy + 3ay + aa)$ comes out = 158·85 cubic inches, and the value thereof 3l. 3s. 11d.

Thus the problem is reſolved by Mr. *T. Baxtonden*, Mr. *R. Butler*, Mr. *W. Davies*, Mr. *R. Mallock*, *Philarithmus*, *Penovius*, *P. M.* Mr. *W. Spicer*, and Mr. *G. Witchell*.—— From this laſt method it appears, that, let the thickneſs of the metal be ever ſo great, the external diameter and depth (as well as the internal) will be, accurately, in the ratio of two to one.

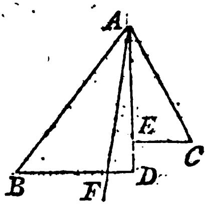
VI. QUESTION 453 *anſwered by* Mr. Thomas Moſs.



Let BC, joining the extremes of the given lines AB and AC, be biſected by AE; upon which conceive a ſemicircle FGA to be deſcribed, and conceive FG to be perpendicular to ED, and GH to BC: Then it is evident, that $DE \times FG$, or $BC \times GH$, will be = the area $BCED$; which (as BAC is given) muſt be a maximum (by the queſt.), and therefore GH (as BC is invariable) muſt alſo be a maximum; and this, ſince G is always in the circumference of the circle, muſt neceſſarily happen when HG paſſes through the center O; in which caſe the angle $OFG = \frac{1}{2}FOH = \frac{1}{2}OFM$ (FM being ſuppoſed perpendicular to BC): Whence this *Conſtruction*. Having joined A, F, and made FM perpendicular to BC, draw FG to biſect the angle AFM; then, through A, draw DE perpendicular to FG, and the thing is done.——The numerical calculation is, from hence, very eaſy; whence the angle DAB comes out $= 54° 24\frac{1}{2}'$, and the ſum of the two triangles ($ADB + ACE$) $= 131{\cdot}53$.

P. M. (of *Durham*) has alſo favoured us with a very neat geometrical conſtruction of this problem, which does not eſſentially differ from that above exhibited.

An algebraical Solution to the ſame, by Philarithmus.

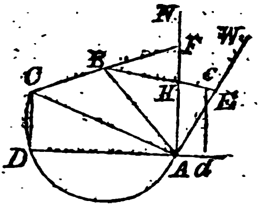


Let AD be the line ſought, and let AF biſect the given angle BAC; call $2BAF$ (60°) A; $2FAD$, V; tab. rad. 1; AB (20), a; AC (14), b; ſin. A, s; coſ. A, c; ſin. V, x; coſ. V, y; then by trigonometry $\frac{2BD \times DA}{aa}$ = ſin. $2BAD$ (or s. $\overline{A+V}$) $= sy + cx$, and $\frac{2CE \times EA}{bb}$ = s. $2CAE$ (or s. $\overline{A-V}$) $= sy - cx$; therefore $\frac{BD \times DA}{2}$, or the $\triangle ABD = \overline{sy+cx} \times \frac{aa}{4}$, and $\frac{CE \times EA}{2}$, or the △ $ACE = \overline{sy - cx} \times \frac{bb}{4}$, and their ſum $= sy \times \frac{aa+bb}{4} + cx \times \frac{aa-bb}{4}$, a minimum; therefore $s\dot{y} \times \overline{aa+bb} + c\dot{x} \times \overline{aa-bb} = 0$; whence, ſubſtituting $-\frac{x\dot{x}}{y}$ for $\dot{y}$, and dividing by $-s\dot{x}$, and tranſpoſing $\frac{x}{y} \times \overline{aa+bb} = \frac{c}{s} \times \overline{aa-bb}$; or $aa+bb : aa-bb$ :: cot. A : tan. V; whence V is given $= 11° 10'7'$, and FAD, its half, $= 5° 35'35'$, and the greateſt ſum of the triangles 131·532.

Thus (or in a manner very little different) the anſwer is alſo given by Meſſrs. *Barker*, *Baxtonden*, *Birchoverenſis*, *Brown*, *Butler*, *Davies*, *Fowler*, *George*, *Kingſton*, *Penovius*, *Spicer*, and *Witchell*.

VII. QUESTION 454 *anſwered by Mr.* O'Cavanah.

Draw AB and BC equal to the two given diſtances (60 and 45), ſo that the ſupplement (ABE) of the angle contained by them, ſhall be equal to (45°) double the given angle included between the wind and the meridian; join AC, and let a ſemicircle be deſcribed thereon, in which apply CD (= 40) = the given difference of latitudes; draw AN parallel thereto; then, if NAW be made = 22° 30', I ſay that BAW will be the angle ſought, which the ſhip makes with the wind.

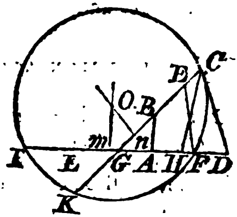


For, having produced CB to meet AN in F, let $BHcE$ be ſo drawn, as to make the $\angle BHA = BCD = BFN$, in which take $Bc = BC$, and let cd be parallel (and conſequently equal) to CD; then AB, Bc, and cd, being (by conſtruction) equal to the three quantities given, it only remains to prove, that the $\angle BEA$, which the ſhip makes with the wind, on her ſecond courſe (Bc), is equal to the $\angle BAE$, on her firſt courſe (AB). Now (by conſtruction) $BFN = BHA$; but $BFN - FBA$ ($2NAW$) $= BAH$; whence $BFN - NAW = BAE$: But BHA (BFN) $- NAW = BEA$; therefore $BAE = BEA$.

CALCULATION. In the triangle ABC we have given AB (= 60), BC (= 45), and the angle ABC (= 135°), to find BAC (= 19° 7′), and AC (= 97·178); from which laſt, and CD (= 40), CAD is found = 24° 18′: Whence BAD = 43° 25′, and BAW = 69° 5′.

Mr. *W. Davies*, and Mr. *Tho. Moſs*, have likewiſe conſtructed this problem: And ſeveral very neat algebraical ſolutions to it, have been received from Mr. *T. Allen*, Mr. *T. Baxtonden*, Mr. *Geo. Brown*, Mr. *W. Kingſton*, Mr. *W. Spicer*, Mr. *W. Stoker*, Mr. *G. Witchell*, and others; which, for want of room, we are obliged to omit.

VIII. QUESTION 455 *anſwered by* P. M. *of* Durham.

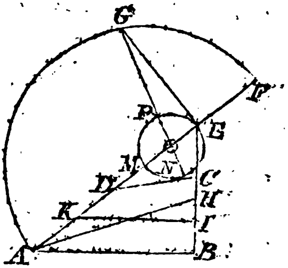


From the angle C draw CH to biſect the given trapezium $ABCD$ (p. 4. 6. Simpſon's Elem. Geom.); produce BC, AD indefinitely, cutting each other in G; from which point, on CG produced, take $GK = GH$, and, on DG produced, take $GL = GB$; draw two right lines biſecting BG, LA perpendicularly in n and m, and interſecting each other in O; round which point O as a center, with the diſtance OC or OK, deſcribe a circle cutting AD (produced both ways if needful) in F and I; from G take $GE = GI$; join E, F, and the thing is done.

DEMONSTRATION. mL and mA are equal (by hypoth.) and alſo mI, and mF (3 Euc. 3.); therefore the remainders IL, AF will be equal; but ſince GE, GI, and GB, GL are equal (by hypoth.) IL, BE, and conſequently AF, BE will be equal. But I ſay moreover, that EF biſects the trapezium, for $CG : GF :: GE\ (GI) : GH\ (GK)$ (35 Euc. 3.) therefore the triangles CGH, EGF are equal (15 Euc. 6.); and, taking away the common triangle AGB, there will remain the quadrilateral $BH = BF$; therefore, becauſe CH biſects the given trapezium BD (by hypoth.) EF will biſect it alſo. *Q. E. D.*

The ſame conſtructed otherwiſe, by Mr. W. Davies.

Draw AH ſo as to make the triangle ABH equal to half the given trapezium $ABCD$ (p. 4, 6. of Simpſ. Geom.); produce AD and BC to meet in E, and, in the former of them, take $AM = EB$, and $EF = EH$; then take EG a mean proportional between AE and EF; and from G, to the middle of EM, draw GO; make $OK = OG$, $BI = AK$, and draw KI, which will divide the given trapezium $ABCD$ into two equal parts.

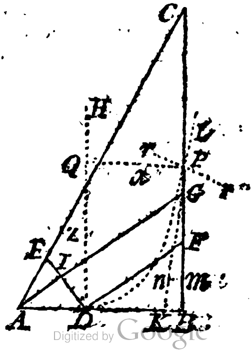


For, if GO be conceived to cut, in P and N, a circle deſcribed on the diameter ME, then will $KE \times KM$ $(= GN \times GP) = GE^2$ (Euc. 36. 3.) $= AE \times EF = AE \times EH$; but, becauſe $AM = EB$ and $AK = BI$, thence is $KM = EI$; therefore $KE \times EI$ $(= KE \times KM) = AE \times EH$; and ſo, the triangle EKI being $= EAH$ (Euc. 15. 6.), the trapezium $ABIK$ is alſo $= ABH = \frac{1}{2} ABCD$. Q. E. D.

In the two ſolutions here exhibited, it will be obſerved, that the authors have conſidered different caſes: For, of the two equal trapeziums, into which the given one is to be divided, the oppoſite ſides of either the one or the other, may, by the queſtion, be taken as equal; according as the interſection of the other oppoſite ſides is ſuppoſed to fall above or below the baſe.

IX. QUESTION 456 *anſwered by* Mr. H. Watſon.

Let ABC repreſent the field, and D, E, F the places of the trees; and let AIG be parallel to DF: Then AIE being $= EDF =$ a right angle, EAI (or CAG) is (= comp. AEI = comp. EAD) $= C$; whence $CG = AG$, and $BC = BG + AG$. From which, as many points as you pleaſe, in the curve bounding the perpendicular (BC) may be determined.



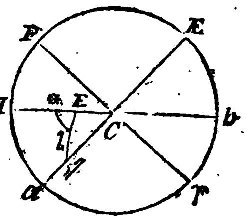
But to determine the leaſt value of BC (calling BF, a; and DB, (x), we have $DB\ (x) : BF + DF$

(a +

$(a+\overline{\sqrt{xx+aa}} :: \overline{AB}\ (a+x) : BG+AG\ (=BG)$ $= \frac{\overline{a+\sqrt{aa+xx}}\times\overline{a+x}}{x}$. Whence, by taking the fluxion, &c. $x^4 = 2a^3x + a^4$. To conſtruct this equation, put $az = xx$, and let the parabola DPL anſwering to $az = xx$ be deſcribed, having its axis DH perpendicular to AD; then if there be taken $DK = a$, Kn (parallel to DH) $= \frac{1}{2}a$, and from the center n, with the radius $\frac{3a}{2}$, an arch rPr of a circle be deſcribed cutting the parabola in P; the perpendicular ſought will paſs through this point P; and, if in the perpendicular, ſo drawn, there be taken $BF = AD$, and the length of AG (parallel to DF) be ſet off from G to C, then ſhall ABC (when AC is drawn) be the triangle required to be conſtructed.

Mr. *M'Roy* (the propoſer) has alſo given an elegant ſolution of this problem, with ſome very ingenious obſervations thereon; the whole of which (though very maſterly) is rather too long for our ſcanty limits, to be given here.—— Mr. *O'Cavanah*, Mr. *Rollinſon*, and Mr. *Witchell* have alſo conſtructed this problem; and ſeveral correſpondents have gone through part of the ſolution.

X. QUESTION 457 *anſwered by* Philarithmus.



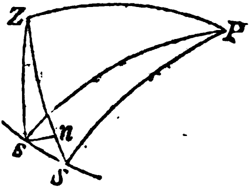
Let E be the point of the ecliptic oppoſite to the ſun's center, and riſing with Arcturus; $*l$ an arc of the circle of longitude paſſing through the ſtar; then (by carrying back the perihelion) the ſun's diſtance from the winter ſolſticial point in 60 days, about the year 850 A. C. (the middle of the century in which Heſiod is uſually placed) will be 60° 10'; whence aſſuming ♎E (= its complement) 29° 50', in the triangle ♎EC there are alſo given the angles at ♎ and C, whence ♎EC is given (= 109° 12'); therefore in the triangle $*lE$ the angle at E being given, and $*l$ (= Arcturus's latitude), El is found = 12° 3', and ♎l = 17° 47', which, added to 19° 53' 52", gives the preceſſion of the equinox from Heſiod's time to 1609, equal to 37° 30' 52", anſwering to the year 997 A. C. or rather 1002 A. C.

But the arc ♎ E ought to be ſhortened at leaſt 1° for the refractions of the ſun and Arcturus, and the ſun's ſemidiameter: Now 1° in the arc ♎ E (or an error of a day in Heſiod's account) leſſens the arc ♎ l, 50′, and conſequently the time 60 years. So that Arcturus's image was in the horizon 60 days after the winter ſolſtice, juſt when the ſun diſappeared about the year 940 A. C. But how far the data are to be depended upon as accurate, I am not able to ſay, conſidering that Heſiod lived before aſtronomy was cultivated among the Greeks as a ſcience, and theſe obſervations of the ſtars were uſed to mark the returns of the ſeaſons in the room of an artificial year, which they wanted. There is one thing indeed might be alledged in favour of thoſe who place him rather later than earlier, that ſuppoſing the obſervations made with all the accuracy they were capable of at firſt, yet it might have been done before Heſiod's time, and retained for the ſake of a round number, whilſt its deviations from truth were not very great, or the cauſe of the alteration known.

Mr. *G. Witchell*, the propoſer of this queſtion, proceeds in the ſolution of it exactly in the ſame manner. He makes the year ſought to be ant. Chr. 1007; which he thinks is near a century and half earlier than the truth; but obſerves, that, if the ſun had been ſet about 8½^h before the appearing of the ſtar, the reſult would give ann. ant. Chr. 870, as Sir Iſaac Newton has it in his chronology. Hence he infers, that chronologers ought not to lay any great ſtreſs, or too much reſt their ſyſtems on ſuch obſervations as theſe.

XI. QUESTION 458 *anſwered by Mr.* G. Witchell.

Let P repreſent the north pole, Z the zenith, and Ss a very ſmall part of the ſun's parallel of declination deſcribed in a given particle of time; ſuppoſing PS, Ps, ZS, Zs to be drawn, and alſo sn perpendicular to ZS. It is evident, that the celerity with which the man (walking uniformly in the direction of his own ſhadow) leaves the meridian, is always proportional to the ſine of the ſun's azimuth PZS; but the ſine of PZS is to the ſine of PSZ (or Ssn) in the conſtant ratio of the ſine of PS to the ſine of PZ: And the ſine of Ssn is, again, to Sn (the increaſe of the ſun's altitude) in the given ratio of the ſine Sns to Ss. Therefore

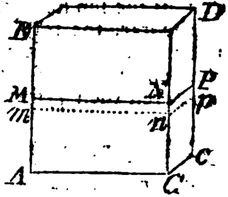


fore the celerity aforeſaid is every where in a conſtant ratio to that with which the ſun's altitude increaſes: And conſequently the whole increaſe of the ſun's altitude, from ſix, to 50 m. after 10 o'clock (found by calculation to be 37° 27′) muſt be to 12 m. (the whole of the man's weſting during that time) in that ſame conſtant proportion. But the increaſe of the ſun's altitude when due eaſt (at which time the man leaves the meridian with his abſolute motion) is at the rate of 8° 53¾′ per hour, or at the rate of 42° 57′ in 4 h. 50 m. (the whole time of walking): Therefore it will be, as 37° 27′ : 42° 57′ :: 12 miles : 13·76 = whole diſtance travelled; being at the rate of 2·85 miles per hour. *Q. E. I.*

Thus alſo (without the leaſt variation in principles) the ſolution is given by Meſſ. *O'Cavanah*, *Philarithmus*, *Rollinſon*, and *Wildbore*.

XII. QUESTION 459 *anſwered by Mr.* Hugh Brown (*the Propoſer*).

Put a = the depth (AB) of the ciſtern, and b = 190·638 = the content, in ale gallons: Then, denoting the three given quantities 2, 5, and 150, by m, n, and p, and calling AM, x; Mm, $-\dot{x}$, and the time of the deſcent through BM, t; we ſhall have $n\sqrt{\frac{x}{a}}$ = the rate (per min.) at which the water runs out; and conſequently $n\sqrt{\frac{x}{a}} - m$ = the rate of the real decreaſe. Hence, as $1 : \dot{t} :: n\sqrt{\frac{x}{a}} - m : -\frac{b\dot{x}}{a}$ (= the decreaſe $MNPpnm$, in the time $\dot{t}$); whence whence $\dot{t} = -\frac{\dot{x}}{a} \times \frac{b}{n\sqrt{\frac{x}{a}} - m}$: Alſo (by the queſt.) we have $mt + b \times \frac{a-x}{a} = p$. But to find the value of t from theſe equations, let y be aſſumed $= \frac{n\sqrt{\frac{x}{a}} - m}{n - m}$ (= the ratio of

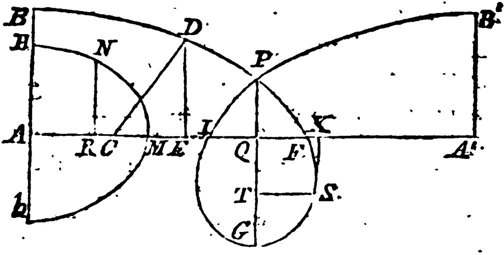


of the decreaſe at *M* to that at *B*) ſo ſhall $\frac{x}{a}=\frac{\overline{\overline{n-m}\times y+m}^{\,2}}{nn}$,

and $\dot{t}=-\frac{2b}{nn}\times\overline{\overline{n-m}\,.\,\dot{y}+\frac{m\dot{y}}{y}}$; and conſequently $t=\frac{2b}{nn}\times\overline{\overline{n-m}\times\overline{1-y}\;m-\times\text{h.}\log.y}$. But, by the other equation, t is $=\frac{p}{m}-\frac{b}{m}+\frac{b}{m}\times\frac{\overline{\overline{n-m}\times y+m}^{\,2}}{nn}$: Which two values being compared together, we get $\overline{\frac{n}{m}-1}\times 2y+\overline{\frac{n}{m}-1}^{\,2}\times\frac{yy}{2}+\text{h.}\log.y=\overline{\frac{n}{m}-1}\times\overline{\frac{n}{2m}+\frac{3}{2}}-\frac{nnp}{2mmb}$: Which, in the preſent caſe, becomes $3y+\frac{9yy}{8}+\text{h.}\log.y=1{\cdot}666$; whence y is found $={\cdot}5954$; and from thence $t=34{\cdot}33=34$m. 20s. the time required.

In a manner very little different, this problem is alſo anſwered by Mr. *G. Brown*, Mr. *R. Butler*, *Philarithmus*, and Mr. *Cha. Wildbore*; whoſe ſolutions are equally conciſe and elegant.

XIII. QUESTION 460 *anſwered by Mr.* O'Cavanah, *of* Dublin.



Let *BDPFGIPB'* be the required curve, deſcribed (in a whole revolution) by the body *D* moving uniformly therein, whilſt the other propoſed body *C* moves uniformly over the right line *AA'*: Conceive *CD*, and alſo the ordinate *DE*, to be drawn; putting *CD* (= *AB*) = *a*, *CE* = *z*, *AC* = *x*, *AE* = *y*, and the given ratio of *BD* to *AC*, as *n* to 1.

Then will $DE = \sqrt{aa-zz}$, $AE = x+z$, and $BD = nx$; whereof the fluxions being $\frac{-z\dot{z}}{\sqrt{aa-zz}}$, $\dot{x}+\dot{z}$, and $n\dot{x}$, we therefore have $\frac{z^2\dot{z}^2}{aa-zz} + \overline{\dot{x}+\dot{z}}|^2 = n^2\dot{x}^2$; whence $\dot{x} = \frac{\dot{z}}{nn-1} + \frac{\dot{z}\sqrt{nnaa-zz}}{\overline{nn-1}\times\sqrt{aa-zz}}$, and $\dot{y}$ ($=\dot{x}+\dot{z}$) $= \frac{nn\dot{z}}{nn-1} + \frac{\dot{z}\sqrt{nnaa-zz}}{\overline{nn-1}\times\sqrt{aa-zz}}$. To aſſign the fluent hereof, let HMb be a ſemi-ellipſis, having its greater ſemi-axis $AM = AB$, and its leſſer (AH) in proportion thereto, as $\sqrt{nn-1}$ is to n; then take $AR = z$ ($= CE$) and make RN parallel to AB; ſo ſhall the arch $HN =$ the fluent of $\frac{\dot{z}\sqrt{nnaa-zz}}{n\sqrt{aa-zz}}$; and we ſhall therefore have $y = \frac{nn\times AR + n\times HN}{nn-1}$; whence the ordinate DE is alſo known.

COROLLARY I. When z becomes $= a$, then y is $= AF = \frac{nn\times AM + n\times HNM}{nn-1}$: But, when the two bodies are arrived at Q and G, and z is (again) $= 0$, then $y = AQ = \frac{n\times HMb}{nn-1}$; whence QF ($= AF - AQ$) is given $= \frac{nn\times AM - n\times HM}{nn-1}$.

COROLLARY II. If $\dot{y}$ ($= \frac{nn\dot{z}}{nn-1} + \frac{\dot{z}\sqrt{nnaa-zz}}{\overline{nn-1}\times\sqrt{aa-zz}}$) be taken $= 0$, then will $z = \frac{na}{\sqrt{nn+1}}$; and the ordinate ($KS$) which touches the curve in (S) will be $= \frac{a}{\sqrt{nn+1}}$ ($= \sqrt{aa-zz}$). And if the value of y correſponding hereunto be found (by the general equation) and from it AQ be ſubtracted, we ſhall thence get the ſemi-breadth QK (or ST) of the nodus $PSGIP$.——As to the value of PQ it may be alſo obtained from the ſame equation, by which $\frac{nn\times AR + n\times HN}{nn-1}$ ($= y$) is given $= \frac{n\times HMb}{nn-1}$ ($= AQ$):

Where, if the value of n be ſuch, that $AR = AM$, then will $n \times AM = HM$, or $n : 1 :: HM : AM$; in which caſe the interſection P will fall in the line AA': But if the value of n be leſs than that determinable from hence (but ſtill greater than unity) the point P will then fall below the line AA', but never by a diſtance greater than $\frac{2884}{10000} \times AB$; nor can ST be ever leſs than $\frac{266}{1000} \times AB$.

When n is ſuppoſed leſs than unity, the radical quantity $\sqrt{nnaa - zz}$ will be no longer poſſible than till z becomes $= na$; after which time the conditions of the problem can no longer be fulfilled.

Mr. *Tho. Allen*, Mr. *E. Rollinſon* (the propoſer), Peter Walton, Mr. *Cha. Wildbore*, and *Philarithmus*, have alſo favoured us with very elegant ſolutions to this problem. The laſt of theſe gentlemen (who is a new correſpondent, and whoſe judgment and accuracy we greatly approve) has to his ſolution ſubjoined ſome curious obſervations, differing very little from thoſe above exhibited.

XIV. QUEST. 461 *anſwered by* (*the Propoſer*) P. Walton.

$\frac{\dot{x}}{\sqrt{1 - x^4}}$ being $= \frac{\dot{x}}{\sqrt{1 - x^2} \times \sqrt{1 + x^2}} = \frac{\dot{x}}{\sqrt{1 - x^2}} \times \overline{1 - \frac{1}{2}x^2 + \frac{1.3}{2.4}x^4 - \frac{1.3.5}{2.4.6}x^6 + \&c.}$ it appears, by art. 286 of Mr. Simpſon's Fluxions, that the whole fluent of $\frac{\dot{x}}{\sqrt{1 - x^4}}$ is $= f \times \overline{1 - \frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} - \frac{1^2.3^2.5^2}{2^2.4^2.6^2} + \&c.}$ f being the whole fluent of $\frac{\dot{x}}{\sqrt{1 - x^2}}$; which, it is well known, is $\frac{1}{4}$ of the periphery of a circle whoſe radius is 1.

Now, if x be ſuppoſed $= y^{\frac{1}{2}}$, $\frac{\dot{x}}{\sqrt{1 - x^4}}$ will be $= \frac{\frac{1}{2}y^{-\frac{1}{2}}\dot{y}}{\sqrt{1 - y^2}}$ of which fluxion (by Landen's Math. Lucubrations, p. 146) the whole fluent is $\frac{e + \sqrt{ee - 2f}}{2}$, e denoting $\frac{1}{4}$ of the periphery of an ellipſis, whoſe ſemi-axes are $\sqrt{2}$ and 1.

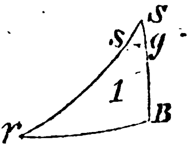
It follows therefore, that the ſeries $1 - \frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} - \frac{1^2.3^2.5^2}{2^2.4^2.6^2} +$ &c. is $= \frac{e + \sqrt{ee - 2f}}{2f}$.

The ſame anſwered by Mr. E. Rollinſon.

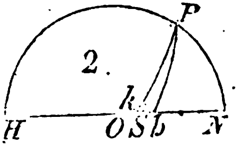
It is well known that $\mathcal{Q} \times : 1 - \frac{1}{2}e + \frac{1.3}{2.4}f - \frac{1.3.5}{2.4.6}g +$ &c. is the fluent of $\frac{\dot{x}}{\sqrt{1-x^2}} \times : 1 - ex^2 + fx^4 - gx^6 +$ &c. when $x = 1$; $\mathcal{Q}$ being the fluent of the firſt term, and $= \frac{1}{4}$ of the periphery of the circle whoſe radius is unity. Hence, if e be taken $= \frac{1}{2}$, $f = \frac{1.3}{2.4}$, $g = \frac{1.3.5}{2.4.6}$, &c. then $\mathcal{Q} \times : 1 - \frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} - \frac{1^2.3^2.5^2}{2^2.4^2.6^2} +$ &c. $=$ flue. $\frac{\dot{x}}{\sqrt{1-xx}} \times : 1 - \frac{1}{2}x + \frac{1.3}{2.4}x^2$ &c. $=$ flu. $\frac{\dot{x}}{\sqrt{1-xx}} \times \overline{1+xx}^{-\frac{1}{2}} =$ flu. $\frac{\dot{x}}{\sqrt{1-x^4}}$; and conſequently $1 - \frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} - \frac{1^2.3^2.5^2}{2^2.4^2.6^2} = \frac{1}{\mathcal{Q}} \times$ flu. $\frac{\dot{x}}{\sqrt{1-x^4}}$ (when $x = 1$). But $\frac{\dot{x}}{\sqrt{1-x^4}} + \frac{x^2\dot{x}}{\sqrt{1-x^4}}$ is known to expreſs the fluxion of an elliptical arch (R), whoſe two ſemi-axes are 1 and $\sqrt{2}$; and it may be eaſily demonſtrated (by the method laid down at p. 76. in Simpſon's Laws of Chance) that, if A and B be aſſumed to denote the fluents of $\frac{x^{pn-1}\dot{x}}{\sqrt{1-x^n}}$ and $\frac{x^{pn+\frac{1}{2}n-1}\dot{x}}{\sqrt{1-x^n}}$ (p and n being any poſitive numbers whatever), the product AB (of theſe fluents) will (when $x = 1$) be $= \frac{2\mathcal{Q}}{pnn}$. Whence, by taking $n = 4$, and $p = \frac{1}{4}$, ſo that A and B may become the reſpective fluents of $\frac{\dot{x}}{\sqrt{1-x^4}}$ and $\frac{x^2\dot{x}}{\sqrt{1-x^4}}$ (whereof the ſum is given above $= R$) we have AB, in this caſe, $= \frac{1}{2}\mathcal{Q}$: From whence, and the equation $A + B = R$, A is found $= \frac{R}{2} + \frac{1}{2}\sqrt{R^2 - 2\mathcal{Q}}$; and $\frac{A}{\mathcal{Q}} = \frac{R}{2\mathcal{Q}} + \frac{1}{2}\sqrt{\frac{RR}{\mathcal{Q}\mathcal{Q}} - \frac{2}{\mathcal{Q}}} =$ the ſeries propoſed.

Some correſpondents have objected againſt this queſtion, as an unfair one; "because there can be no reaſoning from the data, to arrive at a concluſion."——We ſhall not take upon us to decide this point; but ſhall not, however, ſcruple to allow, that problems of a very abſtracted and intricate nature, are leſs eligible in a work like this, than ſome others of a different kind.

The PRIZE QUESTION *anſwered by* Κυβερνήτης.



Let sS repreſent the increaſe of the ſun's longitude (♈ s in the ſpace of ſix hours (or $\frac{1}{4}$ of a day); and let Sg be the increaſe of declination correſponding thereto: Putting $s =$ ſin. ♈, $x =$ ſin. ♈ S, and $u = sS$: Then will $sx =$ ſin. dec. BS, and $\sqrt{1 - ssxx}$ = its coſine: And ſo, again (per ſpherics) tang. ♈ S ($\frac{x}{\sqrt{1-xx}}$) : tang. BS ($\frac{sx}{\sqrt{1-ssxx}}$) :: rad. : coſ. S :: sS (u): $Sg = \frac{su\sqrt{1-xx}}{\sqrt{1-ssxx}}$ = the increaſe of decl. in $\frac{1}{4}$ of a day. Therefore, if m be now put for $\frac{1}{4}$ of the whole periphery of the circle (the rad. being unity) and z for the arch, expreſſing the ſun's aſcenſional difference, we ſhall have $m : m + z :: \frac{su\sqrt{1-xx}}{\sqrt{1-ssxx}} : \frac{su \times \overline{m+z}\sqrt{1-xx}}{m\sqrt{1-ssxx}}$ = the alteration of decl. from noon to ſun-ſetting. Let this (*in Fig.* 2.) be denoted by Sk (ſuppoſing PS to be the ſun's polar diſtance, and S his place in the horizon HON, at ſetting) and let b, c, and d be taken to expreſs the ſine, coſine, and tangent of PN the pole's elevation. Then, the ſine of S being $= \frac{b}{\sqrt{1-ssxx}}$, its coſine will be $= \frac{\sqrt{cc-ssxx}}{\sqrt{1-ssxx}}$; whence $bk = \frac{b}{\sqrt{cc-ssxx}} \times (Sk)$; and conſequently ſin. kPb ($= \frac{\text{rad}}{\text{ſin.}\ PS} \times (bk) =$



$$\frac{b\times(Sk)}{\sqrt{1-ssxx}\times\sqrt{cc-ssxx}}=\frac{bsu\times\overline{m+z}.\sqrt{1-xx}}{m\times\overline{1-ssxx}\times\sqrt{cc-ssxx}};$$

which, by the queſtion, is to be a maximum; and conſequently its log. ($=$ log. $\frac{bs}{m}+$ log. $u+$ log. $\overline{m+z}+\frac{1}{2}$ log. $\overline{1-xx}$ $-$ log. $\overline{1-ssxx}-\frac{1}{2}$ log. $\overline{cc-ssxx}$) a maximum alſo: Whence, in fluxions (putting $t=\frac{s}{c}$) we have $\frac{\dot{u}}{u}+\frac{\dot{z}}{m+z}$ $-\frac{x\dot{x}}{1-xx}+\frac{2ssx\dot{x}}{1-ssxx}+\frac{ttx\dot{x}}{1-ttxx}=0$; but the ſine of z being $\frac{bsx}{c\sqrt{1-ssxx}}$, we have $\dot{z}=\frac{bs\dot{x}}{\overline{1-ssxx}.\sqrt{cc-ssxx}}$ $=\frac{ds\dot{x}}{\overline{1-ssxx}.\sqrt{1-ttxx}}$. And, to determine u and $\dot{u}$ alſo, let e ($=$ ·017) denote the eccentricity of the earth's orbit, and y the ſine complement of the ſun's (or earth's) true anomoly: Then $\overline{1-ey}|^{2}$ (as is well known) will expreſs the rate of the increaſe of this anomoly (or of the ſun's true longitude): Which quantity being therefore proportional to u, we have $\frac{\dot{u}}{u}=\frac{-2e\dot{y}}{1-ey}$. But the difference of the two angles, whoſe ſines are y and x, being given ($=9^\circ\ 14'$) $=$ the diſtance of the aphelion from the ſolſtitial point, we ſhall, by putting the ſin. of $9^\circ\ 14'=p$, and its coſine $=q$, have $y=qx-p\sqrt{1-xx}$; and conſequently $\frac{\dot{u}}{u}$ ($=\frac{-2e\dot{y}}{1-ey}$) $=-\frac{2eq\dot{x}\sqrt{1-xx}-2epx\dot{x}}{\sqrt{1-xx}\times\overline{1-eqx+ep\sqrt{1-xx}}}=-2e\dot{x}$, very near (becauſe the terms both in the numerator and denominator, after the two firſt, may, on account of their ſmallneſs, be neglected). This value, therefore, with that of $\dot{z}$, being ſubſtituted in the general equation, we at length get $-2e+\frac{ds}{\overline{1-ssxx}\sqrt{1-ttxx}\times\overline{m+z}}-\frac{x}{1-xx}+\frac{2ssx}{1-ssxx}$ $+\frac{ttx}{1-ttxx}=0$: From whence, (either by reſolving the whole into a ſeries, or by any other of the known methods of approximation) the value of x may be found, and will come out $=$ ·493 $=$ ſin. of $29^\circ\ 32'$ $=$ ſun's longitude ſought; anſwering to the 19th day of April; the afternoon of that day exceeding the forenoon by $1'\ 4\frac{3}{4}''$.

The

The Eclipſes calculated for 1760, *by* *Mr.* George Witchell.

There will, in the courſe of this year, be four eclipſes, two of the ſun, and as many of the moon; three of which will be viſible to the inhabitants of Great Britain: The calculations whereof from Mayer's Tables, for the meridian and latitude of London, are as follow.

The firſt is a very ſmall eclipſe of the moon, May 29th, in the evening; beginning at 9 h. 14 m. the middle at 9 h. 34 m. the end at 9 h. 54 m. apparent time; the duration being only 40 m. and the quantity eclipſed $\frac{1}{5}$ of a digit.

The ſecond is a ſolar eclipſe, June 13th, in the morning; beginning at 6 h. 33 m. 58 s. the greateſt obſcuration at 7 h. 25 m. 26 s. and the end at 8 h. 12 m. 30 s. apparent time; the duration 1 h. 32 m. 32 s. and the digits eclipſed 4° 33′.—— This eclipſe will be total in the Leſſer Aſia; and the central ſhade will nearly traverſe the ſame route with that celebrated eclipſe of antiquity, May 18th, an. ante Chr. 603, which frightned the Medes and Lydians into a peace, after a five years obſtinate war.

The third is an eclipſe of the moon, Nov. 22d, in the evening; beginning at 7 h. 43 m. the middle at 8 h. 56 m. and the end at 10 h. 3 m. duration 2 h. 25 m. digits eclipſed 6° 10′.

The fourth is an eclipſe of the ſun, Dec. 7th, about two in the afternoon, but inviſible to all theſe northern parts, by reaſon of the moon's great ſouth latitude.

Some other calculations of the ſolar eclipſe (June 13th) by other correſpondents, from different tables, and for different parts of the kingdom, are here ſubjoined.

Calculated by		Beg. h.m.	Mid. h.m.	End h.m.	Dur. h.m.	Dig.
Mr. T. Harris, from Brent's Tab. for	London.	6 37	7 23	8 10	1 33	4 33
	Bugbrook	6 33	7 17	8 4	1 31	4 12
Mr. T. Allen, for Spalding		6 38	7 26	8 17	1 39	5 10
Mr. J. Goodhead, for Nottingham		6 35	7 16	8 8	1 33	4 24

Other calculations of this eclipſe have been received, but ſo very incorrect, and wide of the truth, that it would do the authors no credit to inſert them.

New Queſtions.

I. QUESTION 462, *by Miſs* Ann Nichols.

Two partners, in a venture made,
Gain twice two hundred pounds in trade:
The ſtock of *A*, when they began,
Exceeded *B*'s by eighty-one;
Twice ninety-five *B* gain'd in all.
Now for the ſtock of both I call.

II. QUESTION 463, *by Miſs* S. T.

A drover bought in as many ſheep, of different ſorts, as coſt him 48l. one-third of which he ſold again at 20s. a-piece; one-fourth at 18s. and the reſt at 16s. a-piece; and found his gain, upon the whole, to be 5l. 10s. What number of ſheep did he buy and ſell?

III. QUESTION 464, *by Mr.* Tho. Harris.

Near Ouſe's verdant banks, in Bedfordſhire,
Stands Carlton, bleſt with a ſagacious fair,
In whom at once Minerva's wit is ſeen,
Diana's chaſtneſs, and the graces' mien.
Would you the name of this fair charmer know,
Firſt ſolve th' equations which you'll find below.

* $x+y+z+v=56$, $xx+yy+zz+vv=910$, $xv+2yy-zz=6$, $z=2y$; in which the values of x, y, z, and v denote the places in the alphabet of the four letters that compoſe this amiable maiden's name.

IV. QUESTION 465, *by Mr.* Geo. Brown.

From two given points, to draw two lines to meet in the circumference of a circle given in poſition and magnitude, ſo that the ſum of their ſquares ſhall be the leaſt poſſible.

V. QUESTION 466, *by Mr.* W. Chapman.

There are three circles whoſe diameters are 30, 20, and 25 inches; having their centers all placed in the ſame right line; whereof the diſtance of the firſt from the ſecond is 30, and of the ſecond from the third 28 inches; now you are deſired to determine the diameter and poſition of a fourth circle, that ſhall touch all the three given ones.

VI. QUESTION 467, *by Mr.* W. Spicer.

Surveying of a field, I found the four ſides thereof to be 10, 9, 7, and 6 chains, in a ſucceſſive order; I likewiſe, at the two extremes of the longeſt ſide, took the bearings of the oppoſite angles, which were N. E. by E. and W. S. W. Hence the content of the field is required.

VII. QUESTION 468, *by Mr.* Tho. Barker.

Suppoſing two ſides of a triangle to be given (equal to 25 and 16 feet), whereof the greater is parallel to the horizon; I deſire to know what the length of the third ſide muſt be, ſo that the time of the deſcent of a heavy body along the ſame, ſhall be a minimum?

VIII. QUESTION 469, *by* P. M. *of* Durham.

To deſcribe the circumference of a circle through two given points, and which ſhall cut off from a given circle an arc equal to an arc given.

IX. QUESTION 470, *by Mr.* Patrick O'Cavanah.

On one of the banks of a certain river, ſtand four windmills, all in the ſame right line; whereof the diſtance of the firſt from the ſecond is known to be 150 yards, of the ſecond from the third 180 yards; and of the third from the fourth 200 yards: Being on the oppoſite ſhore, I found by obſervation, that the two middlemoſt of them ſubtended an angle, at my eye, of 15 degrees; and that the angles ſubtended by the firſt and ſecond, and by the third and fourth, were equal, the one to the other. Hence I would know (by means of a geometrical conſtruction) not only the breadth of the river, but alſo my particular diſtance from each of the four objects, in that ſtation.

X. QUESTION 471, *by Mr.* E. Rollinſon.

A gentleman bought an eſtate in houſes for 1500 l. which, being let, brought him in 120 l. per annum, clear of all expences and deductions: At the end of ten years (moſt of the houſes being out of repair, and he not chooſing to be at the expence of fitting them up) he ſold the whole eſtate again for 800 l. The queſtion is, to find what intereſt he made of his money.

[Simpson] 1760.

XI. QUESTION 472, *by Mr. W. Bevil.*

Out of a ſemi-circular piece of ground, the radius of which 12 chains, I am to take a garden with a fence of 10 chains erminating in the ſemi-circle) ſo as to contain the moſt nd poſſible; how many acres will fall to my lot.

XII. QUESTION 473, *by Mr. Cha. Wildbore.*

'Tis propoſed to divide a given number into ſo many parts, and ſo proportioned among themſelves, that the continual product of the firſt, the ſquare of the ſecond, the cube of the third, the biquadrate of the fourth, &c. ſhall be a maximum.

XIII. QUESTION 474, *by Mr. G. Witchell.*

Suppoſing the moon's declination, as well as the latitude of the place to be given, I deſire to know, at what elevation her parallax in altitude increaſes the quickeſt.

PRIZE QUESTION, *by Mr. O'Cavanah.*

Two perſons, *A* and *B*, whoſe chances for winning a ſingle game are as 3 to 2, the former having ſeven guineas, and the latter four, enter into play together, on condition that *A* every game ſhall ſtake two guineas to *B*'s one; and that the play between them ſhall continue till one of them, either by having loſt his whole ſtock, or for want of a complete ſtake, is obliged to give out. The queſtion now is; to find the gain of *B* and the diſadvantage of *A* in this agreement; *and to* point out a general method for the reſolution of all queſtions of this nature.

Note, M. *De Moivre*'s method will not give a true ſolution; nor has there been any rule or method yet laid down by which ſuch queſtions can be reſolved.

The queſtion by the very ingenious *Peter Walton*, requiring the ſum of the ſeries $x+\frac{x^2}{4}+\frac{x^3}{9}+\frac{x^4}{16}$ &c. by means of circular arcs and logarithms, when $x=\frac{1}{2}$, *is of* [illegible] limited, and very intricate nature, and what we could not [illegible] care to give in form among the others, as ſome of our [illegible] reſpondents, we are convinced, would not allow it to be [illegible] one. It is worthy, however, to be obſerved, that it [illegible] in the caſe here propoſed (where $x=\frac{1}{2}$) and in that where $x = 1$ (x being poſitive), in which the ſeries ſeems poſſible to be ſummed in the manner ſpecified. With reſpect to the latter caſe, it is pretty well known, at this time, that the ſum is expreſſed by two-thirds of the ſquare of the ſemi-circumference of the circle whoſe diameter is unity. And if from the half of this ſum, half the ſquare of the h. log. of the number 2 be ſubtracted, the remainder will be the ſum in the other caſe, where $x = \frac{1}{2}$. How this is derived, is left to be conſidered (agreeably to the deſign of the propoſer) by thoſe who have a taſte for theſe ſubtile and abſtracted ſpeculations.

1761.

Queſtions anſwered.

I. QUESTION 462 *anſwered.*

SINCE, according to the queſtion, *A*'s ſtock exceeded *B*'s by 81 l. and his gain was greater than *B*'s by 20 l. Say therefore as 20 : 81 :: 210 (*A*'s whole gain) 850 l. 10 s. the ſtock of *A*; whence that of *B* is found = 769 l. 10 s.

Thus the ſolution is given by *Amaryllis*, Mr. *S. Burt*, Mr. *J. Collins*, Mr. *W. Hervey*, Mr. *E. D. Hudſon*, Mr. *W. Ingram*, *Leſbia*, Mr. *W. Litſon*, Mr. *R. Locke*, Mr. *R. Marſh*, Mr. *R. Miles*, Mr. *R. Morris*, Mr. *G. Nokes*, *Mall Ormiſhaw*, Mr. *Sim. Pedley*, Mr. *T. Sandling*, Mr. *W. Snaith*, Mr. *R. Spencer*, Mr. *Mat. Ward*, Mr. *T. Wilkin*, and others. ——A great number of algebraical ſolutions to this queſtion have likewiſe been received from Mr. *W. Beer*, Mr. *T. Bromhall*, Mr. *E. Hare*, Mr. *T. Jones*, Miſs *A. Nicholls* (the propoſer), Mr. *J. Lyon*, Mr. *T. Sadler*, Mr. *J. Scholfield*, Mrs. *Eleanor Suggett*, Mr. *T. Wilſon*, and others.

II. QUESTION 463 *anſwered.*

Let $12x$ = the number of ſheep; then, by the queſtion, $4x \times 20 + 3x \times 18 + 5x \times 16 = 1070$, or $214x = 1070$: Therefore $x = 5$, and $12x = 60$, the number of ſheep required.

In this manner it is reſolved by Mr. *T. Bromhall*, Miſs *S. Fenwick*, Mr. *E. Hare*, Mr. *Joſ. Harrington*, Mr. *D. Haſtings*, Mr. *E. Houlſton*, Mr. *G. Langley*, Mr. *R. Mallook*, Mr. *Chr. Meſhan*, Miſs *Nicholls*, Mr. *T. Ree*, Mr. *Alex. Rowe*, Mrs. *E. Suggett*, Mr. *T. Walker*, and ſome others. ——Mr. *Jonath. Aſhton*, *Amaryllis*, Mr. *J. Chapman*, Mr. *W. Comport*, Mr. *Nath. Cory*, Mr. *T. Crump*, Mr. *R. Dening*, Mr. *J. Hitchcock*, *Leſbia*, Mr. *E. Lowe*, Mr. *W. Penn*, *Mall Ormiſhaw*, Mr. *J. Salter*, Mr. *R. Spencer*, Mr. *W. Wells*, Mr. *T. Wilkin*, and Mr. *E. Wright*, likewiſe obliged us with ſolutions to this problem.

III. QUESTION 464 *anſwered by Mr.* Edw. Nott, *of* Stamford.

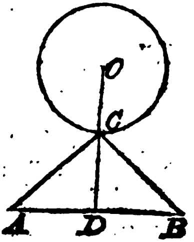
By writing $2y$ for z, the equations become $x+3y+v$ $(=56)=a$, $xx+5yy+vv$ $(=910)=b$, $vx-2yy$ $(=6)$ $=c$: To the ſecond of which add twice the third, and we ſhall have $xx+2vx+vv$, or $\overline{x+v}^2=b+2c-yy=\overline{a-3y}^2$ (by the firſt); whence $10yy-6ay=b+2c-aa$; from which y will be found $=9$, $x=8$, $z=18$, and $v=21$: And the letters of the alphabet correſponding, whereby the young lady's name is formed, are, *I*, *H*, *S*, *W*.

I WISH I cou'd ſuch a fair charmer diſcloſe,
I'd gladly another equation propoſe,
Which the beſt algebraiſt ſhould never untie,
Tho' he puzzled for ever with v, x, and y.

In this ſame manner the ſolution is alſo given by Mr. *T. Allen*, Mr. *T. Atkinſon*, Mr. *T. Baker*, Mr. *T. Barker*, Mr. *E. Barras*, Mr. *E. Batten*, Mr. *T. Bramley*, Mr. *J. Buddles*, Mr. *W. Burtoft*, Mr. *Chr. Cave*, Mr. *J. Collins*, Mr. *T. Cooke*, Mr. *W. Embleton*, Miſs *S. Fenwick*, Mr. *W. Terrill*, Mr. *N. Gerrad*, Mr. *R. Gibbons*, Mr. *J. Hampſon*, Mr. *Joſ. Harrington*, Mr. *D. Haſtings*, Mr. *J. Hicks*, Mr. *Malachy Hitchins*, Mr. *J. Honey*, Mr. *E. Houlſton*, Mr. *S. Kemp*, Mr *E. Lowe*, Mr. *R. Mallock*, Mr. *Chr. Meſhan*, *Neſnihctuh*, Mr. *J. Orchard*, Mr. *T. Roe*, Mr. *A. Rowe*, Mr. *Joſ. Ruſton*, Mr. *T. Sadler*, Mr. *T. Walker*, Mr. *R. Walton*, and Miſs *M. Williams*.

IV. QUESTION 465 *anſwered by Mr.* Davies, *of* Newent, Glouceſterſhire.

Let *O* be the center of the given circle, and let *A* and *B* be the two given points; draw the line *AB* and biſect it in *D*; alſo draw *DO* cutting the circle in the point *C*. So ſhall *AC* and *BC*, when drawn, be the two lines required.

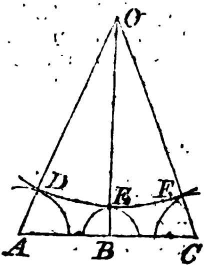


DEMONSTRATION. Becauſe *DC* is a minimum, or the ſhorteſt line that can poſſibly be drawn from *D* to the circumference of the circle, $2DC^2$ muſt therefore be a minimum. But, by a known theorem, $AC^2+BC^2=2AD^2+2DC^2$; therefore ($2AD^2$ being conſtant) AC^2+BC^2 muſt be a minimum. *Q. E. D.*

In the very ſame manner the problem was conſtructed by Mr. *Joſ. Brampton*, Mr. *T. Moſs*, and Mr. *G. Witchell.* ——Meſſ. *T. Allen*, *T. Barker*, *J. Buddles*, *W. Chapman*, *W. Terrill*, *J. Hampſon*, *T. Harris*, *E. Nott*, *Noſnibctuh*, *Steph. Ogle*, *Paul Sharp*, *W. Spicer*, and ſome others, have given neat algebraic ſolutions to it.

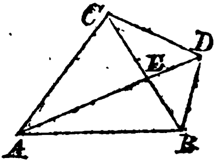
V. QUESTION 466 *anſwered by Mr.* E. Batten.

Let *A*, *B*, and *C* be the centers of the three given circles, and *O* that of the required one. Put $a=AB=30$, $b=BC=28$, $c=AC=58$, $x=OB$, $x+m\ (x+5)=OA$, $x+n\ (x+2\frac{1}{2})=OC$. Now (per lemma, p. 128 of Mr. Simpſon's Select Exercifes) $\overline{x+m}^2\times b+\overline{x+n}^2\times a=\overline{xx+ab}\times c$; that is, $\overline{b+a}.xx+\overline{2bm+2an}.x+bmm+ann=cxx+abc$; or, $\overline{2bm+2an}.x+bmm+ann=abc$; whence $x=\frac{abc-bmm-ann}{2bm+2an}=111\frac{41}{172}$; and conſequently the radius *OE* of the required circle $=101\frac{41}{172}=101{\cdot}238$, &c.



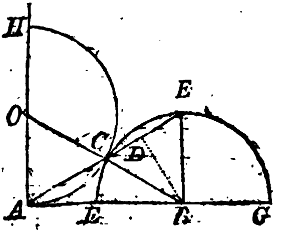
In the ſame, or in a manner very little different, the anſwer is alſo brought out by Meſſ. *T. Allen*, *W. Chapman*, *L. Charlton*, *W. Davies*, *J. Hampſon*, *T. Harris*, *S. Kemp*, *W. Kingſton*, *E. Nott*, *P. Sharp*, *W. Spicer*, and *T. Walker*.*

VI. QUESTION 467 *anſwered.*



The beſt ſolutions that have been received to this queſtion are derived in virtue of a theorem in the Diary for 1758, where it is demonſtrated, that the quadruple of the area of any trapezium *ABCD* is equal to $AB^2+CD^2-BC^2-AD^2 \times \frac{\text{tang. } E}{\text{rad.}}$: From whence (ſeeing the angle *E*, as well as the ſides of the trapezium, are all given, by the queſtion) the area will be found: And in this manner is actually determined by Mr. *W. Davies*, Mr. *D. Haſtings*, Mr. *T. Harris*, Mr. *Malachy Hitchins*, Mr. *W. Kingſton*, Mr. *W. Spicer*; and is found to be $= 8 \times \text{tang. } 11° \, 15' =$ 1'5912992 ſquare chains.——If the ſecond bearing were to be W. N. W. inſtead of W. S. W. (as it was miſprinted) the angle made by the two diagonals would then be 5 points (inſtead of one); and the area of the trapezium 11'9728 ſquare chains.

VII. QUESTION 468 *anſwered by Mr.* T. Barker, *the Propoſer.*



CONSTRUCTION. With a radius *BE* equal to the leſſer of the two given ſides, let a ſemi-circle *EFG* be deſcribed; and, having made *BE* perpendicular to the longer ſide *AB*, draw *AF*, cutting the circle in *C*, from whence draw *CB*: Then ſhall *ABC* be the triangle ſought.

DEMONSTRATION. Make *AH* perpendicular to *AB*, and produce *BC* to meet it in *O*; and with the radius *OA* or

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* A *Conſtruction* of this prob. may be ſeen in Lawſon's Apollonius on Tangencies.

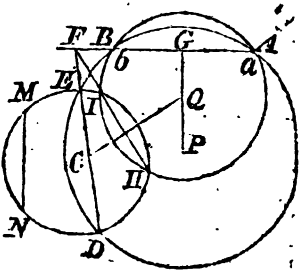
OC (for they are equal, becauſe *BF* and *BC* are equal) let the ſemi-circle *ACH* be deſcribed. Then the time of deſcent along the chord *CA* will, it is well known, be equal to the time of deſcent along the diameter *HA*; which will be the ſhorteſt poſſible, becauſe the ſemi-circle *ACH* only touches the given one *ECFG*, and has therefore its diameter leſs than any other ſemi-circle that can be deſcribed on *AH* produced, to cut the given ſemi-circle *ECFG*. *Q.E.D.*

CALCULATION. In the right-angled triangle *ABF*, are given $AB = 25$, and $BF = 16$; whence the angle $BAF = 32° 27'$: Then in the triangle *ACB* are given *AB*, *BC*, and the angle *A*; whence $AC = 12{\cdot}425$.

But the required ſide *AC* may be otherwiſe found, independent of trigonometry: For, if *BD* be made perpendicular to *AF*, then, by ſimilar triangles, $FD = \frac{BF^2}{AF}$; and conſequently $AC \; (= AF - 2FD) = \frac{AF^2 - 2BF^2}{AF} = \frac{AB^2 - BF^2}{\sqrt{AB^2 + BF^2}} = 12{\cdot}425$.

In the very ſame manner the problem is conſtructed, and determined, by Mr. *J. Brampton*, Mr. *W. Davies*, Mr. *R. Gibbons*, Mr. *J. Hicks*, Mr. *T. Harris*, and Mr. *G. Witchell*. ——Very neat algebraic ſolutions to it have alſo been received from Mr. *T. Allen*, Mr. *J. Buddles*, Mr. *L. Charlton*, Mr. *T. Cooke*, Mr. *W. Embleton*, Miſs *Sally Fenwick*, Mr. *D. Haſtings*, Mr. *J. Honey*, Mr. *S. Kemp*, Mr. *E. Nott*, Mr. *P. Sharp*, Mr. *W. Spicer*, and others.

VIII. QUESTION 469 *anſwered by Mr.* Lionel Charlton, *of* Whitby.

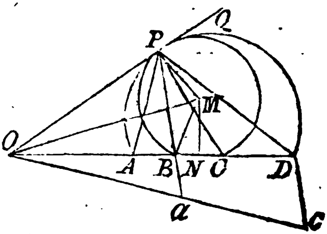


Having drawn *GP* to biſect, at right angles, the line *AB* joining the given points, from any point in it, as *P*, let the circumference of a circle be deſcribed through *A* and *B*, to cut the given circle *EMND* in any two points *D* and *E*; through which points draw *DEF* to meet *AB* produced in *F*; then draw *FH* at an equal diſtance from the center *C* with the chord MN of the arch given; and, perpendicular thereto, draw CQ, meeting GP in Q: So ſhall Q be the center of the circle required.

Demonstration. From the center Q, through H and I, let the circumference of a circle be deſcribed, cutting AF in b and a. Then (by Euc. 36. 3.) $Fa \times Fb = FH \times FI = FD \times FE = FA \times FB$: But (by Euc. 6. 2.) $Fa \times Fb = FG^2 - bG^2$, and $FA \times FB = FG^2 - BG^2$; therefore $FG^2 - bG^2 = FG^2 - BG^2$; and ſo, bG being $= BG$, the points b, B coincide, and the circumference of the circle deſcribed from the center Q, through H and I, likewiſe paſſes through B and A. *Q. E. D.*

IX. QUESTION 470 *anſwered by Mr.* E. Rollinſon.

Let A, B, C, D be the places of the four objects; and upon BC let a ſegment of a circle BPC be deſcribed to contain the given angle of 15°: Make Ba and Dc parallel to each other, taking the former $= BA$, and the latter $= DC$; and then draw caO to meet DA, produced, in O; from whence draw OQ to touch the circle in P, which is the point ſought.



Demonstration. Conceive the circumference of a circle to be deſcribed through A, P, and D. By conſtruction and ſim. △s, $Ba\,(BA) : OB :: Dc\,(DC) : OD$; whence, by diviſion, $OA : OB :: OC : OD$, and conſequently $OA \times OD = OB \times OC = OP^2$ (Euc. 36. 3.) whence it is evident, that OQ alſo touches the circle APD in the point P: But (by Euc. 32. 3.) the angle $APO = ADP$, and $BPO = BCP$; and conſequently $APB\ (BPO - APO = BCP - ADP) = CPD$. *Q. E. D.*

Calculation. Having, from the center M, let fall the perpendicular MN, and drawn MB, MP, MO, &c. it will be as $Dc - Ba$ (50) : BD (380) : : Ba (150) : $BO = 1140$; whence $NO = 1230$. Alſo as NB (90) : NO (1230) : : tang. NMB (15°) : tang. $NMO = 74° \ 43' \frac{1}{2}$; whence $NOM = 15° \ 16\frac{1}{2}$, and $BMO = 59° \ 43\frac{1}{2}$. Again, as ſin. MBN 75°) : rad. (: : $MN : MB\,(MP)$) : : ſin. NOM : ſin. $POM =$

15° 49½'; whence $PMO = 74° 10½'$, PMB $(= PMO + BMO) = 133° 54'$, and BCP $(= \frac{1}{2}PMB) = 66° 57'$: From which the reſt are readily found, viz. $PB = 639{\cdot}93$, $PC = 688{\cdot}61$, APB (or CPB) $= 13° 29\frac{5}{8}'$, $PA = 636{\cdot}49$, $PD = 788{\cdot}66$, and the perpendicular diſtance of P from AD (= breadth of the river) $= 633{\cdot}62$.

Mr. *W. Davies*, Mr. *D. Haſtings*, Mr. *W. Kingſton*, Mr. *E. Nott*, and Mr. *G. Witchell* have conſtructed this problem in the ſame manner.——*P. M.* of *Durham* has alſo obliged us with exceeding neat conſtructions of this, and of the 4th, 7th, and 8th problems; which, had they come to hand in due time, would have obtained a place ſuitable to their merit.

X. QUESTION 471 *anſwered by Mr.* J. Honey, *of* Redruth, Cornwall.

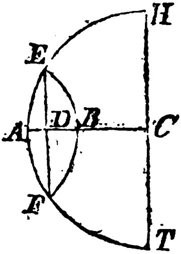
Let $a = 1500$l. $b = 800$l. $c = 120$l. and x the required rate of intereſt: Then will $ax =$ principal and intereſt, and $ax - c =$ amount after the firſt payment is deducted. And in the ſame manner we have $ax^{10} - cx^9 - cx^8 - cx^7 - cx^6 - cx^5 - cx^4 - cx^3 - cx^2 - cx - c = b$ (per queſt.) or $ax^{10} - c \times \frac{x^{10} - 1}{x - 1} = b$; therefore $ax^{11} - \overline{a - c} \times x^{10} - bx + b + c = 0$. Solved, $x = 1{\cdot}04142$, &c. and the rate of intereſt required 4l. 2s. 10d. per cent.

The ſame anſwered by Mr. W. Kingſton, *of* Bath.

Let x be the rate: Then the amount of 1500l. capital; at the end of 10 years, will be $1500x^{10}$; and an annuity of 120l. forborn the ſame time, will amount to $120 \times \frac{x^{10} - 1}{x - 1}$. Therefore, we have $1500x^{10} = 120 \times \frac{x^{10} - 1}{x - 1} + 800$ (by the queſtion), or $x^{10} - {\cdot}08 \times \frac{x^{10} - 1}{x - 1} = \frac{8}{15}$. Put $x = 1 + v$; then x^{10} $(= \overline{1 + v}|^{10})$ being $= 1 + 10v + 45v^2 + 120v^3 + 210v^4$, &c. our equation, by proper ſubſtitution, will become $6{\cdot}4v + 35{\cdot}4v^2 + 113{\cdot}2v^3$, &c. $= \frac{1}{3}$; whence, by reverting the ſeries, v is found $= {\cdot}04142$: And the rate of intereſt ſought is 4l. 2s. 10d. per cent.

Mr. *T. Barker*, Mr. *W. Chapman*, Mr. *R. Dening*, Mr. *T. Harris*, and ſome others, ſent ſolutions to this problem.

XI. QUESTION 472 *anſwered by Mr.* G. Witchell.



It is demonſtrable, even by common geometry (ſee Simpſon's Elem. p. 207, 2d edit.) that the given length, or boundary *EBF* (whatever the length the right line *EF* is to have) muſt form the arch of a circle. In order therefore to the reſolution of this, and other problems of the like nature, it will be very convenient to have ready at hand, a proper ſeries, or near approximation for the area of a circular ſegment (*EBF*) expreſſed in terms of the arch *BE* (a), and its ſine *DE* (s), independent of the radius. Such an approximation is the following one, viz. $2s\sqrt{\frac{2}{3}a\times\overline{a-s}} = EBF$: From whence, by making s variable (and $a = 5$, as given by the queſt.) we have $\frac{\frac{4}{3}aa\dot{s}-2as\dot{s}}{\sqrt{\frac{2}{3}a\times\overline{a-s}}}$ = flux. of the ſegment *EBF*. But the fluxion of the other ſegment *EAF*, if the radius *CA* (= 12) be denoted by r, will, it is well known, be $=\frac{2s^2\dot{s}}{\sqrt{rr-ss}}$. Therefore, when the ſum of both ſegments is a maximum, we ſhall have $\frac{\frac{4}{3}aa\dot{s}-2as\dot{s}}{\sqrt{\frac{2}{3}a\times\overline{a-s}}}+\frac{2s^2\dot{s}}{\sqrt{rr-ss}}=0$; and conſequently $\frac{3a}{2}\times\overline{s-\frac{2}{3}a}^2\times\overline{rr-ss}-s^4\times\overline{a-s}=0$: Whence s is found $=3{\cdot}844$; from which the area of the ſegment *EBF* comes out $=15{\cdot}09$, and that of the ſegment *EAF* $=3{\cdot}26$: And conſequently the whole of both $=18{\cdot}35$ ſquare chains.*

XII. QUESTION 473 *anſwered by Lieut.* Henry Watſon.

Let a repreſent the quantity given, and x the required number of parts into which it is to be divided; and let the firſt (or leaſt) part be denoted by y. Then it is very eaſy to demonſtrate, that the 2d part will be $= 2y$, the 3d part $= 3y$, &c. Therefore we have $y + 2y + 3y + 4y \ldots\ldots + xy$ (or $x \times \frac{x+1}{2} \times y$) $= a$: And conſequently $\frac{2a}{x \times \overline{x+1}}$. Now in order to find ſuch an integer for the value of x, we ſhall make $y \times \overline{2y}^{\,2} \times \overline{3y}^{\,3} \ldots\ldots \times \overline{xy}^{\,x}$, or its equal, $2^2 \times 3^3 \times 4^4 \ldots\ldots \times x^x \times \overline{\frac{2a}{x\cdot\overline{x+1}}}^{\,\frac{x\cdot\overline{x+1}}{2}}$, a maximum, let $z - 1$, and z be wrote ſucceſſively therein, inſtead of x, and let the two quantities thus ariſing, be made equal to each other: So ſhall

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* Although approximating theorems ought not generally to be uſed when they are to be put into fluxions, yet they will ſometimes anſwer pretty well, as is the caſe here.

$$2^2 \times 3^3 \times 4^4 \ldots\ldots \overline{z-1}^{\,z-1} \times \overline{\frac{2a}{\overline{z-1}\cdot z}}^{\,\overline{z-1}\cdot\frac{1}{2}z}$$

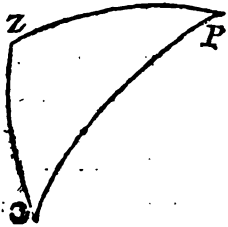
$$= 2^2 \times 3^3 \times 4^4 \ldots\ldots z^z \times \overline{\frac{2a}{z\cdot\overline{z+1}}}^{\,\frac{1}{2}z\cdot\overline{z+1}}$$; which, if the whole be divided by $2^2 \times 3^3 \times 4^4 \ldots\ldots \times \overline{z-1}^{\,z-1}$, will will give $\overline{\frac{2a}{\overline{z-1}\cdot z}}^{\,\overline{z-1}\cdot\frac{1}{2}z} = z^z \times \overline{\frac{2a}{z\cdot\overline{z+1}}}^{\,\frac{1}{2}z\cdot\overline{z+1}}$: And, if the indices be now divided by $\frac{1}{2}z$, we ſhall have $\overline{\frac{2a}{\overline{z-1}\cdot z}}^{\,z-1} = z^2 \times \overline{\frac{2a}{z\times\overline{z+1}}}^{\,z+1}$; whence, by reduction, $\frac{\overline{z+1}^{\,z+1}}{\overline{z-1}^{\,z-1}} = 4aa$, or $2 \log. z + \overline{z+1} \log. \overline{1+\frac{1}{z}} - \overline{z-1} \log. \overline{1-\frac{1}{z}} = 2 \log. 2a$; from either of which equations the value of z may be found: And the next inferior integer thereunto will be the number of parts required.

COROLLARY. When a is a large number, z being alſo large, $\overline{z+1} \times \log. \overline{1+\frac{1}{z}} - \overline{z-1} \times \log. \overline{1-\frac{1}{z}}$ will then be $= 2$, very near: And conſequently $2 \log. z + 2 = 2 \log. 2a$; or, $\log. z + 1 = \log. 2a$; or, by putting $c = (2{\cdot}71828$, &c. the number whoſe h. log. is 1), we have $\log. z + \log. c = \log. 2a$: Therefore $cz = 2a$, and $z = \frac{2a}{c}$.

A very curious and explicit ſolution to this problem has been received from Mr. *Rich. Holding*; which we omit (with regret) as being rather too long for our ſcanty limits. This gentleman's farther correſpondence will be very acceptable.——Mr. *Witchell*'s ſolution does not eſſentially differ from that given above. The propoſer (Mr. *Wildbore*) has given a fluxionary ſolution: Which are all the true anſwers that have been received to this problem.

XIII. QUESTION 474 *anſwered by Mr.* T. Allen, *of* Spalding.

It is evident that the parallax increaſes the quickeſt, when its fluxion bears the greateſt ratio poſſible to the fluxion of the angle at the pole *P*.



Put *a* and *b* for the ſine and coſine of *P* ☽ the moon's polar diſtance, *c* and *d* equal to the ſine and coſine of *PZ* (the comp. of lat.) $n =$ horizontal parallax, and $x =$ coſine of *P*; then will $\sqrt{1-\overline{bd+acx}^{2}} =$ ſine *Z* ☽ the moon's zenith diſtance, and $n\sqrt{1-\overline{bd+acx}^{2}} =$ her parallax in altitude; whoſe fluxion $\frac{-nac\dot{x}\times\overline{bd+acx}}{\sqrt{1-\overline{bd+acx}^{2}}}$ divided by $\frac{-\dot{x}}{\sqrt{1-xx}}$ (the fluxion of the angle *P*) gives $\frac{nac\sqrt{1-xx}\times\overline{bd+acx}}{\sqrt{1-\overline{bd+acx}^{2}}}$, a max. The fluxion of the log. of which being made $=0$ (making at the ſame time $r=bd$, and $s=ac$) we have $-\frac{x}{1-xx}+\frac{s}{r+sx}+s\times\frac{r+sx}{1-\overline{r+sx}^{2}}=0$: From whence the value of *x* (and conſequently the moon's altitude) may be found.

The ſame anſwered by Mr. Witchell, *the Propoſer.*

This gentleman ſubſtitutes *x* for the coſine of the moon's zenith diſtance; and then (the notation of the known quantities being here made to agree with that in the ſolution above given, by Mr. *Allen*) he finds (by a known theorem) the coſine of the angle at the pole to be $= \frac{x-r}{s}$; and, conſequently its ſine $= \frac{\sqrt{ss-rr+2rx-xx}}{s}$; whence (by dividing the fluxion of the one by the other) the fluxion of the angle itſelf is found $= \frac{-\dot{x}}{\sqrt{ss-rr+2rx-xx}}$; which (by the queſt.) being to $\frac{-nx\dot{x}}{\sqrt{1-xx}}$ (the flux. of the parallax $n\sqrt{1-xx}$) in the leaſt ratio poſſible, $\frac{nnxx \times \overline{ss-rr+2rx-xx}}{1-xx}$ muſt therefore be a maximum; whence, by taking the fluxion, &c. we have $x^4 - rx^3 - 2x^2 + 3rx + ss - rr = 0$; from which x will be known.

PRIZE QUESTION *anſwered by the Propoſer.*

Let the number of the pieces ſtaked at each game by *A* and *B*, be denoted by r and s; and let their chances for winning each game, be in the ratio of a to b: Moreover, let Δ denote the expectation of *A*, when (in the courſe of the play) he has any number (q) of pieces in his poſſeſſion; and let Δ', or $\overline{\Delta}$ be his expectation, after one more game, when he will have either $q+s$, or $q-r$ pieces, according as he wins or loſes. If he wins the game (whereof the probability is $\frac{a}{a+b}$) he will have $q+s$ pieces, and his expectation will then be Δ', which therefore, multiplied by $\frac{a}{a+b}$, gives $\frac{a}{a+b} \times \Delta'$ for his preſent expectation on that event. But if he loſes the game (whereof the probability is $\frac{b}{a+b}$) he will only have $q-r$ pieces, and his expectation being then denoted by $\overline{\Delta}$, his preſent expectation on this event will therefore be $= \frac{b}{a+b} \times \overline{\Delta}$: And conſequently $\frac{a}{a+b} \times \Delta' + \frac{b}{a+b} \times \overline{\Delta}$ will be $= \Delta$ his whole (preſent) expectation. Whence, putting $n = \frac{b}{a}$ and $m = n+1$, we have $\Delta' = m\Delta - n\overline{\Delta}$.

Let

Let now, 1, C, D, E, F, G, &c. denote the reſpective expectations of A, in the caſe propoſed, when he has 1, 2, 3, 4, 5, 6, &c. pieces in poſſeſſion: Then, r being in this caſe $=2$, and $s=1$, it is manifeſt, from the equation $\Delta' = m\Delta - n\triangle$, that the value of each new term of the ſeries 1, C, D, E, F, G, &c. will be equal to the laſt (or preceding) term multiplied by m, minus the laſt but two drawn into n. And ſo theſe values are derived, one from another, as in the annexed ſcheme.

$$\left|\begin{array}{c|l|l}
1 & 1 = & 1 \\
2 & C = & C \\
3 & D = & mC \\
4 & E = & m^2C - n \\
5 & F = & \overline{m^3 - n} \times C - mn \\
6 & G = & \overline{m^4 - 2mn} \times C - m^2n \\
7 & H = & \overline{m^5 - 3m^2n} \times C - m^3n + nn \\
8 & I = & \overline{m^6 - 4m^3n + n^2} \times C - m^4n + 2mnn
\end{array}\right.$$

&c. &c.

And, univerſally, if r be taken to denote the number of any term of this ſeries, reckoned from the beginning, the term itſelf will be truly expreſſed by $C \times \overline{m^{r-2} - \overline{r-4}\,.\,m^{r-5}n + \frac{r-6}{1}\,.\,\frac{r-7}{2}m^{r-8}nn - \frac{r-8}{1}\,.\,\frac{r-9}{2}\,.\,\frac{r-10}{3}m^{r-11}n^3}$, &c. $- m^{r-4}n + \overline{r-6}\,.\,m^{r-7}nn - \frac{r-8}{1}\,.\,\frac{r-9}{2}m^{r-10}n^3 + \frac{r-10}{1}\,.\,\frac{r-11}{2}\,.\,\frac{r-12}{3}m^{r-13}n^4$, &c. Which, when r is taken $=11$ (the whole number of pieces of both A and B) will become $C \times \overline{m^9 - 7m^6n + 10m^3n^2 - n^3} - m^7n + 5m^4n^2 - 3mn^3 = 11$; becauſe, if A ſhould have all the 11 pieces in poſſeſſion, the play will then be at an end, and he will then have $\frac{5}{3}$ that ſum, certain. Now, if in this eqaation there be wrote $\frac{5}{3}$ and $\frac{2}{3}$ inſtead of their equals m and n, the value of C will be found $=\frac{376923}{383543}$.

But the required expectation of A, when he has 7 pieces in poſſeſſion, is found above to be $\overline{m^5 - 3m^2n} \times C - m^3n + nn = \frac{1775C - 642}{243} = \frac{1739933}{383543} = 4{\cdot}5364$, &c. Therefore his required loſs, or diſadvantage, will be $= 7 - 4{\cdot}5364 = 2{\cdot}4636$, &c. $=$ 2l. 11s. 3¼d. *Q. E. I.*

After the ſame manner the loſs of A and the gain of B may be determined in any other caſe: But it muſt be obſerved,

ſerved, that, when *B*, as well as *A*, ſtakes more than one piece at a time, the values of the ſeveral terms in the propoſed ſeries of expectations will then be expreſſed by means of as many of the leading terms, as there are units in (*s*) the ſtake of *B*. And if the *s* laſt terms of the ſeries ſo expreſſed, be made reſpectively equal to the numbers which they anſwer to (or ſtand againſt), then as many (ſimple) equations will from thence be obtained, as there are unknown quantities *C*, *D*, *E*, &c. to be determined.

To exemplify the proceſs here pointed out, would, I apprehend, take up too much room: For which reaſon I am alſo obliged to omit the invention of a general approximation for the reſolution of all problems of this kind; which comes exceeding near the truth in thoſe caſes where it is moſt wanted, that is, where the number of ſtakes is great.

The Eclipſes calculated for 1761.

There will, in the courſe of this year, be ſix eclipſes, four of the ſun, and two of the moon; whereof one of the latter only will be viſible to the inhabitants of Great Britain

The firſt is a very ſmall eclipſe of the ſun, May 4th, between five and ſix in the afternoon, viſible only in great ſouth latitudes.

The ſecond is a viſible and total eclipſe of the moon, May 18th, of which ſome calculations, from different tables, and by different correſpondents, are here ſubjoined.

Calculated by	For	Beg. Ecl. h. m.	Beg. T.D. h. m.	Mid. Eclip. h. m.	End T. D. h. m.	End Eclip.
Mr. G. Witchell, from Mayer's Tables,	London	8 22	9 29	10 16	11 2	12 9
Mr. J. Metcalfe, from Brent's Tables,	London	8 24	9 31	10 20	11 8	12 15
	Wentworth	8 20	9 27	10 16	11 4	12 11
Mr. T. Harris, from Brent, by 3 equ. only	London	8 32	9 38	10 27	11 16	12 23
	Bugbrook	8 28	9 34	10 23	11 12	12 19
Mr. T. Allen,	Spalding	8 24	9 31	10 17	11 3	12 9
Mr. W. Terrill, from Leadbet. Tables	London	8 21	——	10 15	——	12 9
	Redruth	8 1	——	9 55	——	11 49
Mr. W. Chapman, from Dunthorn's Tables	Leiceſter	8 26	9 33	10 21	11 9	12 16
Mr. W. Reeves, from Halley's Tables	Burton on the Water.	8 17	9 24	10 10	10 56	12 3

The third is a very ſmall ſolar eclipſe, June 3d, between 1 and 2 in the morning; viſible only within the artic circle.

The fourth is another of the ſame ſort, Oct. 27th, between 10 and 11 at night.

The fifth is a total eclipſe of the moon, Nov. 12th, about noon, and conſequently inviſible to us.

The ſixth and laſt is of the ſun, Nov. 26th, about 2 in the afternoon; but inviſible here, becauſe of the moon's great ſouth latitude.

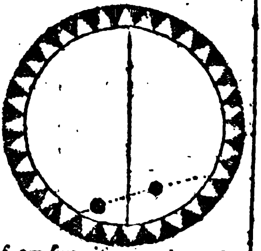
Beſides theſe eclipſes of the two great luminaries, there will happen on June 6th, the long-talked-of, and much-expected tranſit of the planet Venus over the ſun's diſk: Some calculations of which are as follow.

Calculated by	Begin. Tranſ. h. m.	Tot. Im. h. m.	Middle Tranſ. h. m.	Beg. Im. h. m	End Tra. h. m	N. Ap. ♀ to ☉'s Cent. ′ ″
Mr. G. Witchell	2 26	2 45	5 47	8 41	9 7	9 37
M. La Caille	1 37	2 0	5 4	7 56	8 32	8 57
Mr. T. Walker, from Hal. Tab.	2 12		5 27		8 41	
Mr. T. Allen	2 24		5 35		8 46	
Mr. T. Harris	2 4	2 27	5 23	8 20	8 43	9 39

Mr. *Witchell* (whoſe ſkill and great exactneſs in calculations of this ſort we cannot enough commend) takes notice, that in his computations (inſerted in the preceding page) no regard is had to parallaxes. He has however, obliged us with another calculation (and a type) for the city of London, on ſuppoſition that the ſun's horizontal parallax is 10 ſeconds; which is as follows.

Eq. time h.	′	″	App. Diſt. Cent. ′	″	
4	0	0	12	35	
5	0	0	10	28	
5	48	58	9	58	leaſt diſt. cent.
6	0	0	9	59	
7	0	0	11	2	
8	0	0	13	14	
8	41	31	15	12	begin. of emerſion
9	0	0	16	11	
	4	13	16	24	end
			31	36	diam. of ♀
			1	12	diam. of ☉

Type for London



from ſun-riſing to the end.

Theſe numbers are the reſult of a calculation, in which the ſun's place was computed by Mayer's tables (as agreeing beſt with obſervation) and that of ♀ from Halley's, except in the place of her node: For, by comparing Horrox's obſervation, Nov. 24th, 1639, with one taken at Paris by M. La Caille, Dec. 21ſt, 1746, it appeared that the motion of her nodes from the equinox, in 100 Julian years, was 52′ 47″; whereas in Halley's tables it is only 51′ 40″. Upon this authority I have ventured to add 2′ 9″ to the place of the node; which is all the correction that the theory of ♀ in theſe tables ſeems to want.——It may not perhaps be improper to add here a ſhort theorem, whereby the ſun's parallax may be found by an obſervation of this tranſit, made in one place only; leaving it to the gentlemen who are converſant in obſervations to determine how far it is practicable. Obſerve the apparent motion of ♀ ☌ ☉ during the time of the tranſit, and let a denote the difference between that and the true motion for the ſame time, found by theory; let alſo the apparent diſtance of ♀ from the zenith, and the angle made by her path with the vertical circle paſſing through her center, be determined both at the beginning and end of the obſervation, and put b = ſine of the 1ſt zenith diſtance, c = ſine of the 2d, d = coſine of the 1ſt vertical angle, e = coſine of the 2d, and let the given ratio of the diſtances of the earth and Venus from the ſun be as f to g: Then will $\frac{a \times \overline{f-g}}{g \times \overline{bd-ce}}$ = ſun's horizontal parallax.

G. Witchell.

New Queſtions.

I. QUESTION 475, *by Miſs* Ann Nicholls.

Old John, who had in credit liv'd,
Tho' now reduc'd, a ſum receiv'd:
This lucky hit's no ſooner found,
Than clam'rous duns come ſwarming round:
To th' landlard,—baker,—many more,
John paid in all pounds ninety-four.
Half what remain'd—a friend he lent—
On Joan and ſelf, one-fifth he ſpent;
And when of all theſe ſums bereft,
One-tenth o'th' ſum receiv'd had left.
— Now ſhew your ſkill, ye learned fair,
And in your next that ſum declare.

II. QUES-

II. Question 476 *by Mr.* J. Hampſon.

To find that number, which being any how divided into two unequal parts, the greater part added to the ſquare of the leſſer, ſhall be equal to the leſſer part added to the ſquare of the greater.

III. Question 477, *by Mr.* Tho. Barker.

Near me two lovely maids reſide,
Bleſt with each grace—their ſex's pride;
But ſhould ſuch charms fail to engage,
Without the gold, in this wiſe age,
The two equations plac'd below,*
Will, when reſolv'd, their fortunes ſhew.

* viz. $\left\{\begin{array}{l} xy = 125x + 300y \\ yy - xx = 90000 \end{array}\right\}$ to be ſolved by a quadratic.

IV. Question 478, *by Mr.* W. Chapman.

Having, at a certain (unknown) diſtance, taken the angle of elevation of a ſteeple, I advanced 60 yards nearer (upon level ground) and then obſerved the elevation to be the complement of the former to a right angle: Advancing 20 yards ſtill nearer, the elevation now appeared to be juſt the double of the firſt. Hence the ſteeple's height is required.

V. Question 479, *by Mr.* S. Kemp.

Having given the the vertical angle of a triangle (104°) and alſo the length (24) of a line dividing it in the given ratio of 5 to 4, and terminating in the oppoſite ſide; to determine the triangle ſo that the area thereof ſhall be a minimum.

VI. Question 480, *by Mr.* W. Spicer.

The perimeter of a triangle being given (120 feet) and the vertical angle (70°); to determine all the ſides thereof, ſo that the triangle itſelf ſhall be the greateſt poſſible.

VII. Question 481, *by* P. M. *of* Durham.

Two right lines, and alſo a third of any order, being given in poſition, to draw another right line intercepted between the line of the indeterminate order and one of the given right lines, alſo cutting the other, ſo as to make given angles at the interſection, and have its ſegments made thereby in a given ratio.

VIII. Ques-

VIII. QUESTION 482, *by Mr.* Rich. Mallock.

The area of a triangle being given = 126; the ſum of its three ſides = 54, and the ſum of their ſquares = 1010; to determine the triangle.

IX. QUESTION 483, *by Mr.* J. Brampton.

There is a pond in the form of a right-angled triangle, which is intended by the owner for a decoy. Going to ſurvey it, I found it ſo ſurrounded by buſhes, to a conſiderable diſtance, that the following meaſures were all that I could take: On the baſe produced, I meaſured from the acute angle, 4 chains: Here I could ſee a tall poplar, which grew on the bank of the hypothenuſe; I took its bearing from the chain-line 20°: When I got to the tree I could not ſee my former ſtation, but found that the perpendicular of the triangle ſubtended a right angle there: Then I meaſured from the tree to the angle oppoſite the baſe, 5 chains. 'Tis required from theſe meaſures to plot the triangle.

X. QUESTIION 484, *by Mr.* Rich. Gibbons.

A ſhell being thrown from a mortar, at an elevation of 30°, the report of its fall was heard at the mortar, juſt 20 ſeconds after the exploſion: Hence to find the length of the range.

XI. QUESTION 485, *by Mr.* Hugh Brown.

A and *B* borrow 400l. each, for a certain ſtated time: At the expiration of which, *A*, who agreed to allow compound intereſt, had 463 l. 1s. to pay; but the debt of *B* (who was to pay ſimple intereſt only) amounted but to 460 l. The time and rate of intereſt are required.

XII. QUESTION 486, *by Mr.* T. Harris.

In ſixty-ſix,* the time declare,
When day and twilight equal are.

* *Degrees of north latitude.*

XIII. QUEST. 487, *by Miſs* Ann Nicholls, *of* Hadham.

Two places, *A* and *B*, are known to lie both under the ſame meridian: And it is obſerved, that on Jan. 20, the ſun riſes 36 minutes ſooner at *A* than at *B*; and that on May 30, the he riſes 40 minutes earlier at *B* than at *A*: From which data, I demand the latitude of both places.

XIV. QUESTION 488, *by Mr.* G. Witchell.

To determine the equation of the curve, whoſe ſubtangent is every where, equal to the ſum of the abſciſſa and ordinate correſponding; and to find the area thereof, when the two quantities laſt mentioned are equal, and given.

The PRIZE QUESTION, *by Mr.* E. Rollinſon.

To determine the orbit that a planet will deſcribe, when, beſides its proper gravitation to the ſun, it is urged in the direction of the radius vector by a perturbating force, which is every where in proportion to the ſun's attraction, as the coſine of the angle deſcribed about his center, from the commencement of the motion, to any given multiple of the radius.

1762.

Queſtions anſwered.

I. QUESTION 475 *anſwered by Mr.* Tho. Sadler.

THE ſum receiv'd by John and Joan—
One hundred forty pounds and one;
And how diſpos'd on you will find,
From the ſolution here ſubjoin'd.

If x be ſuppoſed $=$ the whole ſum John received, then will $\frac{x-94}{2}+\frac{x-94}{5}+\frac{x}{10}+94$ or $\frac{8x+282}{10}=x$, per the conditions of the queſtion; whence $x=141$ l. the ſum required.

Mr. *George Salmon*, late of Mr. *B. Donn*'s ſchool, and ſeveral others, obſerve, that this queſtion is ambiguous, it being doubtful whether $\frac{1}{5}$ of the whole ſum received) or only $\frac{1}{5}$ of what remained after the 94 l. was paid, was the part thereof ſpent on Joan and himſelf, and accordingly find the whole ſum received to be either 141 or 235 pounds? and agreeably to one or other of theſe two meanings it has been anſwered by Meſſrs. *J. Aſkew*, *Tho. Atkinſon*, *T. Baker*, *Tho. Barker*, *Richard Baſſel*, *E. Batten*, *John* and *Wm. Bell*, *Wm. Beer*, *Birchoverenſis*, *T. Bromhall*, *John Buddle*, *R. Butler*, *Wm. Chapman*, *John Collins*, *James Croom*, *Richard Dening*, *John Endon*, *Wm. Embleton*, *Humphrey Fry*, *Richard Gibbons*; *John Goodhead*, *Char. Green*, *John Hampſon*, *Wm. Hardy*, *Thomas Harris*, *Wm. Harvey*, *Da. Haſtings*, *Geo. Hicks*, *John Hitchcock*, *Richard Holden*, *Samuel Kemp*, *Edward Kimpton*, *Wm. Kingſton*, *James Launders*, *John Lyon*, *Rich. Mallock*, *Robert Marſh*, *Cha. Meſban*, *R. Miles*, *James Milner*, *John Newland*, *Geo. Nokes*, *Noſnibctub*, *James Paty*, *Sim. Pedley*, *Wm. Penn*, *John Potter*, *Geo. Reed*, *Tho. Robinſon*, *Tho. Sandling*, *Rich. Spencer*. *Wm. Spicer*, *Wm. Stoker*, *Eleanor Suggett*, *John Swan*, *Wm. Toms*, *Tho. Walker*, *W. Wells*, and *Others*.

II. QUESTION 476 *anſwered by* Birchoverenſis.

Let a and b repreſent any two given unequal numbers, and ſuppoſe ax denotes the greater, and bx the leſſer part of the number required; then will be had $aaxx + bx = bbxx + ax$, by the queſtion; whence $x = \frac{a-b}{aa-bb} = \frac{1}{a+b}$, and conſequently $\frac{a}{a+b}$ and $\frac{b}{a+b}$ will be the two unequal parts, whoſe ſum will always be $= 1$, the number required.

Mr *Tho. Barker* puts $x+y$ for the greater, and $x-y$ for the leſſer part of the required number, and thence forms the equation $\overline{x+y}|^2 + x - y = \overline{x-y}|^2 + x + y$, according to the nature of the queſtion; whence he finds $x = \frac{1}{2}$, and conſequently the ſum of the two parts $x+y$ and $x-y$ equal to 1, the number required; the ſame as before.

True ſolutions to this queſtion have likewiſe been received from Mr. *Tho. Atkinſon*, Mr. *T. Baker*, Mr. *Rich. Baſſell*, Mr. *E. Batten*, Mr. *T. Bromhall*, Mr. *John Buddle*, Mr. *R. Butler*, Mr. *Tho. Boſworth*, Mr. *Wm. Chapman*, Mr. *John Collins*, Mr. *James Croom*, Mr. *Rich. Dening*, Mr. *Wm. Embleton*, Mr. *Humphrey Fry*, Mr. *Richard Gibbons*, Mr. *John Goodhead*, Mr. *Cha. Green*, Mr. *John Hampſon*, Mr. *Wm Hardy*, Mr. *Tho. Harris*, Mr. *Da. Haſ-*

Haſtings, Mr. *Geo. Hicki*, Mr. *Rich. Holden*, Mr *Sam. Kemp*, Mr. *Edw. Kimpton*, Mr. *Wm. Kingſton*, Mr. *James Launders*, Mr. *Cha. Meſban*, Mr. *Rich. Miles*, *Noſnihctuh*, Mr. *Steph. Ogle*, Mr. *Wm. Penn*, Mr. *John Potter*, Mr. *Tho. Robinſon*, Mr. *Geo. Salmon*, Mr. *Wm Spicer*, Mr. *Wm. Stoker*, Mrs. *Eleanor Suggett*, Mr. *Wm. Toms*, Mr. *Tho. Walker*, Mr. *W. Wells*, and ſeveral others.

III. QUESTION 477 *anſwered by Mr.* Hugh Brown, *of* Woolwich.

Put $300 = a$; and then $xy = 125x + 300y = \frac{5}{12}ax + ay$; and $yy - xx = 90000 = aa$. Whence, ſubſtituting $\frac{5ax}{12 \times \overline{x - a}}$ for y, we ſhall have* $144x^4 - 288ax^3 + 263aaxx - 288a^3x + 144a^4 = 0$, or $x^4 - 2ax^3 + \frac{263}{144}aaxx - 2a^3x + a^4 = 0$; and, dividing every term by $aaxx$, and putting $z = \frac{2x}{a} + \frac{2a}{x}$ (according to directions given on p. 156 of Simpſon's Algebra, 2d edition) we ſhall have $\frac{z^2}{4} - 2 - z + \frac{263}{144} = 0$, or $z - 4z = \frac{100}{144}$; whence z will be found $= 4\frac{1}{6} = \frac{2x}{a} + \frac{2a}{x} = \frac{2x}{300} + \frac{2 \times 300}{x}$; and conſequently $x^2 - 2\frac{1}{12} \times 300x = -90000$; from the reſolution of which quadratic equation x comes out $= 400$; and from thence y will be found $= 500$.

The ſame anſwered by P. M. *of* Durham.

By tranſpoſition $x \times \overline{y - 125} = 300y$, and $x^2 \times \overline{y - 125}|^2 = 300y^2$; but $x^2 = y^2 - 300^2$; therefore, by ſubſtitution, $\overline{y^2 - 300^2} \times \overline{y - 125}|^2 = 300y^2$; which, expanded, gives $y^4 - 250y^3 - 2.\,300^2y^2 + 125^2y^2 + 2.\,125.\,300^2y - 125^2.\,300^2 = 0 = \overline{y^2 - 125y - 300^2}|^2 - 300^4 - 125^2.\,300^2$; whence $y^2 - 125y - 300^2 = 300 \times \sqrt{300^2 + 125^2} = 300 \times 325$, and $y^2 - 125y = 300 \times 625$; which quadratic equation ſolved, gives $y = 500$, and conſequently $x = 400$.

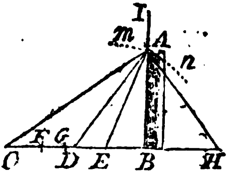
The ſame anſwered by Mr. Wm. Embleton.

Put 300 $= a$, and 125 $= b$; then will $xy = bx + ay$, and $yy - xx = aa$; whence $y = \frac{bx}{x-b} = \sqrt{xx+aa}$, and $bbxx = \overline{x-a}|^2 \times \overline{xx+aa} = x^4 - 2ax^3 + 2aaxx - 2a^3x + aa$: Whence, adding $aaxx$ to each ſide of this equation, and extracting the ſquare root, there will reſult $x\sqrt{aa+bb} = xx - ax + aa$; from which quadratic equation x and y will readily be found equal to 400 and 500 reſpectively.——Mr. *John Goodhead* anſwered it exactly in the ſame manner as this.——We have likewiſe received three other very different ſolutions to it, by quadratic equations only, from Mr. *Tho. Atkinſon,* Mr. *Cha. Green,* and Mr. *Geo. Salmon,* which we are ſorry our narrow limits will not permit us to inſert.

We have alſo received true and conciſe anſwers to this queſtion from Mr. *T. Baker,* Mr. *Tho. Barker* the propoſer), *Birchoverenſis,* Mr. *John Buddle,* Mr. *Robert Butler,* Mr. *Wm. Chapman,* Mr. *John Eadon,* Mr. *Rich. Gibbons,* Mr. *Tho. Harris,* Mr. *Geo. Hicks,* Mr. *Edward Kimpton,* Mr. *James Launders,* Mr. *John Lyon,* Mr. *Richard Mallock,* Mr. *Chr. Meſban,* *Noſnihctub,* Mr. *John Potter,* Mr. *J. Randles,* Mr. *Geo. Reed,* Mr. *Tho. Robinſon,* Mr. *W. Spicer,* Mrs. *Eleanor Suggett,* Mr. *Wm. Toms,* Mr. *Tho. Walker,* and others.

IV. QUESTION 478 *anſwered by* P. M. *of* Durham.

Imagine the thing done, and BA to be the ſteeple whoſe height is ſought, and $C, D, E,$ the three given ſtations; and ſuppoſe the points C, A; D, A; and E, A, to be joined.



Then, ſince the $\angle AEB$ is double the $\angle ACB$ (by hypoth.) and equal to the $\angle$s ACB, CAE (32 Euc. 1.), they muſt therefore be equal to each other, and conſequently the ſides $AE, CE,$ ſubtending them equal. Moreover, ſince the right-angled triangles $ABC, ABD,$ have the acute $\angle$s BAC, ADB, equal (by hypoth.) they will be ſimilar, and ſo $CB : AB :: AB : BD$; whence $CB \times BD (= AB^2 = AE^2 - EB^2 = CE^2 - EB^2 =) CB \times CE - CB \times BE$; whence $BD = CE - BE$, take away

DE, which is common, and there will remain $BE = CD - BE$, or the double of $BE = CD$; whence is derived the following

CONSTRUCTION. Biſect *CD* in *F*, and *FE* in *G*, and produce *CE* till $GB = GC$; and about the center *E*, with the radius *EC*, let the arc of a circle be deſcribed, and at the point *B* erect a perpendicular cutting the ſaid arc in *A*, and *BA* will repreſent the height of the ſteeple required.—— From this conſtruction *BA* is readily found = 74·162, &c. yards.

Much after the ſame manner this queſtion was conſtructed by Mr. *Wm. Embleton.*

The ſame conſtructed otherwiſe, by Mr. Da. Haſtings.

The three given ſtations being ſuppoſed to be at *C*, *D*, and *E*, as before, produce *CE* till *EH* becomes equal thereto, and biſect *DH* with the perpendicular *BI*, and from the center *E*, with the radius *EC* (*EH*), let the arc *mn* be deſcribed cutting *BI* in *A*, and *BA* will be the height of the ſteeple required.

DEMONSTRATION. Let the points *C*, *A*; *D*, *A*; *E*, *A*; and *H*, *A*, be joined. Then, *CE* being $= EA = EH$, by conſtruction, a ſemi-circle, deſcribed about the diameter *CH*, will paſs through the point *A*, and conſequently *CAH* being a right angle, the $\angle ADH$ will be the complement of the $\angle C$ to a right angle; and the $\angle AEB$ being = the $\angle C + $ the $\angle CAE$, will be = twice the angle *C*, the triangle *CEA* being iſoſceles.

Mr. *Robert Butler* conſtructed it according to this laſt method, nearly.

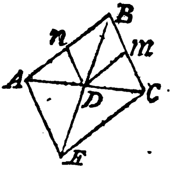
The ſame anſwered (algebraically) by Mr. R. Mallock, Writing-maſter *in* Lyme-Regis, Dorſet.

Let *CE* (*EA*) $= a$, $EB = z$, and x and $y =$ the ſine and coſine of the $\angle ACB$ (rad. 1.); then will $2xy$ and $yy - xx$ be the ſine and coſine of the $\angle AEB$; whence, by trigonometry, will be readily found $BA = \frac{x}{y} \times \overline{z+a} = \frac{y}{x} \times z + \frac{a}{4} = \frac{2xyz}{yy - xx}$; and conſequently $z = \frac{3a}{8} = 30$, and from thence *BA* (the ſteeple's height) is directly found = 74·16198 yards.

Meſſ. *W. Chapman*, *Bircboverenſis*, *Tho. Harris*, *Wm. Kingſton*, *Paul Sharp*, and *W. Spicer*, ſubſtitute exactly the ſame as above, and, by an eaſy and very ſhort proceſs, find $2xx = \frac{5}{8} = 0{\cdot}625 =$ the verſed ſine of 67° 58′ 32″ (double the $\angle ACB$); from whence they determine the ſteeple's height the ſame as above.

This queſtion was likewiſe anſwered by Mr. *Tho. Atkinſon*, Mr. *Tho. Barker*, Mr. *E. Batten*, Mr. *Sam. Beeken*, Mr. *Tho. Boſworth*, Mr. *John Buddle*, Mr. *John Eadon*, Mr. *Rich. Gibbons*, Mr. *John Goodhead*, Mr. *Cha. Green*, Mr. *John Hampſon*, Mr. *Wm. Hardy*, Mr. *Rich. Holden*, Mr. *Sam. Kemp*, Mr. *Edw. Kimpton*, Mr. *Ja. Launders*, Mr. *R. Miles*, Mr. *Ja. Milner*, Mr. *J. Randles*, Mr. *Geo. Reed*, Mr. *Tho. Robinſon*, Mr. *Wm. Stoker*, Mrs. *Eleanor Suggett*, Mr. *John Swan*, Mr. *Wm. Tons*, and ſeveral others.

V. QUEST. 479 *anſwered by Mr.* Rich. Gibbons.



CONSTRUCTION. Conſtitute the angle $ABC =$ the given vertical angle (104°), and divide it in the given ratio of 5 to 4 by the line $BE =$ double the given dividing line (24), and draw EA, EC, parallel to BC and BA reſpectively, and meeting them in the points A and C, and then ABC will be the triangle required.

DEMONSTRATION. $ABCE$ being a parallelogram, by conſtruction, the diagonals BE, AC, will biſect each other (by theor. 12. 2d of Simpſon's Geom. 2d edition), and conſequently BD will be the given dividing line; and, DA being $= DC$, the triangle ABC will be a minimum (by theor. 8. p. 199 and 200 of Simpſon's Elem. of Geom. aforeſaid). *Q.E.D.*

The ſame conſtructed otherwiſe by Mr. Wm. Embleton.

The angle ABC being made $=$ the given vertical angle, and BD the given line dividing it in the given ratio of 5 to 4, as before, draw Dm parallel to BA meeting BC in m, and take $BA =$ twice Dm; then, through the points A and D, let the line AC be drawn meeting BC in C, and ABC will be the triangle required.——For Dm being $= \frac{1}{2} AB$ (by conſtruction) CD will be $= \frac{1}{2} CA$, by the ſim. triangles CDm and CAB, and conſequently the triangle ABC a minimum (by theor. 8. p. 199 and 200 of Simpſon's Elem. of Geom.) 2d edit.)

According to this laſt method, nearly, it was alſo conſtructed by Mr. *Da. Haſtings*.

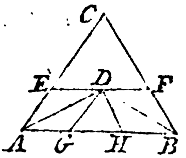
We have likewiſe received neat and elegant geometrical conſtructions to this queſtion from *P. M.* of *Durham*, Mr. *Rob. Butler*, Mr. *Cha. Green*, and ſeveral others.*

An algebraic Solution to the ſame by Mr. T. Bromhall, *at Mr.* Allen's *School, at* Spalding, *in* Lincolnſhire.

This young gentleman puts $Dm = a$, Dn (parallel to BC) $= b$, $Cm = x$, and $s =$ ſine of vertical angle ABC; then he readily finds $AB = a + \frac{ba}{x}$, and $\overline{2b + x + \frac{bb}{x}} \times \frac{1}{2}as =$ area of the required triangle, which is to be a minimum; In fluxions, $x^2\dot{x} - b^2\dot{x} = 0$; whence $x = b$, or $Bn = An$, and conſequently $AD = DC$, the ſame as demonſtrated above. ——Much after the ſame manner it was anſwered by Mr. *John Eadon*, Writing-maſter, at the free ſchool in *Sheffield*, *Yorkſhire*.

Algebraic ſolutions to this queſtion have likewiſe been received from Mr. *Tho. Barker*, Mr. *E. Batten*, *Birchoverenſis*, Mr. *Tho. Boſworth*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *John Goodhead*, Mr. *John Hampſon*, Mr. *Wm. Hardy*, Mr. *Tho. Harris*, Mr. *Rich. Holden*, Mr. *Sam. Kemp*, Mr. *Wm. Kingſton*, Mr. *James Launders*, Mr. *Rich. Mallock*, Mr. *Geo. Reed*, Mr. *Tho. Robinſon*, Mr. *Paul Sharp*, Mr. *W. Spicer*, Mr. *Wm. Stoker*, Mr. *Wm. Toms*, Mr. *Tho. Walker*, and others.

VI. QUESTION 480 *conſtructed by Mr.* Da. Haſtings.



It evidently appears, from theor. 6. p. 198 of Simpſon's Elem. of Geom. 2d edition, that the triangle required will be iſoſceles; therefore upon AB, equal to the given perimeter, or ſum of the three ſides, conſtitute an iſoſceles triangle with the given vertical angle ACB (= 70°), and biſect the angles CAB, CBA, with the right lines AD, BD, meeting each other in D, and let the points A, D

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* The above cannot be eſteemed geometrical conſtructions, as the method of dividing an angle in the ratio of 4 to 5, is not known.

and B, D be joined; then draw DG and DH parallel to CA and CB reſpectively, and CDH will be the triangle required; which is too evident to need any further demonſtration.

P. M. of *Durham*, obſerves, that it is very eaſy to demonſtrate, geometrically, that the two ſides comprehending the given vertical angle will be equal; and therefore, premiſing that, he determines the triangle by the following

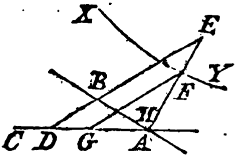
CONSTRUCTION. Make $ACB =$ the given vertical angle, and CA and CB each $=$ half the given perimeter; join A, B; and biſect the angles CAB, CBA, by right lines meeting in D; through the point D draw EF parallel to AB, meeting the ſides CA, CB, in the points E, F; and CEF will be the triangle required.——For the $\angle EDA$ ($= DAB$, 29 E. 1.) $= \angle EAD$ (by hyp.) therefore $AE = DE$ (6 Euc. 1.) and conſequently $CA = CE + ED$; and in the ſame manner CB will be proved $= CF + FD$; therefore $CE + CF + EF = CA + CB$, *i.e.* equal to the given perimeter; whence, &c.

The ſame ſolved algebraically by Birchoverenſis.

Let $s =$ ſine of the given vertical angle ECF (70°), $m = 120$ (the given perimeter), $a + x$ and $a - x$ the two ſides comprehending the vertical angle; then will $m - 2a =$ the third ſide EF, and $\overline{aa - xx} \times \frac{1}{2} s =$ the area of the required triangle, which it is evident by inſpection, will be the greateſt when $x = 0$, or the triangle ECF is iſoſceles; and hence the remaining part of the queſtion is eaſily ſolved by plain trigonometry, it being as the ſum of the natural ſines of the three angles of any plain triangle is to its perimeter, ſo is the natural ſine of any one of thoſe angles to the ſide correſponding or oppoſite thereto; whence the ſides EC (CF) and EF are readily found $= 38{\cdot}1297$, &c. and $43{\cdot}74059$, reſpectively.——Mr. *E. Batten* and Mr. *Sam. Kemp* anſwer it exactly according to this method.

Mr. *R. Butler*, Mr. *Wm. Embleton*, Mr. *Francis King*, and ſeveral others, have given neat conſtructions to this queſtion; and Mr. *Tho. Barker*, Mr. *Tho. Boſworth*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *John Eadon*, Mr. *Rich. Gibbons*, Mr. *John Goodhead*, Mr. *Cha. Green*, Mr. *John Hampſon*, Mr. *Wm. Hardy*, Mr. *Tho. Harris*, Mr. *Rich. Holden*, Mr. *Wm. Kingſton*, Mr. *James Launders*, Mr. *Rich. Mallock*, *Noſnibctub*, Mr. *Geo. Reed*, Mr. *Paul Sharp*, Mr. *W. Spicer*, Mr. *Wm. Stoker*, &c. have ſent neat and conciſe algebraic ſolutions to it.

VII. QUESTION 481 *conſtructed by* P. M. *of* Durham (*the Propoſer.*)



Let *XY* be the line of the indeterminate order; *AB*, *AC*, the two right lines given by poſition and meeting in *A*; in one of them *AB*, with which the right line, required, is to make given angles, aſſume a point *B*, through which draw a right line, making the given angles with *AB*, and meeting *AC* in *D*; produce *DB* to *E*, ſo that *DB* may be to *BE* in the given ratio; through *E*, *A*, draw a right line (produced if neceſſary) to cut *XY* in *F*; and through *F* draw a line parallel to *ED*, cutting *AB* and *AC* in *H* and *G*; and the thing is done.

For the right line *AB* falling upon the parallels *DE*, *GF*, intercepted by the ſame right lines *AD*, *AE*, will cut them ſimilarly in *B* and *H* (2 Euc. 6.) and make equal angles with each (29 Euc. 1.); but it makes the given angles with *DE*, and cuts it in the given ratio in *B*; therefore alſo *GF*, intercepted by *XY*, and one of the given right lines *AC*, is cut by the other *AB*, in *H*, in the given ratio, and makes given angles therewith at the interſection.

NUM. CALC. will vary according to the property of the indeterminate line *XY*.

Mr. *R. Butler*'s conſtruction is nearly the ſame as the above.

VIII. QUESTION 482 *anſwered by Mr.* R. Butler.

Let $2p = 54$; $s = 1010$; $a = 126$; and x, y, and z denote the three ſides of the triangle required. Then, by a known theor. $p \times \overline{p-x} \times \overline{p-y} \times \overline{p-z} = aa$; whence, putting $2p - x - y$ for its equal z, we ſhall have $\overline{x-p} \times yy + \overline{2pp + xx - 3px} \times y + 2ppx - pxx - p^3 = \frac{aa}{p}$. Moreover, $\overline{2p-x-y}|^2 + xx + yy = 4pp - 4px - 4py + 2xy + 2xx + 2yy$ being $= s$, per the queſtion, we get $yy = \frac{1}{2}s - 2pp + 2px - xx + \overline{2p-x} \times y$; which, ſubſtituted for yy in the preceding equation, gives $\frac{1}{2}s - 2pp + 2px -$

$$xx + \overline{2p-x} \times y = \frac{2ppx - pxx - p^3 - \frac{aa}{p}}{p-x} + \frac{2pp - 3px + xx}{p-x} \times y;$$

$\times y$; but $\frac{2pp - 3px + xx}{p - x} \times y$ is $= \overline{2p - x} \times y$; and therefore our equation becomes $\frac{1}{2}s - 2pp + 2px - xx =$

$$\frac{2ppx - pxx - p^3 - \frac{aa}{p}}{p - x},$$ or $x^3 - 54x^2 + 953x = 5460$; the three roots of which cubic equation, viz. 13, 20, and 21, will expreſs the three ſides of the triangle required, as x ſtands indifferently for e'er a one of them.

P. M. of *Durham*, anſwered this queſtion much in the ſame manner as the above; as likewiſe did Mr. *J. Hampſon*, Mr. *Wm. Chapman*, Mr. *Wm. Spicer*, and ſome others.

The ſame anſwered by Mr. Paul Sharp.

This gentleman puts $a = 126$, $2b = 1010$, $2c = 54$, $x = \frac{1}{2}$ ſum of any two ſides of the required triangle, and $y = \frac{1}{2}$ their difference; then will $x + y$, $x - y$, and $2c - 2x$, repreſent the three ſides themſelves, and $6xx - 8cx + 2yy + 4cc = 2b$ (per queſt.): Whence $yy = b - 2cc + 4cx - 3xx$. Moreover, $\overline{c - x + y} \times \overline{c - x - y} \times \overline{2x - c} \times c$ is $= aa$; whence

$$yy = \frac{\frac{aa}{c} + c^3 - 2x^3 + 5cx^2 - 4c^2x}{c - 2x} = b - 2cc + 4cx$$

$- 3xx$; which, reduced and converted into numbers, becomes $x^3 - 54x^2 + 967{\cdot}25x = 5750{\cdot}25$; whence x will be found $= 17$, and $y = 4$, and the ſides of the required triangle equal to 13, 20, and 21.

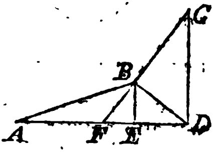
According to this laſt method of ſolution this queſtion was alſo anſwered by Mr. *Tho. Atkinſon*, Mr. *Tho. Barker*, Mr. *E. Batten*, *Birchoveronſis*, Mr. *John Buddle*, Mr. *Wm. Embleton*, Mr. *R. Gibbons*, Mr. *John Goodhead*, Mr. *Tho. Harris*, Mr. *Geo. Hicks*, Mr. *Rich. Holden*, Mr. *Sam. Kemp*, Mr. *Wm. Kingſton*, Mr. *James Launders*, Mr. *John Potter*, Mr. *Tho. Robinſon*, Mrs. *Eleanor Suggett*, Mr. *Tho. Walker*, Mr. *James Wollward*, and ſeveral others.

The ſame anſwered by Noſnihctuh.

The area of any plane triangle being $=$ a rectangle under half its perimeter and the radius of its inſcribed circle, it follows, that $\frac{126}{27} = 4{\cdot}6$ will be the radius of the inſcribed circle, in the caſe of this queſtion, the ſquare of which put $= b$;

$= b$; $1010 = n$; $27 = c$; and let x denote half the ſum, and y half the difference of the two ſegments of any one of the ſides made by the radius of the inſcribed circle, drawn to the point of contact thereof, and then will the three ſides of the required triangle be repreſented by $c + y - x$, $2x$, and $c - x - y$; whence (by a theorem on page 116 of Simpſon's Select Exerciſes) $\frac{\overline{x+y} \times \overline{x-y} \times \overline{c-2x}}{x+y+x-y+c-2x} = \frac{cx^2 - cy^2 - 2x^3 + 2xy^2}{c} = b$, and conſequently $y^2 = \frac{cb - cx^2 + 2x^3}{2x - c}$. But $\overline{c+y-x}|^2 + \overline{2x}|^2 + \overline{c-x-y}|^2 = n$ (per the queſt.); whence $y^2 = \frac{n}{2} - c^2 + 2cx - 3x^2 = \frac{cb - cx^2 + 2x^3}{2x - c}$; hence, in numbers, we derive the equation $x^3 - 27x^2 + 235{\cdot}25x - 682{\cdot}5 = 0$; from which (by either of the two firſt methods explained in Sect. 12. of Simpſon's Algebra) x will be found $= 10$, and $y = 4$, and the three ſides of the triangle required 13, 20 and 21.

IX. QUESTION 483 *anſwered by Mr.* John Hampſon.



Let *FDC* repreſent the triangular pond required; in which *DB* is perpendicular upon *FC*, and *AF* and *BC* equal 4 and 5 chains reſpectively, and the angle *BAF* = 20°.

Put $a = AF = 4$; $b = BC = 5$; m and n equal the ſine and co-ſine of the angle *BAF* (20°); $x =$ perpendicular *CD*, and $y = BF$.

Then, per ſimilarity of triangles, will be found $BE = \frac{by}{x}$, $FE = \frac{y}{x}\sqrt{xx - bb}$, and conſequently $\frac{ax + y\sqrt{vv - bb}}{x} = AE$; whence, by Trigonom. $m : \frac{by}{x} :: n : \frac{ax + y\sqrt{xx - bb}}{x}$; from whence is had $bny = amx + my\sqrt{xx - bb}$; but, from the ſimilarity of the right angled triangles *FDC*, *CDB*, will be found $by + bb = xx$; from the reſolution of which two equations x be found $= 6{\cdot}0634$; and conſequently *FD* and *FC* d to 4·15947 and 7·85296, &c. reſpectively.

The

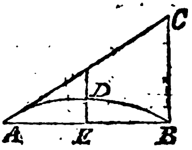
The ſame anſwered by Mr. Rich. Gibbons.

Firſt, ſuppoſing the triangles *ABD*, *AFB*, to be ſimilar, will be found the angle ABF ($= \angle ADB$) $= 35°$, and from thence the logarithm of *BC* too little by 0·0119032; next, ſuppoſing the ſame angle $ABF = 36°$. it gives the logarithm of *BC* too much by 0·0099480: Whence, as the ſum of the errors is to 60′, the difference of the ſuppoſitions, ſo is the firſt error to 32′ 42″; which being added to 35° (the firſt ſuppoſition) gives 35° 32′ 42″ = the true meaſure of the angle *ABF*; whence *AF*, *BC*, and all the angles being given, and now become known, the triangular pond *FDC* may from thence be eaſily plotted and determined.

This queſtion has likewiſe been anſwered by Mr. *Tho. Barker*, *Birchoverenſis*, Mr. *Rob. Butler*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *John Goodhead*, Mr. *Tho. Harris*, Mr. *Edw. Kimpton*, Mr. *Rich. Mallock*, Mr. *Tho. Robinſon*, Mr. *Paul Sharp*, Mr. *W. Spicer*, Mrs. *Eleanor Suggett*, Mr. *Tho. Walker*, Mr. *W. Wells*, and others.

X. QUESTION 484 *anſwered by Mr.* Thomas Allen, *of* Spalding.

Put t = tang. of the angle of elevation *BAC* (= 30°); $a = 1142$ feet (the velocity of ſound per ſecond); $s = 16\frac{1}{12}$ feet (the diſtance a falling body will deſcend in a ſecond); and $x = AB$, the horizontal range required.



Then will $tx = BC$, and $\sqrt{\frac{tx}{s}}$ = the time of flight; and $\frac{x}{a}$ = the time of ſounds moving from *B* to *A*: Therefore $\frac{x}{a} + \sqrt{\frac{tx}{s}} = 20''$ (by the queſtion); whence x is found = 6033·42 feet, the range required.

If p, (the parameter of the parabolic curve *ADB*) = 10450·7, and $\frac{1}{2}x = y$; then will $\frac{2y}{p}\sqrt{\frac{1}{4}pp + yy} + \frac{p}{2} \times$ hyp. log. $\frac{y + \sqrt{\frac{1}{4}pp + yy}}{\frac{1}{2}a}$ = 6353·4 feet, the length of the track *ADB*.

The

The ſame anſwered by Mr. W. Chapman.

Let $s = 16\frac{1}{12}$ feet (the diſtance a falling body deſcends in a ſecond); $b = 1142$ feet (the ſpace paſſed through by ſound in a ſecond); p and q equal the ſine and coſine of 30° (the angle of elevation), and $x =$ the amplitude, or horizontal range required.

Then will $\frac{px}{q} = BC$; and $s : 1''^2 :: \frac{px}{q} : \frac{px}{sq} =$ the ſquare of the time of flight; whence $c - \sqrt{\frac{px}{sq}}$ will be $=$ the time of the return of the ſound (putting $20'' = c$); and conſequently $1'' : b :: c - \sqrt{\frac{px}{sq}} : x$; whence $x = bc - b\sqrt{\frac{px}{sq}}$; from which equation x will be found $= 6033·44$ feet, the amplitude required.——Much after the ſame manner this queſtion was anſwered by *P. M.* of *Durham.*

Meſſrs. *Tho. Barker*, *Rob. Butler*, *Birchovcrenſis*, *John Buddle*, *Wm. Embleton*, *John Goodhead*, *Charles Green*, *Wm. Hardy*, *Tho. Harris*, *Rich. Holden*, *Edw. Kimpton*, *Wm. Kingſton*, *Noſnihctuh*, *Stephen Ogle*, *Paul Sharp*, *Rich. Spencer*, *Wm. Spicer*, and ſeveral others have likewiſe anſwered this queſtion, by methods not greatly different from thoſe above.

XI. QUESTION 485 *anſwered by Mr.* Tho. Barker.

Let $x =$ amount of 1 l. in one year, and $y =$ the number of years required; then will $400x^y = 463·05$ l. and $400y \times \overline{x-1} = 60$ l. (by the queſt. and the nature of compound and ſimple intereſt), whence $y = \frac{3}{\overline{x-1} \times 20}$; and conſequently

$$x^{\frac{3}{\overline{x-1} \times 20}} = \frac{463·05}{400} = 1·15762, \&c.$$

from which equation x will be found $= 1·05$; and hence it appears, that 5 per cent. per annum, and three years, are the rate of intereſt and time required.

The ſame anſwered by Mr. Tho. Harris.

Put $P = 400$ l. $A = 463·05$ l. $a = 460$ l. $t =$ time, and $r =$

rate of intereſt required; then will $trP + P = a$ (by the nature of ſimple intereſt); whence $t = \frac{a-P}{rP} = \frac{b}{r}$ (putting $b = \frac{a-P}{P} = \cdot 15$): Whence, by compound intereſt, we have $P \times \overline{1+r}|^{\frac{b}{r}} = A$, the debt of A; from which equation, by the help of logarithms, r will be found $= 0\cdot 05$: Whence the time required appears to be three years, and the rate 5 per cent. per annum.

The ſame anſwered by Mr. Rich. Holden.

Let $P = 400$l. $A = 463\cdot 05$ l. $a = 460$ l. $r =$ the amount of 1 l. in one year, and $t =$ the time required; then will $PR^t = A$, per compound intereſt, and $P + Pt \times \overline{R-1}$ $= a$, per ſimple intereſt: Whence will be found $R = \overline{\frac{A}{P}}|^{\frac{1}{t}} = \frac{a-P}{Pt} + 1$; and hence, by a table of logarithms and a few trials, t is found = three years, and the rate 5 per cent.

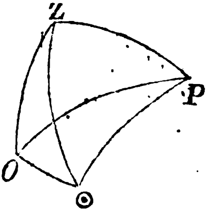
P. M. of *Durham*, and Mr. *Thomas Allen* of *Spalding*, put $a = 463\cdot 05$ l. $b = 460$l. $p = 400$ l. $t =$ number of years or time required, and $r =$ the required rate of intereſt of 1 l. for one year; and then they find $p \times \overline{1+r}|^t = a$, and $p \times \overline{1+tr} = b$ (by the nature of compound and ſimple intereſt): Whence $\frac{\text{l}.a - \text{l}.p}{\text{l}.\overline{1+r}} = (t =) \frac{b-p}{pr}$, and conſequently $\overline{\text{l}.a - \text{l}.p} \times \frac{pr}{b-p} = \text{l}.\overline{1+r}$ $(= \frac{1}{m} \times r - \frac{r^2}{2} + \frac{r^3}{3} -$ &c. m being the modulus); and hence, either by a table of logarithms, or reverſion, or approximation of ſeries, the value of r will be found $= 0\cdot 05$. Therefore the rate of intereſt appears to be 5 l. per cent. and the time of the loan = three years.

True anſwers to this queſtion have likewiſe been received from *Birchoverenſis*, Mr. *Wm. Beer*, Mr. *Tho. Boſworth*, Mr. *John Buddle*, Mr. *Rob. Butler*, Mr. *Wm. Chapman*, Mr. *John Collins*, Mr. *Rich. Denning*, Mr. *John Eadon*, Mr. *Wm. Embleton*, Mr. *Cha. Green*, Mr. *Tho. Harris*, Mr. *Geo.*

Geo. Hicks, Mr. *John Hitchcock*, Mr. *Sam. Kemp*, Mr. *Wm. Kingſton*, Mr. *Ja. Launders*, Mr. *Rich. Miles*, Mr. *Wm. Penn*, Mr. *G. Reed*, Mr. *T. Robinſon*, Mr. *G. Salmon*, Mr. *P. Sharp*, Mr. *W. Spicer*, Mrs. *Eleanor Sugget*, Mr. *Tho. Walker*, Mr. *W. Wells*, and many others.

XII. QUESTION 486 *anſwered by Mr.* Charles Green, *at* Greenwich Obſervatory, *and Mr.* Thomas Harris, *of* Bugbrook, *near* Northampton.

Let x and $-y$ equal the ſine and coſine of $P\odot$ (PO) the ſun's diſt. from the north pole P; s and c equal the ſine and coſine of ZP (= 24°) the complement of the given latitude; and $-d$ = the coſine of $Z\odot$ (= 108°); then by a well-known theorem in ſpherics and the nature of the queſtion, we ſhall have $\frac{cy}{sx}$ and $\frac{cy-d}{sx}$ = to the coſines of the angle ZPO and $ZP\odot$ reſpectively; but the angle $ZP\odot$ is double the angle ZPO (by the queſtion), and therefore $\frac{2ccyy}{ssxx}-1$ (the coſine of double the angle ZPO) will be $=\frac{cy-d}{sx}$; from the reſolution of which equation the value of y will be found = 0·3094, anſwering to Jan. 28, and Nov. 12, nearly.



Mr. *R. Butler*'s proceſs is different, but his final equation and concluſion is exactly the ſame as the above.

This queſtion was likewiſe anſwered by Mr. *Tho. Allen*, *Birchoverenſis*, Mr. *Wm. Chapman*, Mr. *Rich. Holden*, Mr. *J. Randles*, Mr. *W. Spicer*, and ſeveral others.

XIII. QUESTION 487 *anſwered by Mr.* W. Kingſton.

Let d = tang. 21° 54′ (the ſun's declination May 30.); a = tang. 20° 11′ (the ſun's declination Jan. 20); m and n equal the ſine and coſine of 10°; s and c the ſine and coſine of 9°; and let the ſine and coſine of the aſcenſional difference in the latitude of the place A, on May 30, and Jan. 20, be denoted by x and y, and z and v reſpectively; then will $nx+my$, and $cz+sv$, expreſs the ſines of the aſcenſional difference in the latitude of the place *B*, on the ſaid two days reſpectively. Then, per ſpherics, $\frac{z}{a} = \frac{x}{d}$ = the tangent of the latitude of the place *A*, and $\frac{nx+my}{d} = \frac{cz+sv}{a}$ = the tangent of the latitude of the place *B*. From the firſt of theſe equations z is found $= \frac{ax}{d}$, and conſequently v $(\sqrt{1-zz}) = \frac{\sqrt{dd-aaxx}}{d}$; which values ſubſtitute for z and v in the ſecond equation, and it becomes $\frac{nx+my}{d} = \frac{acx+s\sqrt{dd-aaxx}}{da}$; whence $a\times\overline{n-c}\times x + amy = s\sqrt{dd-aaxx}$; and conſequently (by involution, &c.) $\frac{nn-cn}{m}\times xx + \frac{aamm-ddss}{2aam} = \overline{c-n}\times xy$, or $bxx+p = qxy$ (putting $\frac{nn-cn}{m} = b$, $\frac{aamm-ddss}{2aam} = p$, and $c-n = q$); and this involved again, in order to exterminate y, becomes $bbx^4+2bpxx+pp = qqxxyy = qqxx-qqx^4$, or $\overline{bb+qq}\times x^4 + \overline{2bp-qq}\times xx = -pp$; whence $x^4 - 2rx^2 = -t$ (putting $\frac{2bp-qq}{bb+qq} = -2r$ and $\frac{pp}{bb+qq} = t$), and conſequently $2xx = 2r \mp 2\sqrt{rr-t}$ = the verſed ſine of double the aſcenſ. difference at the place *A* on May 30; from whence the latitudes of the places *A* and *B* are readily found equal to 45° 32′ and 54° 26′, reſpectively.

Anſwers to this queſtion have likewiſe been received from *Birchoverenſis*, Mr. *R. Butler*, Mr. *C. Green*, Mr. *T. Harris*, Mr. *R. Holden*, Mr. *Ed. Kimpton*, Mr. *Rich. Mallock*, Mr. *Paul Sharp*, Mr. *W. Spicer*, Mr. *Wm. Stokes*, Mrs. *Eleanor Sugget*, Mr. *Tho. Walker*, and many others.

XIV. QUESTION 488 *anſwered by Mr.* W. Spencer, *of* Stannington, *near* Sheffield, *in* Yorkſhire.

Let x = the abſciſſa, and y = the ordinate of the required curve; then will $\frac{y\dot{x}}{\dot{y}} = x+y$, (per the queſt.) and conſequently $\frac{y\dot{x}-x\dot{y}}{yy} = \frac{\dot{y}}{y}$: The fluent of which gives $\frac{x}{y}$ = hyp.

log. y; whence $x =$ hyp. log. y^y; the equation of the curve required.

But, to find the area, we have $y\dot{x}$ (the general expreſſion for the fluxion of the area) $= y\dot{y} + x\dot{y}$ (from above); whence fluent of $y\dot{x} = \frac{yy}{2} + xy -$ fluent of $y\dot{x}$, or twice the fluent of $y\dot{x}$ (= twice the required area) $= \frac{yy}{2} + xy$, and conſequently the area itſelf $= \frac{yy}{4} + \frac{xy}{2} - \frac{1}{4} = \frac{3yy}{4} - \frac{1}{4}$* (when $x = y$).

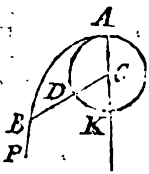
P. M. of *Durham*, after having found the equation of the curve and area the very ſame as above exhibited, adds, moreover, that when $x = y$ ($\frac{x}{y}$ being then $= 1 =$ h. log. y), y will be $= 1 + 1 + \frac{1}{2} + \frac{1}{2.3}$, &c. $= 2{\cdot}71828$, &c. and conſequently the area required (in that caſe) $= 5{\cdot}36831$.

This queſtion is alſo anſwered by *Birchoverenſis*, Mr. *R. Butler*, Mr. *W. Chapman*, Mr. *W. Embleton*, Mr. *C. Green*, Mr. *T. Harris*, and others.

The PRIZE QUESTION *anſwered*.

Suppoſe AP to repreſent part of the required orbit of the planet commencing motion at A, at the given diſtance AC from the center of force C, and put

The radius AC (CD) of the circle $ADK = 1$,
The radius vector CB —— —— $= x$,
The arch AD, meaſuring the angle ACD, deſcribed about the center of force C $= z$,
The meaſure of the celerity with which the area ACB increaſes $= u$,
The meaſure of the centripetal force tending to the center C $= Q$;



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* The quantity $\frac{1}{4}$ is the proper correction of the fluent, and is thus found: In the equation of the curve ($x =$ hyp. log. y^y or $= y \times$ hyp. log. y) when $x = 0$, then log. $y = 0$, and $y = 1$; but when $x = 0$, the area ought to be $= 0$; and ſince, when $x = 0$, and $y = 1$, the area $\frac{yy}{4} + \frac{xy}{2}$ is $= \frac{1}{4}$, $\therefore \frac{1}{4}$ is the correction to be ſubtracted.

and then the centrifugal force at *B* and *A* will be found to to be $\frac{4u^2}{x^3}$ and $4u^2$ reſpectively: And $\frac{\ddot{w}}{\dot{z}\dot{z}} = 1 - w - \frac{Q}{4u^2 \times \overline{1-w}^2}$, per p. 139 and 140 of Simpſon's Miſcellan. Tracts (w being put $= -\frac{1}{x} + 1$, and no force being ſuppoſed to act here, beſides that tending to the center of force *C*, and conſequently u a conſtant quantity).

If now the value of the centripetal force at *A* be to the centrifugal force *there* in any given ratio of $\overline{1-e}$ to 1, then will the centripetal force at *A* be $= 4uu \times \overline{1-e}$, and conſequently that at *B* $= \overline{1-e} \times 4uu \times \overline{1-w}^2$ (the centripetal force being, in this caſe, as the ſquare of the diſtance inverſely); whence we ſhall have $\overline{1-e} \times 4u^2 \times \overline{1-w}^2 \times \frac{\text{coſ. } z}{n} =$ the perturbating force acting on the planet at *B* (n being the given multiple of the radius) and conſequently $\overline{1-e} \times \frac{n - \text{coſ. } z}{n} \times 4u^2 \times \overline{1-w}^2 =$ the whole force urging it towards the center of force *C* in the direction of the radius vector *CB*, or the whole force whereby it tends to the center *C*, in that direction; and this value ſubſtituted for Q in the above equation, it becomes $\frac{\ddot{w}}{\dot{z}\dot{z}} = -w + e + \frac{\overline{1-e} \times \text{coſ. } z}{n}$: From the reſolution of which equation, w will be found $= e + \frac{\overline{1-e} \times \overline{B + \frac{z}{2}} \times \text{ſin. } z}{n}$ (*B* being any conſtant quantity at pleaſure); from whence the orbit *AP*, and every thing elſe required, may be readily determined, &c.

All our contributors who have ſolved this queſtion agree ſo nearly in the above method of ſolution, that we cannot attribute it to any one in particular; but the 1ſt prize of 12 diaries fell to the lot of Mr. *Morland*, and the 2d of 8 to Αυτόματικος.

The Eclipſes calculated for 1762.

In the revolution of this year will happen four eclipſes, two of the ſun, and as many of the moon; three of which are viſible to theſe nations.

The firſt is an inviſible ſolar eclipſe, April 24th, between five and ſix in the morning.

The ſecond is a partial eclipſe of the moon, May 8th, in the morning, of which ſome calculations are as follow.

Calculated by	Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. ° ′
Mr. W. Chapman, for Leiceſter	2 24	3 56	5 29	3 5	9 50
Mr. C. Green, for London	2 28	4 0	5 33	3 5	9 55
Mr. Metcalfe, for London	2 25	3 58	5 32	3 7	9 49
Mr. Metcalfe, for Wentworth	2 19	3 53	5 26	3 7	9 49
Mr. T. Harris, for London	2 28	4 0	5 33	3 5	9 54
Mr. Allen, for Spalding, Lincoln.	2 26	3 58	5 31	3 5	9 54
Mr. E. Greenſted, for London	2 27	4 0	5 33	3 6	9 58

Mr. *Metcalfe* obſerves that the end of this eclipſe cannot be ſeen here, as the moon ſets that morning above an hour before.

The third is a viſible ſolar eclipſe, October 17th, in the morning, of which we have received the following calculations.

Calculated by	Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. ° ′
Mr. W. Chapman, from Manuſcript Tables, for Leiceſter	6 55	7 46	8 44	1 49	6 12
The ſame Gent. for Peterſburgh	9 51	10 56	12 9	2 18	11 47
Mr J. Walker, of Colnbridge, in Yorkſhire, for London	6 59	7 50	8 48	1 49	5 20
Mr J. Walker, of Colnbridge, in Yorkſhire, for Colnbridge	6 53	7 45	8 41	1 42	5 13
Mr. C. Green, for London	6 57	7 49	8 47	1 50	5 59
Mr. J. Metcalfe, for London	6 52	7 42	8 36	1 44	4 56
Mr. J. Metcalfe, for Wentworth	6 45	7 36	8 32	1 47	5 4[illegible]
Mr. T. Harris, for London	6 57	7 49	8 47	1 50	5 53
Mr. T. Allen, for Spalding	6 55	7 47	8 45	1 50	5 59
Mr. E. Greenſted, for London	7 0	7 53	8 46	1 46	6 0

The general Phænomena of the Solar Eclipſe, Oct. 17th,
by Mr. T. Harris.

	Lat. N.		Longitude		☉ Limb
	°	′	°	′	
The eclipſe begins at ſun-riſe	56	18	1	10 W.	Upper
Riſes centrally eclipſed —	75	10	4	18 E.	Total
Centrally eclipſed at noon	52	2	50	19 E.	Total
Centrally ecliyſed in the 90°	38	12	63	30 E.	Total
Sets centrally eclipſed —	23	39	109	2 E.	Total
The end at ſun-ſet —	2	50	95	23 E.	Upper
The contact of the two limbs	9	48	50	19 E.	Upper
Eclipſed in the meridian and the horizon near 10 digits	80	40	50	19 E.	Lower

The laſt eclipſe is of the moon, Nov. 1ſt, in the evening, of which our correſpondents exhibit the following account.

Calculated by	Beg.		Mid.		End		Dur.		Dig.	
	h.	m.	h.	m.	h.	m.	h.	m.	°	′
Mr. W. Chapman, for Leiceſter	7	18	8	41	10	4	2	46	6	48
Mr. C. Green, for London	7	22	8	45	10	8	2	46	6	46
Mr. J. Metcalfe, of Wentworth, for London	7	10	8	34	9	59	2	49	7	25
Mr. J. Metcalfe, of Wentworth, for Wentworth	7	5	8	29	9	54	2	49	7	25
Mr. T. Harris, for London	7	22	8	45	10	8	2	46	6	46
Mr. T. Allen, for Spalding	7	20	8	43	10	6	2	46	6	45
Mr. E. Greenſted, for London	7	20	8	42	10	4	2	44	6	43

New Queſtions.

I. QUESTION 489, *by Mr.* T. Baker; *in which Queſt. 3. in the laſt Year's Diary is anſwered.*

By a quadratic it appears right plain,
Five hundred y——four hundred x explain:
Now, ſir, if in return you'll tell to me,
The age and fortune of my charming ſhe;
Which from the giv'n equation * will appear:
I'll ſtrive to do as much for you next year.

$$* \left.\begin{array}{l} xy = 59700 + 12y \\ x - y = 2375 \end{array}\right\} \begin{array}{l} x = \text{her fortune.} \\ y = \text{her age.} \end{array}$$

II. QUESTION 490, *by Mr.* Tho. Atkinſon.

Given the ſum of the natural ſines of the acute angles in a right-angled plane triangle $= 1\frac{2}{5}$ (radius being equal unity) and the baſe or longer leg of the triangle is $=40$; required the hypothenuſe and perpendicular of the ſaid triangle.

III. QUESTION 491, *by Mr.* John Hampſon.

Required to find two ſuch non-quadrate numbers, whoſe product ſhall be a ſquare, and the ſaid product added to the ſquare of either number ſhall be a ſquare alſo.

IV. QUESTION 492, *by Mr.* Edw. Kimpton.

Obſerved three ſtars *A*, *B*, and *C*, all in the arc of a great circle; the diſtance of *A* and *B* was found to be $=10°$, and that of *B* and $C=26°$: The difference of the azimuths of *A* and *C* was 90°; and the middlemoſt *B* was the neareſt diſtance poſſible to the zenith; it is required, from this data, the altitudes of the three ſtars.

V. QUESTION 493, *by* Noſnihctuh.

To draw a right line through a given point betwixt two right lines given by poſition, ſo that the rectangle under the parts thereof, intercepted by that point and thoſe lines, may be a minimum.

VI. QUESTION 494, *by Mr.* Joſeph Fiſher.

The gravity of ſalt water being to that of freſh, as 138 is to 135; in what latitude will a gallon of freſh water be to a gallon of ſalt water under the equinoctial, as 136 is to 138?

VII. QUESTION 495, *by Mr.* W. Spicer.

The legs of a plane triangle being given equal to 30 and 40 reſpectively; and if a line be drawn from the vertical angle to the middle of the oppoſite ſide, the rectangle of the ſaid line and baſe is a maximum; required the biſecting line and baſe.

VIII. QUESTION 496, *by* P. M. *of* Durham.

The greateſt plane triangle, having a given angle and any other limit, is that whoſe ſides about the given angle are equal; required the demonſtration.

IX. QUESTION 497, *by Mr.* Stephen Ogle.

Required to draw a right line through the focus of any given Apollonian parabola, ſo as to divide the area thereof into two ſuch parts as ſhall obtain a given ratio.

X. Questiion 498, *by Mr.* Tho. Barker.

Given $BE = 45$, $ED = 21$, and the time of deſcent of a heavy body through $BA =$ time of deſcent through BC; required the lengths of the inclined planes AB, BC, and alſo their poſition with regard to the vertical line, or plane, BD, ſo that the rectangle under AE, CD (perpendiculars upon BD) may be the leaſt poſſible. [*See the fig. to the ſolution.*]

XI. Question 499, *by Mr.* Cha. Green.

Obſerved the ſun's true altitude, at Greenwich obſervatory, to be 45° 41′, and one hour afterwards found it increaſed by juſt 8° 4′; required the day and hour when theſe obſervations were made.

XII. Question 500, *by Mr.* Tho. Harris.

'Tis juſt one-half o'th' time till * noon,
When rad'ant Sol due Eaſt does ſhine
Upon the twenty-firſt of June;
Say † where——and Phillis ſhall be thine.

* From ſun-riſe. † In what north lalitude.

XIII. Question 501, *by Mr.* T. Moſs.

From the given point P, to draw, geometrically, a right line, cutting the ſides of the given triangle ABC, ſo that the ſegments AE, BF, ſhall obtain a given proportion. [*See the fig. to the ſolution.*]

XIV. Question 502, *by Mr.* T. Allen, *of* Spalding.

In the equation $\frac{\dot{y}}{y} - \frac{\dot{x}}{x} = \frac{x^{m}x}{ay^{n}}$; it is required to determine the relation of the fluents x and y, ſuppoſing that when $x = a$, y is alſo $= a$.

The Prize Question, *by Mr.* G. Witchell.

It is known, that the ſine of the moon's parallax, at any apparent altitude, is in proportion to the ſine of the horizontal parallax, as the coſine of the ſaid altitude is to the radius: But it is frequent, in ſuch operations, to uſe the true altitude inſtead of the apparent. Now I ſhould be glad to know the quantity of the greateſt error that can ariſe from hence, and what the true altitude will at that time be, on ſuppoſition that the horizontal parallax is juſt 1 degree.

1763.

Queſtions anſwered.

I. QUESTION 489 *anſwered by* Mr. Tho. Sadler.

THE age of your moſt charming ſhe,
Twenty-five years appears to be:
Her fortune likewiſe I have found
Two thouſand and four hundred pound.

For multiplying the latter of the two given equations by y, and ſubtracting it from the former, there reſults $yy + 2363y = 59700$; from whence y will be found $= \frac{\sqrt{4 \cdot 59700 + 2363^2} - 2363}{2} = 25$ years, the lady's age; and thence x will come out $= 2400$ l. her fortune.

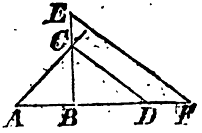
Mr. *R. Gibbons* thinks that a great age for ſuch a fortune to lie upon hand; and Mr. *Malachy Hitchins* ſeems partly of the ſame opinion, when he ſays;

Twenty-five years her age I've found,
Her worth juſt twice twelve hundred pound:
Dear *Baker*, pray don't tarry longer;
I'd rather have one eight years younger.

Much after the ſame manner it is anſwered by Meſſrs. *J. Aiſtrope, Tho. Baker (the propoſer.), J. Barker, Tho. Barker, E. Batten, Tho. Boſworth, T. Bromhall, T. Bramley, J. Buddle, R. Butler, W. Chapman, T. Davidſon, R. Dening, J. Doleman, J. Eadon, J. Giles, John Hampſon, W. Hardy, E. Hare,* jun. *Sam. Harlock, W. Harvey, J. Hitchcock, S. Hodges, T. Hopkinſon, J. Hudſon, S. Kemp, E. Kimpton, W. Kingſton, J. Launders, W. Matthewſon, T. Orme, J. Potter, J. Probert, W. Rawle, T. Robinſon, Alex. Rowe, T. Sanderſon, P. Sharp, C. Shooter, R. Spencer, W. Spicer, J. Stokes, Elean. Suggett, R. T. Tho. Walker, W. Walton, Matt. Ward, W. Wells,* and many others.

II. QUES-

II. QUESTION 490 *anſwered by Mr.* Wm. Embleton.



CONSTRUCTION. Through *A*, the extremity of the right line *AD* ($=1\frac{2}{5}$) draw *AC*, making an angle therewith = 45°, and about the other extreme *D*, as a center, with the radius (1), deſcribe an arc of a circle cutting *AC* in *C*; draw *CB* perpendicular upon *AD*, and produce *BD* till it becomes = *BF* (40); then draw *FE* parallel to *DC*, meeting *BC* produced in *E*, and *EBF* will be the triangle required.

DEMONSTRATION. The angle *BAC* being = 45°, and *B* a right angle (by conſtruction) *BC* will be = *AB*, and conſequently the ſum of *BC* and *BD* (the ſines of the angles *F* and *E*, to radius $DC=1$) will be $= AD = 1\frac{2}{5}$, and *BF* being = 40 (per conſtruction) the whole is manifeſt, &c.

CALCULATION. As rad. (*CD*) : $s\angle A$ (45°) :: *AD* (the given ſum of the ſigns) : $s\angle ACD$, the exceſs of which above *ACB* (45°) is = *BEF*, the greater of the two acute angles required.

Mr. *R. Butler* and Mr. *Da. Kinnebrook* conſtructed it in the ſame manner exactly.

An Algebraic Solution to the ſame by Mr. John Hudſon, *Land-Surveyor.*

Let s ($=\frac{7}{10}$) = half the given ſum of the ſigns, and x = half their difference; then will $s+x$ = ſine of the greater, and $s-x$ = that of the leſſer of the angles *E*, *F*; whence, per. 47 Euc. 1, $\overline{s+x}|^2+\overline{s-x}|^2$, or $2s^2+2x^2=1$; and conſequently $x=\sqrt{\frac{1-2s^2}{2}}=\frac{1}{10}$; whence the ſines of the ſaid two angles appear to be $\frac{6}{10}$ and $\frac{8}{10}$, and the leg *EB* and hypothenuſe *EF*, 30 and 50 reſpectively.

According to this method, nearly, it is anſwered by Meſſrs. *J. Barber*, *Tho. Barker*, *E. Batten*, *T. Boſworth*, *T. Bromhall* (of Mr. *Allen*'s ſchool) *T. Bramley*, *J. Buddle*, *W. Chapman*, *R. Dening*, *J. Giles*, *R. Gibbons*, *J. Hampſon*, *W. Hardy*, *E. Hare*, jun. *S. Harlock*, *Hitchins*, *S. Hodges*, *T. Hopkinſon*, *S. Kemp*, *E. Kimpton*, *W. Kingſton*, *J. Launders*, *R. Mallock*, *J. Potter*, *T. Robinſon*, *Alex.*

Rowe, T. Sanderſon, P. Sharp, W. Spicer, J. Stokes, Elean. Sugget, T. Walker, W. Walton, M. Ward, W. Wells, and others.

III. Queſtion 491 *anſwered by Mr.* R. Mallock.

Conſtitute a right angle *ACD* [*See fig. laſt queſtion*] with two lines *AC*, *CD*, reſpectively meaſuring the ſides of any two known ſquares at pleaſure, and let the points *A*, *D* be joined; draw *CB* perpendicular upon *AD*, and *AB* and *DB* will be the meaſures of the correſponding numbers required. For $BD \times AD$ or $BD^2 + BD \times AB = DC^2$, and $AB \times AD$ or $AB^2 + AB \times BD = AC^2$, by a well-known property of right angled triangles, and $AB \times BD$ ($= BC^2$, by Euc. 6. 8.) $= \frac{AC^2 \times CD^2}{AD^2}$, which is manifeſtly a ſquare number, as AC^2 and CD^2 are ſuch, by ſuppoſition, &c.

Example. Suppoſe $AC = 3$, and $DC = 4$; then will $AD = 5$ (per 47. Euc. 1.) and thence will be found *AB* and $DB = \frac{9}{5}$ and $\frac{16}{5}$ reſpectively; the firſt equimultiples of which numbers, in integers, are 9 and 16, but, being ſquare numbers, they are not for the purpoſe: But the next pair of integers, 18 and 32, (found by multiplying each of the preceding ones by 2) will anſwer the conditions of the queſtion; and, by proceeding in this manner, innumerable other anſwers may be diſcovered.

The ſame anſwered otherwiſe by Mr. John Hampſon, *the Propoſer.*

Let bba and a repreſent the two numbers required, the product whereof $aabb$ will evidently be a ſquare number; but, per nature of the queſtion, $b^4aa + bbaa$ and $aa + bbaa$ muſt each be a ſquare number, and conſequently $bb + 1$ = a ſquare number (as $b^4aa + bbaa$ and $aa + bbaa$ are evidently $= bbaa \times \overline{bb + 1}$ and $aa \times \overline{bb + 1}$ reſpectively) which ſuppoſe $= \overline{b - c}|^2$; whence b will be found $= \frac{cc - 1}{2c}$; but as a may be taken at pleaſure, ſuppoſe it $= 4ccd$, and then $bba = d \times \overline{cc - 1}|^2$; from whence may be found any number of anſwers in whole numbers at pleaſure, making c not leſs than 2, and d any non-quadrate number.

Example. If c and d are each $= 2$, then will $bba = 18$, and $a = 32$, the firſt anſwers; but if $d = 3$, then $bba = 27$, and $a = 48$, the next anſwers: And if $d = 5$, then bba will be $= 45$, and $a = 80$, &c. which anſwers are all in the ratio of 9 to 16. Again, ſuppoſing $c = 3$, and expounding d by 2, 3, 5, 6, 7, &c. ſucceſſively, another ſet of anſwers, each in the ratio of 9 to 16, will be obtained; and thus we may proceed as far as we pleaſe.

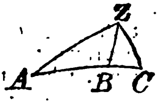
The ſame anſwered otherwiſe by Mr. T. Allen.

Let the required numbers be denoted by x and nx, and then nxx, $\overline{n+1} \times xx$, and $n \times \overline{n+1} \times xx$ being ſquare numbers, by the queſtion, n and $n+1$ muſt therefore be ſquare numbers likewiſe; whence, putting z and $z+a = \sqrt{n}$ and $\sqrt{n+1}$ reſpectively, $2az + aa$ $(= \overline{z+a}^2 - zz)$ will be $= 1$, and conſequently $z = \frac{1-aa}{2a}$, where aa may be any ſquare number leſs than unity, and x any number at pleaſure, &c.

Ingenious ſolutions to this queſtion have likewiſe been received from Mr. *J. Barber*, Mr. *Tho. Baker*, Mr. *J. Buddle*, Mr. *R. Butler*, Mr. *J. Chapman*, Mr. *Steph. Hodges*, Mr. *Da. Kinnebrook*, Mr. *Tho. Sanderſon*, Mrs. *Eleanor Suggett*, Mr. *W. Wells*, and others.

IV. QUESTION 492 *anſwered by Mr.* Rich. Gibbons.

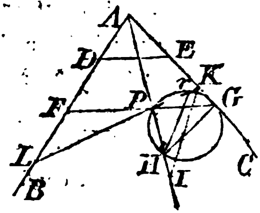
Let A, B, C repreſent the three ſtars, and Z the zenith of the place. Then, the arc ZB being perpendicular upon AC, per the nature of the queſtion, we ſhall, per ſpherics, (prop. 35 Emer. Trig.) have ſine of $AB + BC$ (30°) : ſine of $AZB + BZC$ (90°) :: ſine of $BC - AB$ (10°) : ſine of $BZC - AZB$ $(20^\circ\ 19'\ 20'')$. Therefore $AZB = 34^\circ\ 15'\ 20''$, and $BZC = 55^\circ\ 9'\ 40''$; the altitude of the ſtar $A = 72^\circ\ 18'\ 14''$, of $B = 75^\circ\ 19'\ 32''$, and that of $C = 65^\circ\ 22'\ 23''$.



Mr. *Tho. Allen* and Mr. *E. Hare*, jun. anſwer it in this manner exactly.

We have alſo been favoured with elegant and conciſe algebraic ſolutions to this queſtion from Mr. *Tho. Boſworth*, Mr. *J. Buddle*, Mr. *Rob. Butler*, Mr. *W. Chapman*, Mr. *J. Hampſon*, Mr. *W. Hardy*, Mr. *S. Harlock*, Mr. *Malachy Hitchins*, Mr. *Steph. Hodges*, Mr. *Tho. Hopkinſon*, Mr. *J. Hudſon*, Mr. *W. Kingſton*, Mr. *Da. Kinnebrook*, Mr. *Ja. Launders*, Mr. *Tho Sanderſon*, and others.

V. QUESTION 493 *anſwered by Mr.* Da. Kinnebrook, *and Mr.* T. Moſs.



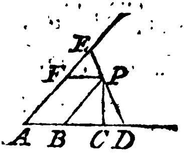
CONSTRUCTION. Let P be the given point; and AB and and AC the two lines given in poſition; in which take $AD =$ AE, and let the points D, E be joined; then through P draw FG parallel to DE, and the thing is done.

DEMONSTRATION. Join the points A, P, and draw the right line GH (meeting AP produced in H) ſo as to make the $\angle PGH$ $= \angle FAP$, and through the points H, P, G, conceive the periphery of a circle to be deſcribed; then the $\angle PGH$ being $= \angle FAP$ (by hypotheſis), and the $\angle APF = \angle HPG$, the $\angle AFP$ will be $= \angle PHG$ (32 Euc. 1); likewiſe the $\angle AFP = \angle AGP$ (by conſtruct.) and conſequently the $\angle AGP = \angle PHG$; wherefore the circle touches the right line AC in the point G (32 Euc. 3). But the triangles APF, HPG, are ſimilar; and therefore $AP : PF :: PG : PH$; whence $PF \times PG = AP \times PH$, and conſequently, when the rectangle $PF \times PG$ is a minimum, its equal $AP \times PH$ or PH (as AP is a conſtant quantity) will be a minimum alſo, which will be in the caſe above-mentioned, viz. when the line FG is drawn ſo as to cut off the $\triangle FAG$ iſoſceles; for let LK be any other right line drawn through the ſaid given point P, and from the point r, where that line and the periphery of the circle interſect, draw rH, and parallel thereto draw KI; then, becauſe AC is a tangent to the circle, PK will evidently be greater than Pr: And therefore, as KI is parallel to Hr, PI will be greater than PH, and conſequently $LP \times PK$ $(= AP \times PI)$ will every where be greater than $FP \times PG$ or its equal $AP \times PH$. *Q. E. D.*

Very ingenious and neat geometrical conſtructions to this queſtion have likewiſe been received from Mr. *Rob. Butler*, Mr. *Rich. Gibbons, P. M.* of *Durham*, and ſeveral others.

The

The ſame ſolved Algebraically by Meſſrs. T. Allen, T. Bromhall, *and* E. Hare, *jun.* &c.



Let P be the given point, and draw BP, FP, parallel to AE and AD reſpectively, and PC perpendicular upon AD. Put $FP = a$, $BC = b$, $PC = c$, and $CD = x$. Then will $\sqrt{cc+xx} = PD$, and per ſimilar triangles, $b+x\ (BD) : \sqrt{cc+xx}\ (PD) :: a\ (FP) : \frac{a\sqrt{cc+xx}}{b+x} = EP$; whence $PE \times PD$ is $= a \times \frac{cc+xx}{b+x}$, which is to be a minimum, per queſtion; and therefore, in fluxions, $2bxx + xxx - ccx = 0$; from whence x is found equal $\sqrt{cc+bb} - b$; and from thence the poſition of the line ED may be determined.

The ſame anſwered otherwiſe by Mr. Wm. Embleton.

Put $PB = a$; $PF = b$; $BC = c$, and $BD = x$; and then will PD be found $= \sqrt{aa+xx-2cx}$, by 9. 2. of Simpſon's Elem. of Plane Geom. 1ſt edition: whence, per ſimilar triangles, BDP, FPE, $x\ (BD) : \sqrt{aa+xx-2cx}\ (DP) :: b\ (PF) : \frac{b}{x}\sqrt{aa+xx-2cx} = PE$; and conſequently $PE \times PD$ will be $= \frac{b}{x} \times \overline{aa+xx-2cx}$, which, according to the queſtion, is to be a minimum: And conſequently, in fluxions, $-\frac{aax}{xx} + x = 0$; whence x is found $= a$, or $AE = AD$; from which the poſition of the required line becomes known.

True and conciſe anſwers to this queſtion have alſo been received from Mr. *Tho. Barker*, Mr. *E. Batten*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *J. Hampſon*, Mr. *Wm. Hardy*, Mr. *Maluchy Hitchins*, Mr. *Steph. Hodges*, Mr. *Tho. Hopkinſon*, Mr. *J. Hudſon*, Mr. *S. Kemp*, Mr. *Wm. Kingſton*, Mr. *J. Launders*, Mr. *Tho. Robinſon*, Mr. *Tho. Sanderſon*, Mr. *P. Sharp*, Mr. *W. Spicer*, and others.

VI. QUESTION 494 *anſwered.*

The numerical data of this queſtion being miſprinted, no ſolution to it has been received. Some, however, obſerve, that it is no more than to find x the ſign of the latitude of the place where a body, weighing w at the equator, ſhall equiponderate with another given homogeneous weight W; which is eaſily found $= \sqrt{\frac{66240}{519} \times \frac{W-w}{w}}$ * (the ratio of the diameters of the earth being as 230 to 231, and the gravity and centrifugal force, at the equator, 1 and $\frac{1}{289}$ reſpectively.

VII. QUESTION 495 *anſwered by Mr.* W. Embleton.

Let ABC repreſent the triangle required, and BD the line biſecting its baſe AC. Now, when $BD \times 2AD$ $(= BD \times AC)$ is a maximum, $AD^2 - 2BD \times AD + BD^2$ or $\overline{AD \backsim BD}|^2$ $(= AD^2 + BD^2$, a conſtant quantity by theo. 11. book 2. Simpſon's Elem. of Plane Geom.

2d

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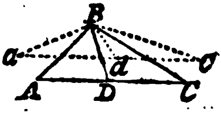
* This expreſſion may be thus found: as 288 : 289 :: w (the weight or grav. at the equator diminiſhed by the centrif. force) : $\frac{298\,w}{288}$ the weight *there* if the centrif. force were taken off, or if the earth did not revolve on its axis; alſo as 230 : 231 :: $\frac{289\,w}{288} : \frac{231 \cdot 289\,w}{230 \cdot 288}$ the weight at the pole; hence $\frac{231 \cdot 289\,w}{230 \cdot 288} - w = \frac{519\,w}{66240}$ is the whole weight gained at the pole: Then, by art. 401 Simpſon's Flux. $1 : xx :: \frac{519\,w}{66240} : \frac{519\,wxx}{66240}$ the weight gained in the lat. whoſe ſine is x, which muſt be $= W - w$; hence $x = \sqrt{\frac{W-w}{519\,w} \cdot 66240}$, as above.

Now the limit of the encreaſe being $\frac{519}{66240} = \frac{1}{128}$ nearly, and the increaſe propoſed in the queſt. being from 135 to 136, or $\frac{1}{135}$ part only; it is evident that it is propoſed within the limit; and, by writing 135 and 136 inſtead of w and W in the above value of x, it becomes $x = \cdot 9723207 =$ the ſine of 76° 29′ 17″ nearly, the angle required.



ition, diminiſhed by the greateſt poſſible of the variable quanti- $BD \times AD$) will manifeſtly minimum, and conſequently o BD a minimum, which, it is nt, can only be when it is $= 0$; ce it clearly follows that the re- d triangle will be right-angled at B, becauſe (ſince is now proved $= BD = DC$) it may be circumſcribed ıe periphery of a ſemi-circle of that radius deſcribed t the center D;——hence by 47 Euc. 1. AC will be l $= 50$; and conſequently $BD = 25$.

ROLLARY. If Ba, Bc, be ſuppoſed any other poſition ıe two given ſides BA, BC, different from that here rmined, then will $AD + BD$ be always greater than $+ Bd$; for $AD^2 + BD^2$ being $= ad^2 + dB^2$, and $) \times DB \sqsubset 2ad \times dB$ (by what has juſt now been de- ſtrated), $AD^2 + 2AD \times DB + DB^2$ will therefore reater than $ad^2 + 2ad \times dB + dB^2$, and conſequently $+ DB \sqsubset ad + dB$.

ccording to this method of ſolution the queſtion is alſo vered by Mr. *Rob. Butler*, Mr. *Rich. Gibbons*, Mr. *Kinnebrook*, *P. M. of Durham*, Mr. *W. Spicer*, and e others.

Algebraical Solution to the ſame by Mr. T. Bromhall.

'ut $AB = 40 = a$, $BC = 30 = b$, $AD = x$, and $BD = y$; n, per 12. 2. Simp. Elem. Plane Geom. 1ſt edit. $2xx +$ $= aa + bb$; whence $y = \sqrt{\dfrac{aa + bb - 2xx}{2}}$, and ıſequently $AC \times BD$ is $= 2x\sqrt{\dfrac{aa + bb - 2xx}{2}}$, which ng to be a maximum per queſt. $4xx \times \dfrac{aa + bb - 2xx}{2}$ ll therefore be a maximum, and conſequently, in fluxions, $axx\dot{x} + 2bbx\dot{x} - 8x^3\dot{x} = 0$; whence $aa + bb - 4xx$ o, and conſequently $2x = \sqrt{aa + bb} = 50 = AC$; d hence it appears that $BD = \frac{1}{2} AC$, and alſo that ABC a right angle.

This queſtion was likewiſe truly anſwered by Mr. *Tho.* *arker*, Mr. *E. Batten*, Mr. *Tho. Boſworth*, Mr. *Tho. Bram-* *y*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *J. Hampſon*,

Mr.

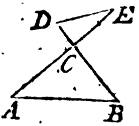
Mr. *Wm. Hardy*, Mr. *Edw. Hare*, jun. Mr. *Sam. Harlock*, Mr. *Malachy Hitchins*, Mr. *Steph. Hadges*, Mr. *Tho. Hopkinſon*, Mr. *J. Hudſon*, Mr. *Sam. Kemp*, Mr. *Wm. Kingſton*, Mr. *Ja. Launders*, Mr. *R. Mallock*, Mr. *Tho. Robinſon*, Mr. *Tho. Saunderſon*, Mr. *P. Sharp*, Mr. *Jo. Stokes*, Mrs. *Eleanor Suggett*, Mr. *Wm. Walton*, and others.

VIII. QUESTION 496 *anſwered by Mr.* Rich. Mallock.

The other given limit, here meant, muſt neceſſarily be ſuch as the baſe, the perimeter, the radius of the circumſcribing circle, or the like, which would confine the increaſe of the required triangle within certain bounds or limits, and affect each of the including ſides exactly alike, and then their ſum may be always denoted by a certain right line determinable by means of the ſaid limit, and conſequently (the area of every plain triangle being = the rectangle under any two of its ſides drawn into half the ſine of their included angle, the triangle ſought will be a maximum, when the ſaid right line is ſo divided, that the rectangle of its two parts may be a maximum (the ſine of the included angle being a conſtant quantity) *i. e.* when they are equal, which is too well known to need a particular demonſtration here.

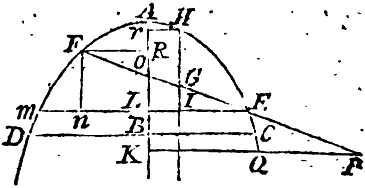
Mr. *J. Hudſon* puts b for the baſe, a for the ſine of the given angle, s and c for the ſine and coſine of half the ſum, and x and y for the ſine and coſine of half the difference of the other two angles, and then the area of the required triangle is readily found $= bb \times \frac{ss - xx}{2a}$, which, it is manifeſt, will continually increaſe, as b increaſes, and x, at the ſame time, decreaſes, and conſequently that it will (under any aſſigned poſſible value of b) be greateſt, when x is leaſt, that is, when x vaniſhes, and the required triangle becomes iſoſceles, whatever the other given limit may be, unleſs it reſtrains the including ſides from becoming equal at the ſame time that, per queſtion, they are ſuppoſed every way capable of being ſo, which would be abſurd indeed, and incompatible with the true nature of the queſtion !——If either of the including ſides, or their difference, or the difference of their ſquares, or the ſum or difference ariſing from either of them, being added to or ſubtracted from n times the other (except when n equals unity), or the like, were to be given, then, they affecting the including ſides not alike, the queſtion, inſtead of being limited thereby, would become either abſurd or infinite.

The ſame anſwered by P. M. *of* Durham (*the Propoſer*). *Taken from the Diary for the Year* 1764.



Aſſume the points *D*, *E*, in the ſides *BC*, *AC*, comprehending the given angle, produced, and let the right line *DE* be drawn, and then the $\triangle DCE$, as alſo each of its ſides, will be given. But the triangles *DCE*, *ACB*, are the halves of parallelograms ſtanding about the common angle at *C*, and parallelograms, ſtanding about a common angle, are directly as the rectangles of their ſides (23 Euc. 6), therefore alſo their halves will be in the ſame ratio, *i. e.* $\triangle DCE : \square DCE :: \triangle ACB : \square ACB$. Whence the $\triangle ACB$ having a given $\angle$ and any other limit whatever, is in a conſtant ratio to the rectangle under the ſides about the ſaid given $\angle$, and conſequently when the $\triangle$ is greateſt, the rectangle will be the greateſt alſo. But the ſides *AC*, *BC*, have a conſtant analogy to the ſines of the oppoſite angles at the baſe, and may mutually be repreſented by each other; therefore, when the rectangle under the ſaid ſides is a maximum, it is manifeſt that the rectangle under the ſines of the angles at the baſe muſt be a maximum alſo: But the ſum of theſe angles being a conſtant quantity, the rectangle under their ſines will be the greateſt when the ſaid angles are equal, *i. e.* when the ſides ſubtending them are of conſequence equal (6 Euc. 1). *Q. E. D.*

IX. QUESTION 497 *anſwered by Mr.* Rob. Butler.



Let *DAC* repreſent the given parabola; and *EF* the given dividing line paſſing through the focus *O*, and let the ordinates *FR*, *EL*, be drawn perpendicular upon the axis *AL*, putting $AO = a$, $FR = x$, and $EL = y$; then, per conics, $\frac{xx}{4a} = AR$, and $\frac{yy}{4a} = AL$ ($4a$ being $=$ the parameter); whence $RO \left(= a - \frac{xx}{4a}\right) = \frac{4aa - xx}{4a}$, and $OL = \frac{yy - 4aa}{4a}$; and therefore $\frac{1}{2}x \times$

$4aa$

$\frac{4aa-xx}{4a}+\frac{2}{3}x\times\frac{xx}{4a}+\frac{2}{3}y\times\frac{yy}{4a}-\frac{1}{2}y\times\frac{yy-4aa}{4a}=$ $\frac{4aax-x^3+4aay-y^3}{8a}+\frac{x^3+y^3}{6a}=$ the area of the figure $FAE=$ a given quantity A, by the nature of the queſtion (becauſe the area of the whole, DAC, and the ratio of the parts $DFEC$, FAE being given, per the queſtion, the parts themſelves will from thence likewiſe be given); but, by ſimilar triangles, $x\ (FR):\frac{4aa-xx}{4a}\ (RO)::y\ (EL):\frac{yy-4aa}{4a}\ (OL)$; whence $4aa=\frac{xyy+yxx}{x+y}=xy$, which value ſubſtituted in the preceding equation of the area, &c. it becomes $\frac{x^3+y^3}{6a}+\frac{yxx+xyy-x^3-y^3}{8a}=A$, or $y^3+3y^2x+3yx^2+x^3=24aA$: And hence, by extraction, $y+x$ comes out $=\overline{24aA}|^{\frac{1}{3}}$; from the ſquare of which ſubtracting the equation $4yx=16aa$ (found from above), and there reſults $yy-2xy+xx=\overline{24aA}|^{\frac{2}{3}}-16aa$, and conſequently $y-x=\sqrt{\overline{24aA}|^{\frac{2}{3}}-16aa}$; and hence y is found $=\frac{1}{2}\sqrt{\overline{24aA}|^{\frac{2}{3}}-16aa}+\frac{1}{2}\times\overline{24aA}|^{\frac{1}{3}}$, and $x=\frac{1}{2}\times\overline{24aA}|^{\frac{1}{3}}-\frac{1}{2}\sqrt{\overline{24aA}|^{\frac{2}{3}}-16aa}$; from either of which equations the poſition of the line EOF is determined.

Mr. *Da. Kinnebrook* anſwered it according to this method of ſolution exactly.

The ſame anſwered otherwiſe by Mr. Wm. Embleton.

Draw Hr perpendicular upon AK, and put $x=Ar$; $p=$ the parameter of the principal diameter or axis AK, and ſuppoſe the right line FE (paſſing through the focus O) to be drawn as is required; and let the area of the ſpace $FAHE$ (which is given by the nature of the queſtion) be denoted by a; then, per conics, HG will be found $=\frac{4x+p}{4}$, $FE=4x+p$, $Hr=\sqrt{px}$, &c. Whence a ($=HG\times\frac{2}{3}FE\times$ s. $\angle HGP$) will appear $=\frac{\sqrt{p}}{6}\times\overline{4x+p}|^{\frac{3}{2}}$: And conſequently $x = \frac{1}{4} \times \overline{\frac{36aa}{p}}\Big|^{\frac{1}{3}} - \frac{p}{4}$; from whence the poſition of the line FE may be readily determined.

Mr. *John Hudſon* ſolved it according to this method very near.

The ſame anſwered more generally by P. M. *of* Durham.

This truly ingenious gentleman ſuppoſes the point P, through which the dividing line is required to paſs, to be given any where in the plane of the given parabola DAC (either within it or without), and likewiſe that the right line PEF is drawn as required, cutting the axis AB in O, and dividing the given parabolic area DAC in the given proportion of m to n.

Then, biſecting EF in G, and drawing the diameter HG through G, and the ordinate Em through E, meeting the axis AB in L, and the diameter HG, produced, in I, the area DAC : area EHF :: $m+n : n$ :: $BC^3 : EI^3$. (cor. 2. 52 Sim. 5); whence EI is given. Draw PK perpendicular to the axis in K, and meeting the curve in Q, as alſo Fn perpendicular to the ordinate Em. Then (by a property of the parabola) $AO \times p = \square ELn$; alſo (by another property) $Fn \times p = \square Enm$: But $AK \times p = QK^2$; therefore $OK \times p = QK^2 - \square ELn$: Whence $QK^2 - \square ELn : \square Enm :: (OK : Fn ::) PK : En$, i. e. $QK^2 - \square ELn = nm \times PK$. Call $PK = a$, $QK = b$, $EI = c$, and $IL = x$, then will $EL = c + x$, $Ln = c - x$, and $nm = 2x$: Wherefore $bb - cc + xx = 2ax$, and conſequently $\overline{a-x}|^2 = aa - bb + cc$.

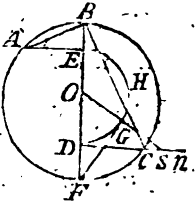
COROLLARY 1. If the point P be in the curve, then $aa - bb = 0$, and $a - x = c$.

COROLLARY 2. But if the point P be in the axis, then $a = 0$, and $xx = cc - bb$.

This queſtion is likewiſe ingeniouſly anſwered by Mr. *J. Hampſon*, Mr. *P. Sharp*, and others.

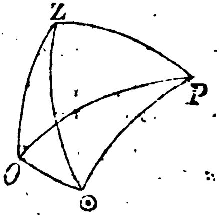
X. QUESTION 498 *anſwered.*

If BD be ſuppoſed to be produced out to F till the time of perpendicular deſcent along it ſhall be equal to the time of deſcent along BA or BC, and the circumference of a circle



circle be deſcribed about BF as a diameter, it will, it is very well known, paſs through the points A and C (ſee art. 204 p. 230 and 131 of Simpſon's Fluxions, 2d edition); whence, DC^2 being $= BD \times DF$ and $AE^2 = BE \times \overline{ED + DF}$, by the property of the circle; we ſhall have $AE^2 \times CD^2 = BE \times BD \times \overline{ED \times DF + DF^2}$, which it is evident, by inſpection, can admit of neither max. nor minimum; but if it were required to be equal to a given quantity (m^2); then, $ED \times DF + DF^2$ $(= DF \times \overline{ED + DF})$ being $= \frac{m^2}{BE \times BD}$ (which ſuppoſe $= nn$) we ſhall, by deſcribing a ſemi-circle EHD upon ED, as a diameter, and drawing a tangent to it ſuch, that the diſtance intercepted between the point of contact G and its interſection F with ED produced may be $= n$, find the value of DF required.——Thus, in Dn, perpendicular to ED, take $Ds = n$; join the points O, s, with a right line, and from the point G, where it interſects the ſaid ſemicircle, draw GF perpendicular to Os, meeting OD produced in F, and the thing is done; for GF is manifeſtly $= Ds$. ($= n$, by conſtruction).

XI. QUESTION 499 *anſwered by Mr.* R. Butler.



Put a and b equal to the coſines of $\odot Z$ and OZ, the co-altitudes at firſt and ſecond obſervations, reſpectively; m and n equal to the ſine and coſine of ZP (the co-latitude) and x and y equal to the ſine and coſine of $\odot P$ or OP (the ſun's diſtance from the north pole); then, per ſpherics, $\frac{a - ny}{mx} =$ coſine of the $\angle \odot IZ$; the ſine of which is $\frac{\sqrt{mmxx - \overline{a - ny}|^2}}{mx}$: Whence, putting p and q equal to the ſine and coſine of $\angle OP\odot$ (15°), the coſine of $\angle ZPO$ will be found $= \frac{qa - nay}{mx} + \frac{p\sqrt{mmxx - \overline{a - ny}^2}}{mx}$; and conſequently $qa - nqy + p\sqrt{mmxx - \overline{a - ny}|^2} + ny = b$ (by Anderſon's theorem);

from whence y is found = ſine of 23½ degrees, anſwering to the 21ſt of June; in the forenoon of which, at the hours of 9 and 10, the obſervations appear to have been made.

The ſame anſwered otherwiſe by Mr. T. Allen.

For the coſines of the given coaltiudes $\odot Z$ and OZ put a and b reſpectively and c and d = the ſine and coſine of ZP, the colatitude; m and n = the ſine and coſine of the given interval of time ($OP\odot = 15°$) between the two obſervations, and x and y = the ſince and coſine of $\odot P$ (OP) the codeclination (radius 1); then will $\frac{b-dy}{cx}$ and $\frac{a-dy}{cx}$ be equal to the coſines of ZPO and $ZP\odot$ reſpectively. Whence, per ſpherics, and the nature of the queſtion, $n \times \frac{a-dy}{cx} + m\sqrt{1-\frac{\overline{a-dy}|^2}{ccxx}} = \frac{b-dy}{cx}$; and hence, by ſubſtituting t for $b-na$, and s for $d-nd$, we get $m \times \sqrt{1-\frac{\overline{a-dy}|^2}{ccxx}} = \frac{t-sy}{cx}$, and conſequently (by involution, &c. and putting $f = ss + mmdd + mmcc$, $g = mmda + ts$, and $-k = mmcc - mmaa - tt$) $fyy - 2gy = -k$; from the reſolution of which y is found $= \frac{g \pm \sqrt{gg-fk}}{f}$, expreſſing the ſine of the declination required; from whence every thing elſe wanted, may be readily determined.

Mr. *Charles Green* (the propoſer) anſwers it by a proceſs not materially different from that above exhibited; and from thence finds x = 9171·796 = coſine of 23° 29′ nearly. Whence the hours of obſervation appear to be 9 and 10 on the 21ſt of June; which agree very near with obſervation: —— but theory and calculation will differ a few ſeconds, nay even minutes, ſometimes, from the niceſt obſervations.

Mr. *Rich. Gibbons*, Mr. *Da. Kinnebrook*, and *P. M.* of *Durham*, by very curious and ſhort proceſſes, have likewiſe brought out the ſame concluſion.

Ingenious anſwers to this queſtion have likewiſe been received from Mr. *T. Barker*, Mr. *J. Buddle*, Mr. *T. Hopkinſon*, Mr. *J. Hudſon*, Mr. *W. Kingſton*, Mr. *J. Potter*, Mr. *Tho. Robinſon*, Mrs. *Eleanor Suggett*, Mr. *W. Sutton*, and others.

XII. QUESTION 500 *anſwered by Mr.* John Potter; *to Mr.* Tho. Harris, *the Propoſer.*

SIR,

The latitude below is ſhewn *,
Yet Phillis ſtill may be your own.

* In 64° 35′ 48″ north latitude.

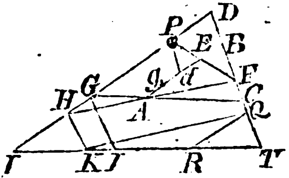
Another Anſwer to the ſame.

Let s and c = the ſine and coſine of $P\odot$ [*See laſt fig.*] or PO (= 66° 31′) the ſun's diſtance from the north pole P; y and x = the ſine and coſine of ZP (the colatitude) and z and u = thoſe of the $\angle ZPO$ (= $\odot PO$ per nature of the queſtion; then will $2uu - 1$ = coſine $\angle ZP\odot$, and per ſpherics, $2uuys - sy + cx$ (= the coſine of $Z\odot$) will be = 0 (per queſtion); whence $2uus = s - \frac{cx}{y}$: And in the right-angled ſpherical triangle OZP, $u : 1 :: \frac{c}{s} : \frac{c}{us} = \frac{x}{y}$ = cotangent of ZP; from which is derived $2uus = \frac{2ccyy}{sxx}$: Whence, by equality, $s - \frac{cx}{y} = \frac{2ccyy}{sxx}$; and conſequently $\frac{x}{y} = 2{\cdot}1057018$ = tang. of 64° 35′ 48″, the latitude required.

According to this method, very nearly, the anſwer is given by Mr. *T. Allen*, Mr. *Tho. Barker*, Mr. *R. Butler*, Mr. *W. Chapman*, Mr. *J. Hampſon*, Mr. *Da. Kinnebrook*, and Mr. *Paul Sharp*——This queſtion is likewiſe ingeniouſly anſwered by Mr. *Tho. Boſworth*, Mr. *John Buddle*, Mr. *Rich. Dening*, Mr. *Rich. Gibbons*, Mr. *Sam. Harlock*, Mr. *Malachy Hitchins*, Mr. *Steph. Hodges*, Mr. *Tho. Hopkinſon*, Mr. *John Hudſon*, Mr. *W. Kingſton*, *P. M.* of *Durham*, Mr. *Wm. Spicer*, Mr. *Ja. Stokes*, Mrs. *Eleanor Suggett*, Mr. *W. Sutton*, Mr. *Matt. Ward*, and others.

XIII. QUESTION 501 *anſwered by Mr.* D. Kinnebrook.

CONSTRUCTION. Through the given point P draw PI parallel to the ſide AB, of the given plane triangle ABC, meeting the other ſide thereof, BC, produced in D, and take



take DI to DT in the given ratio, and let the points I, T, be joined: Take DQ to PI as DT to DI, and draw QR parallel to AB meeting IT in R; through A draw AG parallel to IT meeting ID in G, and from thence let GL be parallel to DT meeting IT in L: Then from R towards I take RK ſo, that $RK \times LK$ may be $= RT \times AG$, and join the points K, Q, and through the point A, and parallel to KQ, let the right line HAF be drawn meeting ID and DT in H and F: Laſtly, join the points P, F, and that will be the line required to be drawn, it cutting AB in E ſo, that $AE : BF :: ID : DT$, the ratio given, by conſtruction.

DEMONSTRATION. By conſtruction, $LK : RT :: AG : RK ::$ (by ſim. triangles) $HG : RQ$: whence, by alternation and equality, $LK : HG :: RT : RQ ::$ (by ſim. triangles) $IT : ID :: IL : IG$; and hence it follows, that the right line HK (when drawn) will be parallel to GL or DT (by corol. theor. 12. 4. of Simp. Elem. of Plane Geom. 2d edit.), and conſequently that, HF being parallel to KQ (by conſtruction), $HKQF$ will be a parallelogram; whence (per ſim. triangles) $HK\ (FQ) : IH :: DT : DI :: DQ : PI$ (by conſtruction), and conſequently $DF\ (DQ - FQ) : PH\ (PI - IH) :: DQ : PI$; but $DF : PH :: BF : AE$ (by ſim. triangles): Whence, by equality, $BF : AE :: DQ : PI$, that is, in the given ratio.

Mr. *R. Butler* has conſtructed this queſtion in the ſame manner, very near.

The ſame anſwered otherwiſe by P. M. *of* Durham, *and* *Mr.* T. Moſs (*the Propoſer*).

Suppoſe the thing done, and PEF to be the poſition of the right line required; draw Pd parallel to BC, and take dg to Pd in the given ratio of AE to BF, and then $AE : gd :: (BF : Pd) :: BE : dE$ (by ſim. triangles), and, compoundly, $AE : AB :: gd : gE$; wherefore AB and gd being both given in length and poſition, it is now only required to produce the given right line Ag to E ſo, that the rectangle under AE and gE may be equal to the given rectangle under AB and gd, which is elegantly done in Simpſon's Select Exerciſes, p. 94.

Mr. *Wm. Embleton* alſo gave a curious conſtruction, which is unluckily omitted.

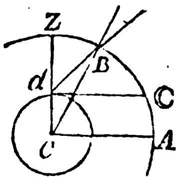
XIV. QUESTION 502 *anſwered by Mr.* W. Spencer.

Multiply each ſide of the given fluxion. equation ($\frac{\dot{y}}{y} - \frac{\dot{x}}{x} = \frac{x^m \dot{x}}{a y^n}$) by $\frac{y^n x^n}{x^{2n}}$, and it becomes $\frac{x^n y^{n-1} \dot{y} - y^n x^{n-1} \dot{x}}{x^{2n}} = \frac{x^{m-n} \dot{x}}{a}$; the equation of the fluents of which is evidently $\frac{y^n}{n x^n} = \frac{x^{m-n+1}}{a \times \overline{m-n+1}}$ or $\frac{1}{n} = \frac{a^{m-n}}{m-n+1}$ when x and y are each $= a$; whence the equation of the fluents truly corrected, according to the conditions of the queſtion, will be $\frac{y^n}{n x^n} - \frac{1}{n} = \frac{x^{m-n+1}}{a \times \overline{m-n+1}} - \frac{a^{m-n}}{m-n+1}$; and conſequently $y^n = \frac{n x^{m+1} + \overline{m-n+1} . a x^n - n a^{m-n+1} x^n}{a . \overline{m-n+1}}$ will be the equation, exhibiting the relation of the fluents x and y required.

COROLLARY. If $n = 1$, then will $\frac{y}{x} = \frac{x^m - a^m + ma}{ma}$, the equation of the fluents in this caſe, &c.

True anſwers to this queſtion have been received from Mr. *T. Allen* (the propoſer), Mr. *R. Butler*, Mr. *Wm. Embleton*, Mr. *Da. Kinnebrook*, *P. M.* of *Durham*, and others.

The PRIZE QUESTION *anſwered by Mr.* T. Allen, *of* Spalding, *in* Lincolnſhire.



Let C repreſent the earth's center, and the arc AB any altitude of the moon, conſidered as variable. Put the ſine of $\angle d☾C$ (the horizontal parallax) $= \cdot 0174524 = a$ (radius 1.) and the ſine of the $\angle ZdB$ (the moon's viſible zenith diſtance) $= x$, and then the ſine and coſine of $\angle dBC$, the true parallax in altitude, will readily be found $= ax$ and $\sqrt{1 - aaxx}$ reſpectively: Whence $x\sqrt{1 - aaxx} - ax\sqrt{1 - xx} =$ ſine of $\angle ZCB$, the true zenith diſtance;

and conſequently, $1 : x\sqrt{1-aaxx} - ax\sqrt{1-xx} :: a : ax\sqrt{1-aaxx} - aax\sqrt{1-xx}$, the ſine of the erroneous parallax. Therefore $ax - ax\sqrt{1-aaxx} + aax\sqrt{1-xx}$, or, which is the ſame thing, $x - \sqrt{xx - aax^4} + a\sqrt{x^2 - x^4}$ is required to be a maximum: In fluxions, $\dot{x} + \frac{2a^2x^2\dot{x} - \dot{x}}{\sqrt{1-aaxx}} + a \times \frac{\dot{x} - 2x^2\dot{x}}{\sqrt{1-xx}} = 0$; whence $\frac{1-2aaxx}{\sqrt{1-aaxx}} + a \times \frac{2xx-1}{\sqrt{1-xx}} = 1$: From the reſolution of which equation the value of x is found $= \cdot 71039 =$ the ſine of 45° 16′ very near. Whence the moon's true zenith diſtance (ZCB) is readily found $=$ 44° 33′ 23″, and the error of parallax $=$ 31″ 40‴; which was required.

This queſtion is likewiſe anſwered by Mr. *R. Butler*, Mr. *Joſ. Fiſher*, Mr. *Cha. Green*, Mr. *D. Kinnebrook*, and others; but the above anſwer, and Mr. *G. Witchell*'s, being the only true ones that came to hand within the limited time, the authors thereof are therefore entitled to the prizes, the former of them to the lot of 12, and the latter to that of 8 Diaries.

The Eclipſes calculated for 1763.

There will be only two eclipſes this year, both of the ſun, and both inviſible to the inhabitants of this part of the earth. They happen in the following order.

The firſt is on April 13th, a little after ten in the morning, and will be very conſpicuous to the Ethiopians, Abyſſinians, &c.

The ſecond, October 7th, about one in the morning, and will be ſeen by thoſe who are that time ſailing on the Pacific Ocean, near the equinoctial.

Mr. *Stephen Hodges*, at the Right Hon. Lord Spencer's, at Althorp, near Northampton, obſerves the ſame as above.

Mr. *J. Metcalfe* has obliged us with with a general calculation of theſe eclipſes, but as they are inviſible here, and we are ſtraitened for room, we muſt omit them.

New Queſtions.

I. QUESTION 503, *by Mr.* Tho. Sadler.

At Marbury a maid doth dwell,
Whoſe wit and beauty moſt excel;
She ſeems to rival ev'ry fair;
They're few that can with her compare:
Her height, age, fortune, you will know
From th' equations propos'd below.

$x + 2y + z = 281{\cdot}5294.$
$xyz = 270988{\cdot}22.$
$xx = 4yy + zz.$

Where $x =$ her fortune.
$y =$ her height in inches.
$z =$ her age—to be ſolved (by a quadratic.

II. QUEST. 504, *by the Rev. Mr.* Wyvell Blennerhaſſett.

A ſhip, at a certain port *A*, obſerves two iſlands *B* and *C*; *B* bears from her N. N. E. and *C*, E. S. E. From thence ſailing E. by N. 5 miles, finds *B* and *C* equally diſtant from her, and continuing the ſame courſe 5 miles farther, ſhe had the ſaid iſlands in a right line; required the port's diſtance from the two iſlands *B*, *C*, and alſo the ſhip's diſtance from them at the ſecond and third obſervations.

III. QUESTION 505, *by Mr.* J. Hudſon, *Land Surveyor.*

There are two places in the ſame parallel of north latitude, differing in longitude 20°, and their diſtance on the parallel is known to exceed their diſtance on the arc of a great circle by 2·335 miles; required the latitude and neareſt diſtance of the ſaid two places, ſuppoſing the earth ſpherical, and 60 miles = a degree.

IV. QUESTION 506, *by* Geometricus.

In two ſimilar right-angled triangles, there is given the baſe of the one, and the perpendicular of the other, in one ſum; and it is propoſed to determine the triangles ſuch, that their hypothenuſes ſhall form the legs of another triangle ſimilar to them; and ſo, that the ſum of their three areas may be of a given magnitude.

V. QUESTION 507, *by Mr.* W. Spicer.

Given the area of a plane triangle = 235, and its vertical angle = 73° 34′, and the ſide of its inſcribed ſquare = 9.6, to deſcribe the triangle.

VI. QUESTION 508, *by Mr.* T. Barker, *of* Weſthall.

Suppoſing *A* and *B* to repreſent two right lines, whereof the ſum is given = *s*, and $\frac{m}{n}$ to denote any given ratio. Required to determine, by a geometrical conſtruction, what *A* and *B* themſelves muſt be, ſo that $A^2 + \frac{m}{n} \times B^2$ may be = *cc*, a given ſquare.

VII. QUESTION 509, *by Mr.* T. Moſs.

A lays *B* one hundred guineas to 1, that in 25 throws with 5 half-pence, he does not throw, preciſely, 4 heads the firſt 5 throws, 6 heads the next 5 throws, 8 the 3d, 10 the 4th, and 12 the laſt 5 throws: Required the reſpective values of theſe two gameſters expectations.

VIII. QUESTION 510, *by Mr.* Rob. Butler.

Suppoſe in latitude 52° north, a ſemi-circular plane, whoſe radius is 20 feet, to be placed in the plane of the meridian, with its baſe downwards; at what time in the afternoon, on 21ſt June, will its ſhadow cover juſt one acre on an horizontal plane?

IX. QUESTION 511, *by Mr.* Malachy Hitchins.

In latitude 50° north, there is a cylinder, whoſe internal diameter is = 6 feet, and length = 12 feet, lying on the horizontal plane, and making an angle with the meridian = 20° eaſt; at what time of the day, on May 10, 1763, will the greateſt part of its internal ſuperficies be enlightened by the ſun; and what ratio will that enlightened ſpace bear to the whole internal ſuperficies?

X. QUESTIION 512, *by Mr.* Joſeph Walker.

Two right lines, drawn from the ſame point, being given both in length and poſition; 'tis required ſo to apply a right line, of a given length, between them, and ſubtending the given angle, that the part of the one of them, next the ſaid angle, may be to the alternate part of the other, in the given ratio of p to q.

XI. QUESTION 513, *by Mr.* T. Moſs.

Let the ratio of the head and bung diameters be what it will, within certain limits, a ſpheroidal caſk may be ſo formed, that the diagonal line, ſuch as is now graduated upon gauging rods, will exhibit the true content of the caſk: 'Tis propoſed to find, by a general method, what thoſe limits are, and how near the head and bung diameters can approach to the ratio of equality, before the above circumſtance fails.

XII. QUESTION 514, *by Mr.* William Embleton.

Suppoſing a triangular priſm, whereof the dimenſions of the ſides are 8, 10, and 12 inches, to be laid in ſtill water; I would know in what poſition, or poſitions, it will reſt, and what part of its ſurface will be immerſed, ſuppoſing its ſpecific gravity to be in proportion to that of the water, as 2 to 3.

XIII. QUESTION 515, *by Mr.* Abr. Botham.

Suppoſe a perfectly elaſtic ball, of 2 inches diameter, to be let fall upon the ſurface of an hemiſphere whoſe diameter is 30 inches, from the height of 5 feet above the plane of the horizon or baſe of the hemiſphere; to find at what diſtance from the axis the ball muſt deſcend, ſo that, after being reflected at the ſurface, it ſhall impinge upon the horizon at the greateſt diſtance poſſible from the center of the hemiſphere.

XIV. QUESTION 516, *by Miſs* Ann Nicholls.

The latitude pray ſhew; the hour o'th' night * too tell,
When the three ſtars below † are in one parallel. ‡

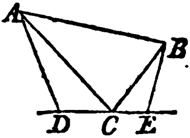
* On Jan. 1, 1763. † Sirius, Cor Leonis, and Aliah. ‡ Of Altitude.

XV. QUESTION 517, *by Mr.* William Spencer.

Given $4\dot{x}^2 + 8y^2\dot{x}^2 + x^2\dot{y}^2 + 4y^4\dot{x}^2 - x^3\dot{y}^2 = 0$, to find the value of x when $y = 2$, ſuppoſing that $x = 2$, when y vaniſhes or becomes $= 0$.

The

The PRIZE QUESTION, *by Mr.* T. Moſs.



Through the angular point *C*, of a given plane triangle *ABC*, to draw, geometrically, a right line ſo, that two others *AD*, *BE*, being drawn, from the given points *A*, *B* to meet the ſame, and make given angles with the right line *AB*, whoſe ſum is leſs than two right angles, their rectangle ($AD \times BE$) may be of a given magnitude.

1764.

Queſtions anſwered.

I. QUESTION 503 *anſwered by* Ruſticus.

TWENTY your charmer's age appears to be—
Height ſixty-five*, as near as I can ſee: * *Inches.*
Her fortune too will eaſily be found,
Nearly a hundred, thirty, and one pound.

The ſame anſwered by Mr. Edw. Griffiths, *Writing-maſter, in* Elleſmere, Shropſhire.

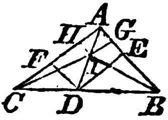
Put $281 \cdot 5294 = a$; $170988 \cdot 22 = b$: and then, by tranſpoſition, involution, &c. will be found $aa - 2ax + xx = 4yy + zz + 4yz = xx + 4yz$ (per equality, &c.): Whence $aa - 2ax = 4yz$, and conſequently $aax - 2axx = 4xyz = 4b$ (per queſt.); from the reſolution of which equation, x comes out $= 131 \cdot 5294$ l. and from thence her age is readily found = 20 years, and her height = 65 inches.——In the ſame manner the anſwer is given by Mr. *J. Hampſon*, who thinks it a very great pity that ſo tall a beauty ſhould have no greater fortune.

This queſtion is likewiſe truly and conciſely anſwered by Meſſ. *Tho. Baker*, *Tho. Barker*, *Geo. Bland*, *T. Bromhall* (both of Mr. Allen's ſchool), *J. Eadon*, *Rob. Forſter*, *Rich. Gibbons*, *Steph. Hodges*, *J. Potter*, *T. Sadler*, (the propoſer), *Tho.*

Tho. Sanderſon, *Wm. Sewell*, *Paul Sharp*, *Wm. Spicer*, *J. Stokes*, *Eleanor Suggett*, *J. Swan*, *Iſaac Tarratt*, *Francis Taylor*, *J. Walker*, *Tho. Walker*. *J. Wood*, *Ja. Young*, and others,

II. QUESTION 504 *anſwered by Mr.* T. Bromhall, *at Mr.* Allen's, *at* Spalding.

Let A repreſent the port; B, C, the two iſlands; AD the ſhip's courſe; and I, D, her places at the ſecond and third obſervations: and let the points A, C; A, B; B, C; C, I, and B, I, be joined, and draw DF and $IH \parallel BA$, and DE and $IG \parallel CA$ reſpectively; and then, by trigonometry will be readily found $DF\ (2IH) = 5{\cdot}55 = b$, and $DE\ (= 2IG) = 8{\cdot}316 = c$.



Put AD ($=$ 10 miles) $= a$, and $CF = x$; and then, per ſimilar triangles, $x : b :: c : \frac{bc}{x} = EB$: Whence $CH = x + \frac{1}{2}c$, $BG = \frac{bc}{x} + \frac{1}{2}b$, and conſequently (47 Euc. 1) $xx + cx + \frac{1}{4}cc + \frac{1}{4}bb = \frac{bbcc}{xx} + \frac{bbc}{x} + \frac{1}{4}bb + \frac{1}{4}cc$ ($= CI^2 = BI^2$, per queſt.) i. e. $xx + cx = \frac{bbcc}{xx} + \frac{bbc}{x}$: Whence, $\overline{x^3 - bbc} \times \overline{c + x}$ ($= x^4 + cx^3 - bbcx - bbcc$) $= 0$, and conſequently $x^3 - bbc = 0$, or $x^3 = bbc$, and $x = \sqrt[3]{bbe} = 6{\cdot}35488$; and hence will be found $AC = 14{\cdot}6695$, $AB = 12{\cdot}8247$, $IC\ (= IB) = 10{\cdot}8730$, $DC = 8{\cdot}4409$, and $DB = 11{\cdot}0441$ miles reſpectively.

In this manner nearly, the anſwer is given by Meſſrs. *Blennerhaſſett* (the propoſer), *Tho. Bramley*, *John Croſs*, *J. Eadon*, *Rich. Gibbons*, *J. Hampſon*, *Steph. Hodges*, *Steph. Ogle*, *Tho. Sanderſon*, *P. Sharp*, *Wm. Spicer*, *Eleanor Suggett*, *Tho. Walker*, *Ja. Young*, and others.

Mr. *Wm. Embleton* conſtructs and ſolves it by means of the ſemi-cubical parabola only, to which it evidently appertains; and *Sangrado*, jun. conſtructs and ſolves it generally, viz. when AI and ID are in any given ratio whatever, by means of the interſection of a circle with a hyperbola deſcribed, to the aſymptotes AC, AB, through the given point D, &c.

From

om the information received from ſeveral correſpon- s, it appears that this queſtion, after the copy of the year's Ladies' Diary was delivered, has been propoſed anſwered in ſeveral of the magazines; an affair the more rizing, as we cannot prevail on ourſelves to think that ngenious propoſer would act ſo diſhonourably, as to any thing he had devoted to the ſervice of the Ladies' y to another place.

I. QUESTION 505 *anſwered by Mr.* Wm. Spicer.

t $a = 600$ (= $10° \times 60 =$ ½ the given difference of lon- le in geographical miles), $b =$ nat. ſine of 10°, $2c =$ 5 (the given exceſs of the diſtance on the parallel above on the arc of a great circle), and $x =$ the coſine of the ude required (rad. = 1); and then ax will = ½ the diſ- e of the two places on the parallel, and $bx =$ the ſine their diſtance on the arc of a great circle: Whence, ing $v =$ the degrees in the ſaid arc of a great circle, ſhall have $60v = ax - c$, per queſt. and conſequently $\frac{60v + c}{a} =$ ſine of 32° 43′ 25″; and hence the latitude ears to be about 57° 16′ 35″, and the neareſt diſtance of two places 646·3635 geographical miles.

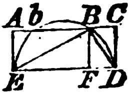
Ir. *J. Hampſon* and Mr. *C. Hutton* put $r = 3437{\cdot}64$ umber of geographical miles in the earth's radius); $y =$ of ½ the required diſtance of the two places on the arc great circle (rad. 1); $\frac{a}{b} - r = d$; all the reſt as above; then, by a very eaſy and ſhort proceſs, find $dy - \frac{ry^3}{2\cdot3}$. $\frac{3\cdot3ry^5}{2\cdot3\cdot4\cdot5} - \frac{3\cdot3\cdot5\cdot5ry^7}{2\cdot3\cdot4\cdot5\cdot6\cdot7}$ — &c. $= c$; from which, by re- ting the ſeries, y comes out = ·09255; and from thence y find every thing elſe nearly the ſame as above.

Mr. *Rich. Gibbons*, Mr. *Paul Sharp*, Mr. *Joſ. Webſter*, ı. and ſome others, have alſo anſwered this queſtion.

IV. QUES-

IV. QUESTION 506 *anſwered by Mr.* Rich. Gibbons.

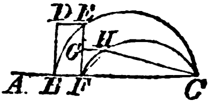
Upon ED, the given ſum of the baſe and perpendicular, conſtitute the rectangle $EC =$ the ſum of the three given areas, and alſo about the ſame, as a diameter, conceive a ſemicircle to be deſcribed, cutting AC in B; join the points E, B, and D, B; and EAB, BCD, and DBE, will be the three triangles required; which is too evident to need a demonſtration.



In this manner exactly it is conſtructed by Mr. *Rob. Butler*, Mr. *Wm. Embleton*, Mr. *Tho. Sanderſon*, *Sangrado*, jun. and others; and Meſſ. *T. Barker*, *Geo. Bland*, *T. Bromhall*, *J. Eadon*, *J. Hampſon*, *B. Robinſon*, *Paul Sharp*, *W. Spicer*, *Eleanor Suggett*, *Tho. Walker*, *W. C. Mathematicus*, &c. have ſent algebraic ſolutions to it.

V. QUESTION 507 *anſwered by Mr.* C. Hutton.

CONSTRUCTION. Produce AB, the ſide of the given inſcribed ſquare, to C, ſo that $AB \times BC$ may be $=$ twice the given area of the triangle, and take BD, $\perp AC$ at B, a mean proportional between AB and BC, and on BC conceive a ſemicircle BEC to be deſcribed, and draw $DE \parallel AC$ meeting the ſame in E, from whence let EF be drawn $\parallel DB$ meeting AC in F: Then, if on FC a ſegment FHC of a circle, to contain the given vertical angle, be deſcribed, and FG be taken $= FB$, and GH drawn $\parallel FC$, meeting the arc FHC in H, and the points H, F, and H, C, be joined, FHC will be the triangle required.



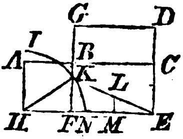
DEMONSTRATION. The $\angle FHC =$ the given angle, per conſtruct. and twice the area of the $\triangle FHC = FC \times FG$ $=$ (by conſtruct.) $FC \times FB =$ (per prop. of the cir.) FE^2 $= BD^2 = AB \times BC =$ twice the given area, by conſtruct. And (by prob. 3d part 2d Simp. Select Exerciſes) $\frac{AB \times BC}{FC + BF}$ $= \frac{AB \times BC}{BC} = AB$, the ſide of the given inſcribed ſquare, by conſtruction. *Q.E.D.*

NOTE, When GH neither cuts nor touches the ſegment HC, this queſt. is, manifeſtly, impoſſible.

Mr. *Wm. Embleton*, Mr. *Steph. Ogle*, and *Sangrado*, jun. conſtruct it nearly in the ſame manner.——Meſſ. *T. Barker*, *T. Bromhall*, *Geo. Bland*, *J. Hampſon*, *Paul Sharp*, &c. have however pointed out ingenious methods whereby it may be ſolved algebraically.

VI. QUESTION 508 *anſwered by Mr.* Wm. Embleton.

CONSTRUCTION. Upon $AC = s$, the given ſum of the two right lines, conſtitute the rectangle $AE = cc$, and take $AB = AH$, and then AB, BC will be the meaſures of A and B reſpectively, and in the given ratio of m to n. For, drawing $BF \parallel AH$, and producing it and EC till they become each $= FE$ (BC), and joining their extremes G, D, then, ſuppoſing $AB : BC :: m : n$, it will be $m : n :: AB \times BC (= BF \times BC = BE) : BC^2$: Whence $BE = \frac{m}{n} \times BC^2$; to which equal magnitudes, the equal magnitudes AF, AB^2, being reſpectively added, it will become $AB^2 + \frac{m}{n} \times BC^2 = AF + BE = AE = cc$, per conſtruction, &c.



NOTE, This queſtion will be found uſeful in the ſolution of ſome difficult problems relating to triangles.

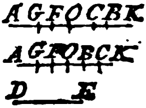
The ſame anſwered by Mr. Da. Kinnebrook.

CONSTRUCTION. Take $HE = s$, the given ſum of the two right lines, and from the center H, with radius c, ſuppoſe the circular arc IN, cutting HE in N, to be deſcribed, and draw LM perpendicular any where upon NE, and ſo as that LM^2 be to $ME^2 :: m : n$, and let the points E, L, be joined by a right line produced, till it meets the ſaid arc in K; from whence draw $KF \perp HE$, and HF, FE will be the meaſures of A and B required.——For, joining the points H, K, $n : m :: FE^2 : FK^2 = \frac{m}{n} \times FE^2$ (by conſtruction and ſim. triangles, EML, EFK), and (by 47 E. 1) $HF^2 + \frac{m}{n} \times FE^2 (= FK^2) = cc (= HK^2$, per conſtruction).

Mr. *Rob. Butler* and Mr. *Tho. Sanderſon* alſo conſtruct it exactly according to this method.

A more general Anſwer to the ſame, by P. M. *of* Durham.

Suppoſe in a right line the two points A, B were given, and it was required to determine another point C therein, ſuch that $m \times AC^2 + n \times BC^2$ might be $= p \times DE^2$ (DE being another given right line.



Biſect AB in O, and divide it in F in the given ratio of m to n; take OK, ſo that $m + n$ may be to p in the duplicate ratio of DE to OK, and, making FG (towards oppoſite parts of F) $= FO$, take GC, ſo that the rectangle under it and CO may be equal to the given rectangle under AK and KB (Simp. Geom. Pr. 6), and C ſhall be the point required.

DEMONSTRATION. For, ſince AB is biſected in O, $\square\, AKB + AO^2 = OK^2$ (6 Euc. 2) and $\square\, GCO = \square\, GOC + OC^2$ (3 Euc. 2) $= 2 \square\, FOC + OC^2$, becauſe $GO = 2 FO$ (hyp.). But $\square\, GCO = AKB$ (hyp.); therefore $2 \square\, FOC + OC^2 + AO^2 = OK^2$, and, multiplying both ſides by $\overline{m + n}$, $m + n \times 2 \square\, FOC + \overline{m + n} \times \overline{OC^2 + AO^2} = \overline{m + n} \times OK^2 = p \times DE^2$ (hyp.). Again, becauſe AB is divided in F in the ratio of m to n, and biſected in O, therefore $m + n : m - n :: (AO : FO ::)$ $\square\, AOC : \square\, FOC$; and conſequently $\overline{m + n} \times \square\, FOC = \overline{m - n} \times \square\, AOC$: Whence $(\overline{m - n} \times 2 \square\, AOC + \overline{m + n} \times \overline{AO^2 + OC^2} = m \times \overline{AO + OC}^2 + n \times \overline{AO - OC}^2 =)$ $m \times AC^2 + n \times BC^2 = p \times DE^2$.

COROLLARY 1. As the point C falls within or without the terms (or points) A, B, the right line AB will be the ſum or difference of the right lines ſought.

COROLLARY 2. The numerical ſolution is had from this equation $\overline{m - n} \times 2 \square\, AOC + \overline{m + n} \times \overline{AO^2 + OC^2} = p \times DE^2$, where p is univerſal, but in the queſtion is limited to the value n.

Neat geometrical conſtructions to this queſtion have alſo been received from Mr. *T. Barker* (the propoſer), *Sangrado*, jun. Mr. *W. Spicer*, and others.

VII. QUESTION 509 *anſwered by Mr.* W. Spencer.

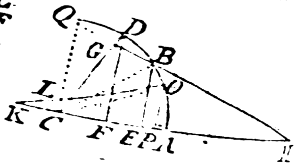
The throwing of m heads, preciſely, in n throws with p halfpence, being evidently the ſame as throwing m heads preciſely in $n \times p$ throws with one half-penny only, it follows, from corol. to prob. 5 on page 12, &c. of Simp. Nat. and Laws of Chance, that, if the 5th, 7th, 9th, 11th, and 13th terms of the binomial $1+1$ raiſed to the 25th power, be each divided by $\overline{1+1}|^{25}$, they will be the reſpective probabilities required, *i. e.* putting $\frac{25}{1} \times \frac{24}{2} \times \frac{23}{3} \times \frac{22}{4} \times \frac{1}{\overline{1+1}|^{25}} = p$; then p, $p \times \frac{21}{5} \times \frac{20}{6}$, $p \times \frac{21}{5} \times \frac{20}{6} \times \frac{19}{7} \times \frac{18}{8}$, $p \times \frac{21}{5} \times \frac{20}{6} \times \frac{19}{7} \times \frac{18}{8} \times \frac{17}{9} \times \frac{16}{10}$, and $p \times \frac{21}{5} \times \frac{20}{6} \times \frac{19}{7} \times \frac{18}{8} \times \frac{17}{9} \times \frac{16}{10} \times \frac{15}{11} \times \frac{14}{12}$ will be the reſpective probabilities of throwing the number of heads as ſpecified in the queſtion, conſidering them as independent one of another: and therefore the probability of their all happening together, according to ſome order or other, will, by corol. to prob. 1ſt of Simp. Laws of Chance aforeſaid, be expreſſed by $p^5 \times \frac{21^4}{5^4} \times \frac{20^4}{6^4} \times \frac{19^3}{7^3} \times \frac{18^3}{8^3} \times \frac{17^2}{9^2} \times \frac{16^2}{10^2} \times \frac{15}{11} \times \frac{14}{12}$; but there being $1 \times 2 \times 3 \times 4 \times 5$ ($= 120$) permutations, or different orders or ways in which 5 ſuch events may all happen together, the probability of their happening in the preciſe order required in the queſtion will therefore be $\frac{p^5 \times 21^4 . 20^4 . 19^3 . 18^3 . 17^2 . 16^2 \;\, 15 . 14}{120 \times 5^4 . 6^4 . 7^3 . 8^3 . 9^2 . 10^2 . 11 . 12}$: From whence the values of *A* and *B*'s expectations may readily be found, &c.

Much after the ſame manner the anſwer is given by Mr. *Wm. Embleton*, Mr. *C. Hutton*, Mr. *Moſs* (the propoſer). *Sangrado*, jun. and ſome others.

VIII. QUESTION 510 *anſwered by* P. M. *of* Durham.

It may be eaſily ſhewn, in the projection of any regular figure, whether right-lined or a conic ſection, that the area of the plane of the figure is to the area of its projection, as the height of the plane above the horizon multiplied by radius, is to the ſhadow of the ſaid height ma[illegible] ſine of the angle, which the ſaid ſhadow makes [illegible] meridian; but this angle is the ſun's azimuth: and [illegible] of the plane, its projection, and alſo its height, are [illegible] And therefore, calling its height $= a$, its ſhadow $=$ [illegible] given ratio of the plane's ſurface to its projection as [illegible] and, likewiſe, denoting the coſines of the given latitude [illegible] declination by c and q, the ſine and verſed ſine of the [illegible] angle ſought by x and v, and the coſine of the difference between the lat. and declin. by d, we ſhall have $m : n$ [illegible] (r being rad.) : $y \times$ s. azim. but s. alt. : coſ. alt. :: [illegible] and coſ. alt. : $x :: q$: s. azim. Whence s. alt. : $x ::$ [illegible] $\times$ s. azim. and conſequently $rr \times$ s. alt. : $qrx ::$ (a[illegible] s. azim. ::) $m : n$; but (by ſpherics) $rr \times$ s. alt. $= rrd - cqv$, and therefore $rrd - cqv : qrx :: m : n$, i.e. $\frac{rrd}{cq} - v : x :: \frac{mr}{c} : n$; whence this eaſy

GEOM. CONSTRUC. Let AB, AD, repreſent the complements of the latitude and declination to the rad. AC $(= r)$, and then $BE = c$, DF $(= q)$, and $CG = d$; produce GB till it cuts the rad. CA produced in H, and take AK a fourth proportional to EH, CH, and AC (12 Euc. 6).



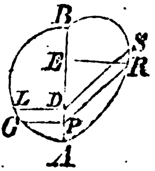
Aſſume KC, CL in the given ratio of $\frac{mr}{c} : n$, and draw $CL \perp KC$ at the point C, and through the points K, L draw a right line, cutting the circle in O, and AO will be the meaſure of the hour angle ſought.

For, producing HG, CL, to meet in Q, and drawing OP the ſine of the arc AO; then, by ſimilar triangles, ($DF : DC :: CG : CQ$, and $DF \times BE : \square DCG ::$ ($BE : CQ :: EH : CH ::$) $AC : AK = \frac{rrd}{cq}$; but KP $(= AK - AP) : OP ::$ ($KC : CA ::$) $\frac{mr}{c} : n$, i.e. because $AK = \frac{rrd}{cq}$, and AP is the verſed ſine of the arc, of which OP is the ſine; therefore $OP = x$, and $AP = v$.

The

The ſame anſwered otherwiſe by Mr. Robert Butler, *(the Propoſer).*

Suppoſe $ARSB$ repreſents the required ſhadow; s = area of the given ſemicircle AGB; a and b = ſine and coſine of the lat. x and y = thoſe of the ſun's alt. z and u = thoſe of his azim. from the north; d = ſine of his decl. (rad. = 1) and $A = 43560$, the feet in an acre: Then, GP being perpendicular to the horizontal plane $ARBA$, $\frac{y}{x} \times PG = PR$ will be the length of its ſhadow; and, drawing DL, DS, indefinitely near, and $\parallel PG$, PR, and meeting the curves in L and S, reſpectively, and $RE \perp AB$, then will $PG \times PD$, and $PD \times ER$, be reſpectively = the areas of the parallelograms PL and PS: but z being the ſine of EPR, RE is $= z \times PR = \frac{zy}{x} \times PG$, and therefore $PD \times ER = \frac{zy}{x} \times PG \times PD$, i. e. the area of the parallelogram PL drawn into $\frac{zy}{x}$ = area of the parallelogram PS, and conſequently $s \times \frac{zy}{x} = A$, let the curve ARB be what it will; but, by ſperics, &c. z is found $= \frac{\sqrt{bbyy - \overline{d - ax}^2}}{by}$, which ſubſtituted above, gives $\frac{s}{bx} \times \sqrt{bbyy - \overline{d - ax}^2} = A$: From whence x is found = ·0111641, and conſequently the ſun's azim. from the weſt = 32° 19′, and the time required = 9′ 16″ paſt 8 o'clock.—— No regard is here had to the ſun's ſemidiameter and refraction; but the anſwer in ſuch caſe will be had, by making d = ſine of 19′ more than what it here is.



IX. QUESTION 511 *anſwered.*

Φιλομαθεματίκος, and ſome others, (anſwering this queſtion) make the time ſought to be about 12′ paſt 11 in the morning, and find the illuminated part = $\frac{1}{9}$ the whole internal ſuperficies nearly; but *Philotechnus* finds the illuminated internal ſurface to be always = a rectangle under the ſine and ſecant of the angle comprehended between the ſolar ray, iſſuing from the center of the ſun, and the diameter of the given cylinder, paſſing through that point of the circumference of its end, whereon the ſaid ray impinges, to radius = the diameter of the ſaid cylinder; from which (as it depends wholly on the ſun's altitude and azimuth conjunctly) being fluxed, put = 0, and reduced, the time required, &c. may be determined.*

X. QUES-

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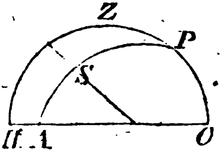
* IX. QUESTION 511.

As the anſwer to this queſtion is partly falſe and partly incomplete, I ſhall ſupply the whole ſolution as below.

I. *To find the Time of the greateſt Illumination.*



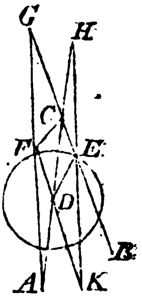
CONSTRUCTION. On the plane of the meridian of the place deſcribe the primative circle, and therein take *Z* the zenith and *P* the pole; draw the horizon *HO*, on which take *HA* = 20° the given horizontal angle made by the meridian and the axe of the cylinder; and through *A* draw the meridian *ASP*. So ſhall *APZ* be the hour angle from noon required when the moſt ſurface is illuminated.

DEMONSTR. It is evident that the curve bounding the illuminated part of the internal ſurface, is always the ſhadow of that half or ſemicircumference of the end of the cylinder which is turned towards the ſun, that is, that ſemicircumference whoſe diam. is perpendicular to the plane drawn through the ſun and the axe of the cylinder. It is alſo evident that the moſt ſurface will be illuminated when the ſolar ray penetrates to the fartheſt diſtance poſſible within the cylinder; and this, it is farther evident, will happen when the ray makes the leaſt poſſible angle with the axe of the cylinder; and this angle being always meaſured by the arc of a great circle paſſing through the ſun and that point *A* of the horizon to which the axe of the cylinder is directed, this arc will therefore be a min. that is, the arc intercepted by *A* and the parallel of declination for the given day, is a minimum; in which caſe it is evident that the arc muſt be perpendicular to the parallel of declination, and will therefore be a meridian paſſing through *A*, as by the Conſtruction.

CALCULATION. In the right-angled triangle *AHP*, it is as the s. of *HP* (130°) ; radius : : tang. *HA* (20°) : tang. hour $\angle P$ =

X. QUESTION 512 *anſwered by Mr.* T. Moſs.

CONSTRUCTION. Suppoſe $DH =$ one of the given lines comprehending the given angle, and from the center D with radius DE (= the given line to be applied) conceive the circumference of a circle to be deſcribed, and make the $\angle HDK =$ the ſupplement of the given angle to 180°, taking $DK : DH :: p : q$, viz. the given ratio, and joining the points H, K, by a right line cutting the circumference of the ſaid circle in E: From whence drawing $EC \parallel DK$, meeting DH in C, and producing DH till $DA = CH$, and CE, till CB becomes = the other given comprehending line, and then, joining the points D, E, the thing is done.—For, by ſim. triangles, $DK : DH :: CE : CH$; and, per conſtruction, $DK : DH :: p : q$; whence, per equality, $CE : DA$ (= CH, per conſtruction) $:: p : q$; and the $\angle BCA = \angle ADK$ (the given angle), and the right lines CA, CB, and DE are = the given lines, by conſtruction.



A

$= \frac{\text{tan. } HA}{\text{ſin. } HP} = \frac{\text{t. } HA}{\text{s. lat.}} =$ tang. of 25° 21′ 47″, which anſwers to 1h. 41m. 39s. and which being taken from 12 hours, there remain 10h. 18m. 21s. or nearly 18 minutes paſt 10 in the morning is the time of the greateſt illumination. And as radius : coſ. HA :: coſ. HP : coſ. PA = 127° 50′ 30″; from which take PS (= 72° 23′) the ſun's polar diſtance, and the remainder 55° 27′ 30″ is = AS the meaſure of the leaſt angle formed by the ſolar ray and the axe of the cylinder.

SCHOLIUM. Hence we may obſerve that the time is the ſame whatever the declination is; and that the time given in the original anſwer is wrong.—It may alſo be remarked, that if the reaſoning in the above, or in what follows, ſhould not be clear to any perſon, he may aſſiſt his imagination by rolling up a rectangular piece of paper into the form of a cylinder, and expoſing it to the ſolar rays, or to the light of a candle.

A CONSTRUCTION *to the ſame by Mr.* Da. Kinnebrook, *and* P. M. *of* Durham.

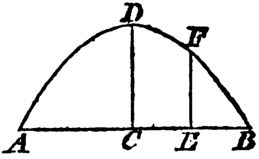
Produce one of the given, comprehending lines BC till CG is to the other of them (AC) in the given ratio of p to q, and let the points A, G, be joined, and about the center C with radius CF (= the given line to be applied) conceive a circular arc to be deſcribed cutting AG in F; from whence draw $FD \parallel CG$ meeting AC in D, and, joining the points F, C, draw $DE \parallel$ thereto, and that will be the poſition of the line required: For, by ſim. triangles, FD (= CE, per conſtruction) : DA : : ($CG : CA$: :) $p : q$, per conſtr. and, DC being a parallelogram (per conſtr.), DE will be = CF = the given line to be applied (per conſtruction).

Meſſ. *Rob. Butler*, *Wm. Embleton*, *Sangrado*, jun. and ſome others, have alſo conſtructed it in an elegant and conciſe manner; and Meſſ. *T. Bromhall*, *J. Hampſon*, *Tho. Sanderſon*, *P. Sharp*, &c. have given curious algebraic ſolutions to it.

XI. QUES-

II. *To find the Quantity of ſurface illuminated.*

Conceive the cylinder to be cut thro' on one ſide in the direction of its axe, and to be ſpread flat out on a plane; then will the illuminated part appear as in the annexed figure, where AB is the ſemicircumference as ſtretched out in a right line; CB is a quadrant; CD, perpendicular to AB, is the projection of the diameter of the end; or it might be conſidered as the ſhadow of the diameter; and, in like manner, every other perpendicular EF is the projection or ſhadow of double the ſine of its correſponding arc BE.



Put now t for the cotang. of the angle formed by CD or by the axe of the cylinder and the ſolar ray, or the cotang. of the arc AS (55° 27′ 30″) in the firſt figure to the radius 1; z = the arc CE, and x = its ſine to the radius r (3) of the cylinder. Then, by plane trigonometry, $2tr = CD$, $\sqrt{rr - xx} = s.BE$ (or coſ. CE), and $2t\sqrt{rr - xx} = EF$. But the fluxion of $CDFE$ is = $\dot{z} \times EF$, and $\dot{z}$ is $= \dfrac{r\dot{x}}{\sqrt{rr - xx}}$; hence $\dot{z} \times EF$ is $= \dfrac{r\dot{x}}{\sqrt{rr - xx}}$

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XI. QUESTION 513 *anſwered by Mr.* T. Moſs, (*the Propoſer*).

Let d = the given diagonal of the ſpheroidal caſk required, x = the bung diameter, the ratio of the head and bung as n to 1, and p = ·7854; and then, by the well-known theorem, its content will be $\frac{2p}{3}\sqrt{dd - \overline{\frac{1+n}{2}}\Big|^{2} \times xx} \times \overline{2xx+nnxx}$: Which, put into fluxions (ſuppoſing n conſtant) and made = 0, and reduced, gives $x = \frac{2d\sqrt{2}}{1+n\sqrt{3}}$ = the bung diameter, in this circumſtance, when the caſk, under the given diagonal d, is the greateſt poſſible; and this value, ſubſtituted for x in the above expreſſion, gives $\frac{16pd^3}{9\sqrt{3}} \times \frac{2+nn}{\overline{1+n}|^{2}}$ = the content of the greateſt ſpheroidal caſk

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$\times 2t\sqrt{rr-xx} = 2tr\dot{x}$; the fluent of which is $2trx$ = the area $CDFE$. And when E arrives at B, then x is $= r$, and the area CDB is $2trr$, the double of which is $ADB = 4trr = tdd$, putting d for the diameter of the cylinder; which, ſince d is = 6, will be $36t$ = 24·78068, the illuminated ſurface required.

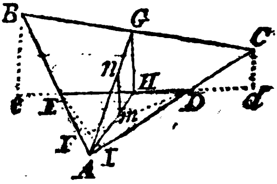
Now, the length of the cylinder being double its diameter, its whole internal ſurface will be $2pdd$, putting p = 3·1416; and therefore the whole is to the illuminated part, as $2p$ to t, that is, as 6·2832 to ·68835, or as 9·12788 to 1.

COROLLARY. From the above it appears that the projection or ſhadow ($2CDFE = 4trx$) of a middle zone of the circular end of the cylinder, *upon its internal ſurface*, is always equal to the ſhadow or projection, *on the plane of the horizon*, of the rectangle under the diameter and diſtance between the two rectilinear ends of the zone; and that the projection of the whole circular end, on the internal ſurface, (that is, the whole illuminated part) is equal to the projection, on the horizontal plane, of a ſquare whoſe ſide is equal to the diameter of the cylinder ——Or, it farther appears, that the projection of the middle zone, is to the rectangle under the ſine and ſecant of the complement of the angle made by the ſolar ray and the axe of the cylinder, to a radius equal to the diameter of the cylinder, as the ſine of the arc CE is to the radius. And, alſo, that the whole illuminated part is equal to a rectangle under the ſine and ſecant of the ſame angle, to the ſame radius equal to the diameter of the cylinder: which laſt is alſo remarked by *Philotechnus* in the original ſolution.

caſk having the given diagonal d, and correſponding to any aſſignable poſſible value of n: And when it is leſs than $d^3 \times \cdot 6283$, the content, as found by the diagonal line, the problem is then manifeſtly impoſſible; ſuppoſe it therefore equal thereto, in order to determine the limit of n (required), *i. e.* ſuppoſe $\frac{16pd^3}{9\sqrt{3}} \times \frac{2+nn}{\overline{1+n}|^2} = d^3 \times \cdot 6283$, or $\frac{2+nn}{\overline{1+n}|^2} = \cdot 7796$; whence n is readily found $= \cdot 898$, &c.——And hence it appears that the preſent conſtructed diagonal line, will not exhibit the true content of a ſpheroidal caſk, when the ratio of the head and bung diameters approaches nearer to an equality than that of ·898 to 1 (or 9 to 10, nearly), in any circumſtance whatever.

XII. QUESTION 514 *anſwered by Mr.* Wm. Embleton.*

Suppoſe ABC repreſents a ſection of the given priſm parallel to its ends, AED the immerſed part thereof, AG, AH, right lines drawn from the angular point A, to the middle points of BC and ED, reſpectively: Then, if the centers of gravity n, m of the triangles ABC, AED (which are known to be $\frac{1}{3}$ of AG and AH from G and H, reſpectively) be joined by the right line nm, the priſm will, by the laws of hydroſtatics (ſee prop. 87 ſect. 9 of Emer. Mechan.) be quieſcent, whenever that line becomes perpendicular to the hori-



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[*] XII. QUESTION 514.

In order to bring this problem to a final ſolution; On DE, produced both ways, demit the perpendiculars Cd, Be, which will therefore be parallel to GH; alſo, on AB and AC, the perpendiculars DF and EI.

Since then GB is $= GC$, and Be, GH, Cd are parallels, He will be $= Hd$; but $HE = HD$, $\because Ee = Dd$. Again, ſince the vertical angles eEB and FED are equal, as alſo dDC and IDE, the two triangles BeE and DFE are ſimilar, as alſo the two CdD and EID; hence $\left\{\begin{matrix} EB : Ee :: ED : EF \\ DC : Dd :: DE : DI \end{matrix}\right\}$, and as the two means are the ſame in both, therefore $EB : DC :: DI : EF$.

Put

zontal line *ED*: But *GH* will then alſo be perpendi-
r to the ſame (by cor. to theor. 12 B. 4 Simp. Geom.
dit.) and ſo the queſtion be reduced to that of dividing
given triangle *ABC* into two given parts by the right
ED, ſo that *GH*, connecting the middle points of *BC*,
ED, may be perpendicular to the latter of them; which
problem purely geometrical; and from whence an an-
in numbers may be readily derived.——This problem
be of uſe in determining the ſtructure of a ſhip, ſo as
e leaſt ſubject to roll, &c.

r. *J. Moreland*'s anſwer is the ſame as the above, very
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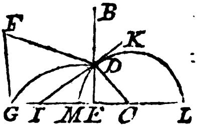
I. QUESTION 515 *anſwered by Mr.* Da. Kinnebrook.



ppoſe the ball to fall from *B*, on the point *D*, of the
n hemiſphere *MDL*, and, after
ction, to deſcribe the curve *DG*,
ing the horizon in the point *G*;
draw *DF*, a tangent to the ſame
: Suppoſe, alſo, *FG*, *DE*, to be
endicular to the horizon, and *KDI*
e a tangent to the hemiſphere at *D*,
put DC ($= 1\frac{1}{4}$ foot) $= r$, $16\frac{1}{12}$ feet $= s$, $DE = x$; and
the direction of the ball *DF*, after reflection, being
as that the $\angle BDK = \angle FDI$ (by the nature of re-
ion), and its velocity therein $= 2\sqrt{4rs - sx}$ (as both
and hemiſphere are perfectly elaſtic), we ſhall, by
laws of falling bodies, trigonometry, &c. find $GC =$
$\frac{\overline{4r - x} \times \overline{2xx - rr}}{r^4} + 1 \times \sqrt{rr - xx}$, a max. (per
queſt.)

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ut, now, $b = AB$, $c = AC$, m and $n =$ ſine and coſ. $\angle A$,
AE, and $y = AD$. Then $nx = AI$, and $ny = AF$; hence
$= x - ny$, and $DI = y - nx$; alſo $BE = b - x$, and CD
$- y$; therefore the laſt proportion becomes $b - x : c - y ::$
$nx : x - ny$, which produces the equation $x^2 - bx + bny$
$^2 - cy + cnx$. But the $\triangle AED = \frac{1}{2} mxy$ is $= a$, a given
nitude, by the queſt. and hydroſtatics; therefore y is $= \frac{2a}{mx}$;
ch being ſubſtituted in the equation, it becomes at laſt $x^4 -$
$\overline{cn} . x^3 + \overline{c + bn} . \frac{2ax}{m^2} - \frac{4a^2}{m} = 0$. From whence the value
may eaſily be found; and thence that of y.

queſt.) from which, thrown into fluxions, and reduced, &c. x comes out $= \cdot 4646$ &c. and hence every thing elſe required may be readily found.

Much after the ſame manner the anſwer is given by Mr. *C. Hutton*, Mr. *J. Moreland*, and ſome others.

XIV. QUESTION 516 *anſwered by Mr.* T. Allen, *of* Spalding.

Put x and y for the ſine and coſine of the co-latitude required, v and z for thoſe of the arc of time that Sirius is then ſhort of the meridian. Alſo, let a and b, c and d, e and f, be the reſpective ſines and coſines of the given polar diſtances of the three given ſtars, Sirius, Cor Leonis, and Aliah; and m and n, p and $q =$ the ſines and coſines of the difference in right aſcenſion betwixt Sirius and the other two of the ſaid ſtars, reſpectively, rad. $= 1$; and then, by ſpherics, the coſines of their zenith diſtances will be found equal to $axz - by$, $cx \times \overline{zn - vm} + dy$, and $fy - eqxz + epxv$, reſpectively; which are all equal, by the queſt. From the firſt and ſecond, y is found $= x \times \frac{az + cmv - cnz}{b + d}$, which, ſubſtituted in the third, and ordered, gives $\frac{z}{v} =$

$$\frac{cm . \overline{b - f} + ep . \overline{b + d}}{a . \overline{d - f} + cn . \overline{b + f} + eq . \overline{b + d}}$$

$=$ tang. of the arc Sirius is ſhort of the meridian at the time required; from whence the hour of the night, and latitude of the place, will be eaſily determined.

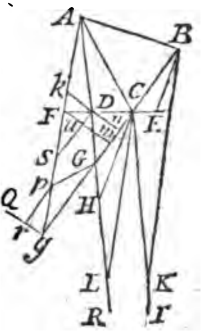
XV. QUESTION 517 *anſwered by Mr.* T. Allen, *and* P. M. *of* Durham.

From the given equation is readily derived $\frac{\dot{x}}{x\sqrt{x - 1}} = \frac{1}{2} \times \frac{\dot{y}}{1 + yy}$; whence, putting $\left\{ \begin{matrix} a \\ b \end{matrix} \right\} =$ circular arc, whoſe radius is unity and $\left\{ \begin{matrix} \text{ſec.} \sqrt{x} \\ \text{tang.}\, y \end{matrix} \right\}$, and $c =$ the arc of $45°$, whoſe ſec. $= \sqrt{2}$, the equation exhibiting the relation of the fluents will be found $2a = \frac{1}{2} b$: Which, corrected, according to the conditions ſpecified in the problem, becomes $2a = \frac{1}{2} b + 2c$; and hence, by a table of natural or logarithmic ſines, &c. the value of a will readily be found $= 1 \cdot 06218$, when $y = 2$; and thence $x = 4 \cdot 21683$, &c. which was required.

In this manner, nearly, the anſwer is alſo given by Meſſ. *R. Butler*, *Wm. Embleton*, *C. Hutton*, *Da. Kinnebrook*, *Wm. Spencer* (the propoſer), and ſome others.

The PRIZE QUESTION *anſwered by Mr.* T. Moſs, (*the Propoſer*).

Imagine DCE to be the poſition of the line required, and produce BC (ſee fig. to queſt.) to meet $Ag \parallel BE$ in g, and then, $AD \times BE$ being a given quantity (per queſt.), it is evident, from ſim. triangles BCE, gCF, that $AD \times Fg$ is a known quantity alſo, it being to the given $\square\ AD \times BE :: Fg : BE$. Moreover, Dk being $\perp Ag$, it is very evident that $Fg \times AD : Fg \times Dk$ in the known ratio of $AD : Dk$; whence it follows that $Fg \times Dk$, or twice the meaſure of the $\triangle\ DFg$, becomes known. Now, ſuppoſe Fm and $Dn \perp Cg$, and produce AD to meet BC produced in G, and then it is manifeſt that $Cg \times \overline{Fm - Dn} =$ twice the meaſure of the $\triangle\ DFg\ (= Fg \times Dk)$; but Cg is known, and conſequently $Fm - Dn$ is known alſo: Hence the problem is reduced to that of drawing a right line from the given point C to cut the right lines Ag, AG, given in poſition, ſo that the difference of the perpendiculars Fm, Dn, falling from the points of interſection F, D, upon Bg, may be $=$ a given right line, *i. e.* equal to twice the given meaſure of the $\triangle\ DFG$ divided by the given line Cg; of which the following is the



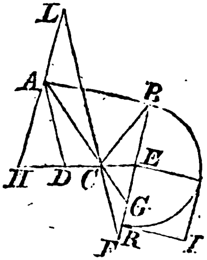
CONSTRUC. In $gQ \perp gB$, take $gr =$ the given difference of the perpendiculars Fm, Dn, and draw $rp \parallel gB$ meeting Ag in p; join p, G, and draw (by prob. 37, p. 242 of Simp. Geom. 2d edit.), from the given point C, a right line cutting Ag, AG, ſo that the ſegments Fp, DG, cut off thereby, may be to each other in the given ratio of Ag to AG, and the thing is done.

DEMONSTRA. Draw $Ds \parallel Cg$ cutting Fm in u; then (by conſt.) $Ag : AG :: Fp : DG$, and, by ſim. triangles, and diviſion of ratios, $Ag : AG :: sg : DG$; whence, by equality, $Fp = sg$, and, taking sp from each, $gp = Fs$, and conſequently $Fu = gr =$ the given difference, by conſtruc. ——This method contains the ſolution of another problem of equal, if not ſuperior, difficulty.

Mr. *Da. Kinnebrook* draws CH and $CI \parallel BE$ and AD reſpectively, and produces AD to meet CH in H, and takes CK in CI, ſo that $CH \times CK$ may be $=$ the given $\square\, AD \times BE$, and, joining B, K, draws $CL \parallel$ thereto, meeting AH produced in L, and then, continuing BC till it meets AL in G, divides AL in D, ſo that $AD \times DL$ may be $= AH \times GL$; from which point D, through C, drawing the right line DCE, and the thing is done.——The demonſtration whereof is extremely natural and eaſy.

The ſame anſwered by Mr. Wm. Embleton, *and Mr.* C. Hutton.

CONSTRUC. Through the given point C, draw $CL \parallel DA$, meeting BE produced in F; and $AL \parallel BE$, in L; produce AC to meet BF in G, and take GR ſo, as that $CL \times GR$ may be $=$ the given $\square\, AD \times BE$, and upon BR deſcribe the ſemicircle BKR: erect $RI \perp BR$, and $=$ to a mean proportional between BF and GR; draw $IK \parallel BR$, meeting the circumference in K, and $KE \parallel RI$ meeting BF in E, the point through which the required line ECD muſt paſs.



DEMONSTRA. Produce LA to meet ECD, produced, in H; and then, RE (by prop. of the circle) being $= \frac{EK^2}{BE} = \frac{RI^2}{BE} = \frac{BF \times RG}{BE}$ (by conſtruc.), and $GE = RE - RG = \frac{BF \times RG}{BE} - RG = \frac{BF - BE}{BE} \times RG = \frac{FE \times RG}{BE}$ (per the fig.), we ſhall (by theor. 20 B. 4 of Simp. Geom. 2d edit.) have $CL : DA$ $(:: HL : HA) :: FE : (GE =) \frac{FE \times RG}{BE}$ (the triangles HCL, ECF, being ſimilar, and the right line ACG making equal angles with the homologous ſides thereof, &c. whence, multiplying extremes and means, $AD \times BE = CL \times GR$ $=$ the given rectangle, by conſtruction. *Q. E. D.*

NOTE, When IK neither cuts nor touches the ſemicircle BKR, this problem is impoſſible.

Meſſ. *Rob. Butler*, *J. Barnard*, *J. Moreland*, *Plus Minus*, *J. Walker*, *T. Walker*, and ſome others, have likewiſe given curious and elegant conſtructions to this queſtion; but the prizes of 12 and 8 Diaries, for the ſolution thereof, have fallen to the lots of Mr. *Rob. Butler*, and Mr. *Wm. Embleton*, reſpectively.

The Eclipſes calculated for 1764.

There will be four eclipſes this year, two viſible and two inviſible, which will happen in the following order: App. t.

1. A viſible lunar eclipſe, March 17th, at night, of which our ingenious correſpondents have obliged us with the following calculations.

Calculated by	Beg.		Mid.		End		Dur.		Dig.	
	h.	m.	h.	m	h.	m.	h.	m.	h.	m.
Mr. T. Hopkinſon, for London	10	48	12	10	13	34	2	46	8	28
Mr. Chapman, for Foxton, Leiceſt	10	39	12	1	13	22	2	43	8	34
Mr. J. Metcalf, for London	10	50	12	13	13	36	2	46	8	31
Wentworth	10	44	12	7	13	30	2	46	8	31
Mr. T. Harris, for London	10	41	12	6	13	29	2	8	8	32
Brington	10	37	12	2	13	25	2	48	8	32
Mr. T. Bramley, for London	10	21	11	42	13	4	2	43	8	9
Mr. J. Webſter, for Loughbor. by Dr. Halley's tables	10	39	12	1	13	22	2	43	8	14
Mr. Hampſon, for Leigh, Lancaſh.	10	33	11	56	13	19	2	46	8	17

2. A great annular eclipſe of the ſun, April 1ſt, in the morning; it will be viſible in moſt parts of Europe, and nearly annular at London.

Calculated by	Begin.		Mid.		End		Dur.		Dig.
	h.	m.	h.	m.	h.	m	h.	m.	
Mr. T. Hopkinſon, for London	9	19	10	46	12	18	2	59	annul.
Mr. Chapman, for Foxton, Leiceſt.	9	15	10	42	12	15	3	0	annul.
Mr. J. Metcalf, for London	9	15	10	43	12	14	2	52	annul.
Wentworth	9	12	10	39	12	9	2	57	annul.
Mr. J. Harris, for London	9	15	10	45	12	20	3	5	annul.
Brington	9	9	10	39	12	14	3	5	annul.
Rome	10	8	11	45	1	3	2	55	7 30
Edinburgh	9	7	10	37	12	12	3	5	11 0
Paris	9	21	10	48	12	23	3	2	11 6
Mr. T. Bramley, for London	9	1	10	27	11	49	2	48	9 51
Mr. J. Webſter, for Loughboro'	9	18	10	23	11	18	2	0	11 9
Mr. Greenſted, for Waterinbury	8	40	10	4	11	30	2	50	10 3
Mr. Hampſon, for Leigh, Lancaſh.	9	10	10	36	12	4	2	54	10 40
Mr. E. Kimpton, for London	9	15	10	44	12	17	3	2	annul.
Mr. Smithurſt, for London	9	2	10	27	11	58	2	56	11 5
Mancheſter	8	53	10	18	11	49	2	56	10 50
Mr. C. Green, for Greenwich	9	2	10	30	12	2	3	0	1 0
Mr. G. Witchell, for London	9	3	10	28	11	59	2	56	10 59

Mr. *Smithurſt* obſerves that the central ſhade of this eclipſe will enter Europe near Cape St. Vincent, and travel through Portugal, Spain, France, Holland, Denmark, and Sweden, till it quits the continent at the North Cape.

The 3d eclipſe will be of the moon, September 10th, about 7 in the morning, and conſequently inviſible to us.

The laſt is a ſolar eclipſe, September 25th, about 5 in the afternoon, but inviſible to theſe northern parts of the world.

Beſides the above, we have alſo been favoured with calculations of the eclipſes from Meſſ. *Stephen Hodges*, *T. Quanbrough*, *Thomas Sanderſon*, *John Stokes*, and others.

New Queſtions.

I. QUESTION 518, *by Mr.* Tho. Sadler.

Old Simon's dead, and Margery is found
A buckſome widow with a thouſand pound;
'Cauſe ſhe hath gold, many a courting go,
Both old and young, the clown and fribbling beau—
She treats them kindly, ladies, you muſt know.
Her age and number of ſweethearts you'll find
From the equations which are here ſubjoin'd.

$$\frac{xy^{\frac{3}{2}}}{\sqrt{60x+5y}} = 71{\cdot}5544. \qquad x = \text{her age.}$$

$$xy - y = 780. \qquad y = \text{her number of ſweethearts.}$$

II. QUESTION 519, *by Mr.* Rich. Gibbons.

Required the values of x and z, when $x^{z^{\frac{1}{2}}} = z^{x^{\frac{2}{3}}} = 100$?

III. QUESTION 520, *by Mr.* Tho. Baker; *addreſſed to Mr.* Mal. Hitchins.

SIR,

Your kind advice I wou'd, with joy, obey'd,
But 'tis too late! I've loſt the lovely maid)

That charming form, I thought wou'd make me bleſt,
Is in another's cold embraces preſt.
Ingenious algebraiſt (well known to fame)
Tell all the world my hateful rival's name.*

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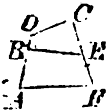
*From the given equations, viz.
$$\left\{\begin{array}{r} m+w+x+y+z=37, \\ w-x+y-z=35, \\ wz+w+z=53, \\ xy+wz=59, \\ \overline{my-x}\times wz=3000. \end{array}\right\}$$
In which the values of m, w, x, y, and z, denote the places, in the alphabet, of the letters compoſing the required name.

IV. QUESTION 521, *by Mr.* Joſeph Walker.

From the equation $\overline{x+1}\times\overline{x^2+1}\times\overline{x^3+1}=30x^3$, to find the value of x, by means of a quadratic only.

IV. QUESTION 522, *by Mr.* Wm. Embleton.

To divide the given trapezium *ADCF* into two parts *AE*, *DE* (geometrically) by the right line *BE*, ſo that $AB : EC :: AE : DE :: m : n$, i.e. in the given ratio of m to n.



VI. QUESTION 523, *by Mr.* Wm. Spicer.

In a quadrangular piece of ground, whereof the ſides are 39, 47, 68, and 57 in a ſucceſſive order, and the diagonal joining the two ſhorteſt of them, 64, 'tis propoſed to form a ſquare fiſh-pond, having its angular points ſituated in the ſides thereof; required a method how to effect the ſame.

VII. QUESTION 524, *by Mr.* Tho. Barker.

Four ſpires ſtanding directly north and ſouth, and at the reſpective diſtances of 2, 3, and 4 miles from each other, were obſerved by a traveller, on a road tending to the north-eaſt, the 1ſt and 2d, and alſo the 3d and 4th, appearing under equal angles, which they alſo did a 2d time, after travelling two miles farther on the ſame road; required his diſtance from them at each obſervation.

VIII. QUESTION 525, *by Mr.* Paul Sharp.

Given the area of a curve $=198{\cdot}9333\times aa$ whoſe equation is $aaxx+y^4-aaxy=2axyy$; required the content of the ſolid generated by the rotation of the ſaid curve about its axis.

IX. QUESTION 526, *by* Geometricus.

Required the nature, area, and deſcription, &c. of the curve, whoſe abſciſſa being the ſame with that of a given circle, its ordinate ſhall be every where equal to the difference between the correſponding ordinate and abſciſſa of the ſaid circle.

X. QUESTION 527, *by Miſs* Ann Nicholls.

On October the 28th, in the latitude 32° north, I obſerved the brighteſt of the Pleiades and Aldebaran, both in the ſame vertical circle; from whence is required the hour of the night, and greateſt latitude poſſible, where ſuch a phænomenon can be obſerved.

XI. QUESTION 528, *by Mr.* Wm. Toft.

At what time on the 10th of June, in the latitude of 12° 30′ north, will the ſun's azimuth be the greateſt?

XII. QUESTION 529, *by* Plus Minus.

In a certain piece of ground ſtand two trees, an oak and an aſh, diſtant from each other 10 chains; the piece is ſurrounded by a ditch of ſo odd a cut, that if you draw lines from any point of it to the trees, and another from tree to tree, a triangle will be formed, ſuch that the half complement of the angle at the aſh to 90°, ſhall be = the angle at the oak; what is the area of the piece?

XIII. QUESTION 530, *by* Philotechnus.

A perſon, aged 35, is deſirous of paying 20l. per annum into the inſurance office, as long as he lives, on condition that his wife, whoſe age is 30, ſhall, after his deceaſe, receive an annuity of 60l. per annum for the remainder of her life after his; required whether the inſurer or the inſured would have the advantage, intereſt of money being at 3 per cent.

XIV. QUEST. 531, *by Mr.* Wm. Spencer, *of* Stannington.

To determine the equation, the area, and the length of the curve, whoſe tangent and ſubtangent have always the ſame given difference (*d*).

XV. QUESTION 532, *by Mr.* Rob. Butler.

To find the ſun's longitude, when his declination alters the faſteſt poſſible.

XVI. QUESTION 533, *by Mr.* T. Allen, *of* Spalding.

Let one end *A* of a ſtraight inflexible rod *AB*, 60 feet long, be laid on an horizontal plane, and the other end *B* on a hemiſphere (whoſe diameter = 30 feet) ſtanding with its baſe on the ſaid plane, and ſuppoſe the end *A* to advance, in a right line, along the plane, until it reaches and touches the ſaid hemiſphere; required the greateſt diſtance the end *B* will have receded from this poſition of the rod, and the area of the curve ſpace deſcribed thereby, and bounded by the ſaid perpendicular poſition of the rod, and another right line drawn from the point *B*, at the commencement of the motion, perpendicular to the ſame.

XVII. QUESTION 534, *by* Curioſus.

Suppoſe one end of a thread *AB*, 100 feet long, is faſtned to the circumference of a circle (or cylinder) whoſe diameter = 4 feet, and, being drawn tight, in the direction of the radius thereof, a ball, weighing 5 pounds, is faſtned to its other end, and impelled, with a velocity of 20 feet per ſecond, in a direction perpendicular to *AB*, by a force acting in the plane of the ſaid circle; required the ball's poſition when it has been 45 ſeconds in motion, the tenſion of the ſtring, &c.

The PRIZE QUESTION, *by* Philalethes Londinenſis.

Suppoſing the ſphere to be projected on the concave ſurface of an infinitely long cylindric tube of paper, &c. touching it every where on the equator, by the eye at the center, and afterwards opened, through any of the projected meridians, and ſpread or ſtretched upon a plane; required the figure of the rumbs in that projection.

NOTE, This is propoſed with an intent to rectify an error in a mathematical tract wrote by the late Rev. Mr. Weſt, of Exeter, and publiſhed anno 1762, which, if unnoticed, might prove of fatal conſequence to the navigator, it being pretended to demonſtrate there, that the rumbs, in this projection, will be right lines; but the contrary will be proved hereafter.

1765.

Queſtions anſwered.

I. QUESTION 518 *anſwered by Mr.* Malachy Hitchins.

MARGERY's age is forty years,
Her ſweethearts are juſt twenty;
How plainly avarice appears
In bringing her ſuch plenty..
Had ſhe ten thouſand charms in ſtore,
But wanted one in money,
I dare affirm, of all the ſcore,
Scarce one would be ſo funny.

The ſame anſwered by Mr. Iſaac Tarratt.

From the 1ſt given equation, ſquared, &c. is derived $x^2y^3 = 60aax + 5aay$ (putting 71·5544 = a, and 780 = b), and, from the ſecond, $x = \frac{b+y}{y}$: Whence $\frac{bb+2by+yy}{yy} \times y^3 = 5aay + \frac{60aab+60aay}{y}$; ſolved, $x = 40$ = her age.
$y = 20$ = her ſweethearts.

A buxſome widow ſure! of wond'rous parts,
Thus to attract, or wound ſo many hearts!
Widow! ſet up a ſchool, inſtruct old maids:
If this thou can'ſt, 'twill be the beſt of trades.

Meſſ. *J. Aſhmore, T. Baker, T. Barker, W. Barnes, T. Bramley, J. Hitchcock, Rd. Mallock, Joſ. Mountfort, J. Probert, B. Rotherham, T. Sadler* (the propoſer), *W. Sewell, Sopor, W. Spicer,* Mrs. *Suggett, T. Walker, J. Young,* and ſeveral others, have likewiſe anſwered this queſtion.

II. QUESTION 519 *anſwered by Mr.* Tho. Walker.

Let X denote the log. of x, Z the log of z, and L that of 100, and then $z^{\frac{1}{3}} \times X = x^{\frac{2}{3}} \times Z = L$ (per the nature of logarithms and the queſt.); whence $Z = \frac{L}{x^{\frac{2}{3}}}$. Aſſume $x = 47·5$, and then $x^{\frac{2}{3}} = 13·115$, and conſequently $\frac{2·000000}{13·115} = Z = 0·152421$ the log. of $1·4215 = z$, according to this aſſumption; but $\overline{1·4215}|^{\frac{5}{2}} \times X$ is $= 1·9986 =$ log. of 99·69, which is too little by 0·31: therefore, now, ſuppoſe x to be $= 47·6$, and then z comes out $= 1·42$, and the 2d error $= 0·16$; whence 0·15 : 0·1 :: 0·16 : 0·106, &c. which added to the laſt aſſumed value of x, gives 47·706, &c. for its true value, very near; and thence z is found $= 1·42$, &c.——And thus, by repeating the operation, we may arrive at any degree of accuracy required.

In this manner, nearly, the anſwer is given by Meſſ. *Tho. Baker*, *Tho. Barker*, *Wm. Barnes*, *S. Beeken*, *R. Gibbons* (the propoſer), *J. Probert*, *Tho. Sadler*, *P. Sharp*, *Iſaac Tarratt*, and others.

III. QUESTION 520 *anſwered by Mr.* Malachy Hitchins.

Could'ſt thou, dear Baker, ſing of Hervey's DEATH,
And not perceive the fleeting ſtate of breath,
Nor know delays to dang'rous iſſues tend,
Ere thou hadſt loſt thy lover and thy friend?
'Tis now in vain thy mournful tale is told!
Thy charmer's loſt!—and her more charming gold!

Mr. *Paul Sharp*, by an eaſy proceſs, derives the two following equations, viz.

$$\overline{43y - xyy - xy - yy - x} \times \overline{59 - xy} = 3000, \text{ and}$$

$$24xy + yxx + xxyy + 6y - 6x - xyy - \frac{\overline{88 + x - y}|^2}{xy - 5} = 233;$$

from which, by the method given by the late truly ingenious and profoundly great mathematician Mr. Thomas Simpſon, on page 82 of his Eſſays, he finds $x = 1$ and $y = 19$; and from thence is readily found $m = 4$, $w = 5$, and $z = 8$; which point out the ingenious propoſer's rival to be DEATH.

This queſtion is alſo anſwered algebraically by Meſſ. *J. Aſhmore*; *T. Baker*, *T. Barker*, *W. Barnes*, *S. Beeken*, *R. Gibbons*, *J. Hitchcock*, *J. Mountfort*, Miſs *Ann Nicholls*, *J. Probert*, *T. Sadler*, *Sopor*, *W. Spicer*, *W. Sewell*, *J. Tarratt*, *W. Toft*, *T. Walker*, *J. Young*, and others.—But as ſolutions this way turn out ſomewhat troubleſome and uncouth, the following method, in point of eaſe and expedition, may perhaps claim the preference, viz.——The 3d given equation ($wz + w + z = 53$) being evidently no more than to find two whole poſitive values of w and z ſuch, that their ſum added to their product may be $= 53$, it appears, from very little conſideration, that they muſt be 5 and 8, and conſequently that their product (wz) is $= 40$; whence it, with certainty, follows (from the 4th given equation) that xy is $= 19$, and conſequently that the number 1 muſt appertain to either x or y, and 19 to the other of them; and thence m is, certainly, found $= 4$ (from the 1ſt given equation): whence it alſo, certainly, follows that $x = 1$, becauſe if it be ſuppoſed $= 19$ (and either 1 or 19 it muſt be) then the laſt given equation will become impoſſible. In like manner it appears that w muſt be $= 5$, becauſe if it be taken $= 8$ (and one of them it muſt be), then the 2d given equation becomes impoſſible. Hence the values of m, w, x, y, and z are found to be 4, 5, 1, 19, and 8, anſwering to the letters (of the alphabet) D, E, A, T, H, reſpectively.

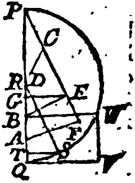
IV. QUESTION 521 *anſwered by Mr.* Thomas Barker.

From the given equation, the following one, viz. $x^6 + x^5 + x^4 - 28x^3 + x^2 + x + 1 = 0$, is readily derived; which, divided by x^3, becomes $x^3 + x^2 + x - 28 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$, or, putting $x + \frac{1}{x} = z$, $\overline{zz + 4z + 10} \times \overline{z - 3}$ ($= z^3 + z^2 - 2z - 30$) $= 0$: Whence $z - 3 = 0$, and $z = 3$; which is the only poſſible value of z, in the equation, and which, ſubſtituted for z in the aſſumed equation $x + \frac{1}{x} = z$, gives $x + \frac{1}{x} = 3$; whence $xx - 3x = -1$, and $x = \frac{3 \pm \sqrt{5}}{2} = \left\{\begin{matrix} 2{\cdot}618 \\ 0{\cdot}382 \end{matrix}\right\}$; both which numbers anſwer the conditions of the queſtion.

Meſſ. *T. Baker*, *R. Gibbons*, *J. Probert*, *T. Sadler*, *W. Spencer*, *J. Tarratt*, *W. Toft*, &c. have likewiſe anſwered it according to this method, nearly.

V. QUES-

V. QUESTION 522 *anſwered by Mr.* C. Hutton.



If the ſides FC, AD, of the given trapezium $ADCF$, be produced to meet in P, the $\triangle PDC$ will be of a given magnitude; and ſince the ſaid trapezium is to be divided in a given ratio, by the right line BE, each of the parts DE and AE will alſo be of a given magnitude; and conſequently the $\triangle PBE$ will be of a given magnitude, which call aa. Then the queſtion is reduced to this, viz. to draw the right line BE ſo, that AB may be to $EC :: m : n$, and the $\triangle BPE =$ the given ſquare aa.

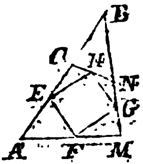
CONSTRUC. In PA produced, take AQ to PC in the given ratio of m to n, and on the diameter PQ deſcribe the ſemicircle PWQ; and from the center R thereof draw the rad. $RS \parallel PF$ and $ST \perp PQ$. Then if QV be made $\perp PQ$, and $=$ a mean proportional between $\frac{m \times PQ}{n}$ and $\frac{aa}{ST}$, and VW and WB be drawn $\parallel PQ$ and QV reſpectively, we ſhall have B for one point through which the required line muſt paſs: and if CE be taken to AB in the given ratio of n to m, E will be the other.

DEMONSTRA. Draw $EG \perp PA$: then, by the ſim. triangles TSR, GEP, $SR : ST :: EP : \frac{EP \times ST}{SR} = EG$; whence (triangles being the halves of their circumſcribing parallelograms, per Elements of Geom.) $\frac{1}{2} PB \times \frac{EP \times ST}{SR}$ or $\frac{PB \times EP \times ST}{PQ}$ ($2RS$ being $=$ the diameter PQ) will be the meaſure of the $\triangle BPE$. But (per conſtruc.) $PB \times \frac{m \times EP}{n}$ $(= PB \times BQ)$ is $= BW^2 = QV^2 = \frac{m \times PQ}{n} \times \frac{aa}{ST}$ (per conſtruction); and conſequently each of theſe equal magnitudes, or ſpaces, being multiplied by $\frac{n \times ST}{m \times PQ}$ (or repeated that number of times), aa will be found $=$ $\frac{PB \times EP \times ST}{PQ} =$ the meaſure of the triangle BPE (the ſame as found above).

VI. QUES-

VI. QUESTION 523 *anſwered by* Mr. C. Hutton.

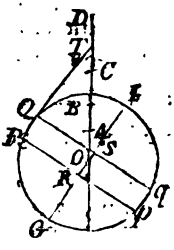
This queſt. may be conſtructed from prob. 41 on page 346 of Simp. Geom. 2d edit. And if the ſides *AC*, *MN*, of the given trapezium *AMNC*, be produced to meet in *B*, the conſtruction here will be the ſame as in that prob. (only obſerving that here *C* and conſequently *B* fall between *A* and *B*); for which reaſon it ſeems needleſs to repeat the conſtruction.



Algebraic ſolutions to this queſtion have been received from Mr. *T. Barker*, the Rev. Mr. *John Croſs* (of *Pocklington* Grammar School, near *York*), Mr. *M. Hitchins*, Mr. *Tho. Sadler*, Mr. *W. Spicer*, Mrs. *Suggett*, Mr. *J. Farratt*, and others: From which (by means of quadratics only) the required ſide of the inſcribed fiſh-pond may be found, and appears to be = 39, nearly; but, for want of room, we are obliged to omit them.

VII. QUESTION 524 *anſwered by* Mr. C. Hutton.

From the conſtruc. on page 238 of Simp. Geom. 2d edit. it appears, that the locus of concourſe of lines drawn from the given points (or places of the ſpires) *A*, *B*, *C*, *D*, ſo that the angles ſubtended by *AB* and *GD* may be equal, will be the circumf. of a circle; which cir. *IPQ*, let be deſcribed by that prob.



Then, to find in which two points a right line, drawn in the given direction (north eaſt), ſhall cut the circumf. of his circle ſo, that the part of it intercepted by thoſe points may be of the given length (2 miles), through the center *O*, draw *GOI* in the given direction, and on it, from *O*, each way, ſet off *OR*, *OS*, each = (1 mile) half the given diſtance of the ſtations: Then through *R* and *S* draw *PR* and *QS* ⊥ *GI*, and the points *P*, *Q*, of their interſection with the circumf. of the ſaid circle, will be the two ſtations (in the road *PT*) required; which is too evident to need any further demonſtration.

Mr. *Tho. Barker* (the propoſer) and Mr. *Malachy Hitchins* have alſo conſtructed this problem; and many contributors have ſent algebraic ſolutions to it.

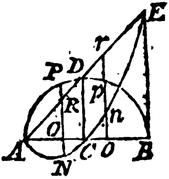
VIII. QUESTION 525 *anſwered by Mr.* Rob. Butler.

By tranſpoſing the given equation, and extracting the ſquare root, it becomes $yy - ax = \pm a\sqrt{xy}$, or, putting $\frac{b^2x}{z^2} = y$, $\frac{b^4x^2}{z^4} - ax = \pm \frac{abx}{z}$; whence $x = \frac{az^4}{b^4} \pm \frac{az^3}{b^3}$, and $y = \frac{az^2}{b^2} \pm \frac{az}{b}$ (where b may be any quantity at pleaſure, and ſo the curve may be conſtructed). Suppoſe $b = 1$; and then $\dot{x} = 4az^3\dot{z} \pm 3az^2\dot{z}$, and $y\dot{x}$ (the flux. of the area) $= 4a^2z^5\dot{z} \pm 7a^2z^4\dot{z} + 3a^2z^3\dot{z}$, and the correct fluent thereof $\overline{\frac{2z^6}{3} \pm \frac{7z^5}{5} + \frac{3z^4}{4} - d} \times aa = 198{\cdot}9333\,aa$ per queſt. (d being $= \frac{2}{3} \pm \frac{7}{5} + \frac{3}{4}$); from whence, when a is expreſſed in numbers, z, and conſequently x and y, may be found in numbers alſo.

In like manner (putting $p = 3{\cdot}1416$) the flux. of the ſolidity ($py y\dot{x}$) is found $= p \times \overline{azz \pm az}^2 \times 4az^3\dot{z} \pm 3az^2\dot{z}$, and conſequently $pa^3 \times \frac{1}{2}z^8 \pm \frac{11}{7}z^7 + \frac{5}{3}z^6 \pm \frac{3}{5}z^5 - n$ = the ſolidity itſelf (n being = the ſum of the coefficients of the powers of z, &c.).

Meſſ. *W. Embleton*, *C. Hutton*, *D. Kinnebrook*, and *P. Sharp* (the propoſer) anſwer it in this manner nearly.

IX. QUESTION 526 *anſwered by Mr.* Da. Kinnebrook.



Let ADB repreſent the given ſemicircle, and ACE the required curve, which, it is eaſy to perceive, will paſs through the center C, of the circle, and meet a tangent to it at B, in the point E, whoſe diſtance from B is = the given diameter AB.

Join the points A, E, and draw the ordinates PO, po, &c. cutting AE in R, r, &c. and AB in O, o, &c. reſpectively, and take ON, on, &c. thereon every where $= PR$, pr, &c. and N, n, &c. will be the points in the required curve.——For $AB = BE$ (per nature of the queſt. and conſtruc.) and conſequently $AO = OR$, &c.

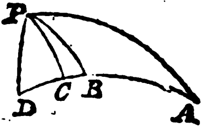
(per ſim. triangles) whence $PO \backsim AO = PR = NO$, &c. (per conſtruction). *Q. E. D.*

Put, now, AO (or Ao, &c.) $= x$, $AB = 2a$, NO (or, no, &c.) $= y$, and then (per prop. of cir. and queſt.) $\sqrt{2ax - xx} = PO$, and y (NO, &c.) $= \sqrt{2ax - xx} \backsim x$, and $\overline{x \pm y}|^2 = 2ax - xx = \frac{2a\sqrt{2} - x\sqrt{2}}{2aa} \times aax\sqrt{2}$: Whence $\overline{x \pm y}|^2 : x\sqrt{2} \times \overline{2a\sqrt{2} - x\sqrt{2}} :: aa : 2aa$, i. e. $NR^2 : AR \times RE :: CD^2 : AD^2$; and hence it appears, that the required curve ACE is an ellipſe, whoſe center is D (the rad. DC being perpendicular upon the diameter AB), and AE a diameter thereof, the conjugate to that diameter being DC.——And if an infinite number of parallels to PN be conceived to be drawn, the circular and elliptic areas $APDR$, $ANCO$ will be divided into an infinite number of parallelograms, whoſe baſes PR, NO, &c. and altitudes are equal; and therefore the circular area $APDR =$ the elliptic area $ANCO$.——In like manner the area DEB is proved to be $=$ the area CEB; and thence the ſemiellipſe $ANCED$ appears to be $=$ the ſemicircle $ACBD$.

In this manner, nearly, the anſwer is given by Mr. *Rob. Butler*, and Mr. *C. Hutton*: But Mr. *T. Allen*, after proving the nature and deſcription, &c. of the required curve to be the ſame as above, puts AO, or Ao, &c. $= x$, the correſponding ordinate ON, or on, &c. $= y$, the rad. $AC = a$, and finds the fluxions of the ſpaces ANO, $Cno = \dot{x}\sqrt{2ax - xx} - x\dot{x}$ and $x\dot{x} - \dot{x}\sqrt{2ax - xx}$ reſpectively; and the fluent of the former of them (when $x = a$) is $=$ the meaſure of the circ. quad. $ADC - \frac{1}{2}AC^2$, and the correct fluent of the latter $= \frac{xx - aa}{2}$ — the meaſure of the circ. ſeg. $CDpo =$ (when $x = 2a$) $\frac{3AC^2}{2}$ — the meaſure of the circ. quad. $CDB =$ the meaſure of the whole ſpace CBE; to which if that of the ſpace ANC ($=$ the meaſure of the circ. quad. $ADC - \frac{1}{2}AC^2$) be added, the area of the whole curve ſpace $ANCEBC$ will be found $= AC^2$; *A thing ſomewhat remarkable!*

X. QUES-

X. QUESTION 527 *anſwered by Mr.* T. Allen.

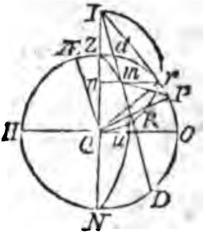


Let *A* and *B* be the places of Aldebaran and the brighteſt of the Pleiades reſpectively, *P* the pole, *PC* the meridian, and *CA* the vertical circle paſſing through the two ſtars. In the ſpherical triangle *ABP*, are given *AP* the polar diſtance of Aldebaran = 73° 58′ 54″, *BP* that of the brighteſt of the Pleiades = 66° 58′ 54″, and the included angle, he diff. in right aſcenſion of the two ſtars = 12° 13′ 30″, whence the ∠ *PBC* is found = 60° 7′ 15″; then in the ſpherical triangle *PBC* are given *PB* and the ∠ *PBC* as above, and *PC* the comp. of the given lat. = 58°, from whence the ∠ *CPB* is found = 28° 55′ 40″, which is the arc that the brighteſt of the Pleiades is ſhort of the meridian at the time required.

Laſtly, the greateſt lat. in which this phænomenon can be obſerved, is, when the ∠ *PCB* becomes = *PDB* a right angle. Therefore, rad. : ſine of *PB* : : ſine of the angle *PBD* : coſine of 37° 14′ 53″, the greateſt lat. required.

Meſſ. *R. Butler*, *W. Embleton*, *D. Kinnebrook*, *Tho. Sanderſon*, *Joſ. Webſter*, and ſome others, anſwer it in this manner very near.

XI. QUESTION 528 *anſwered by Mr.* Wm. Toft, (*the Propoſer*).



In the orthographic projection of the ſphere, on the plane of the meridian, the horizon *HO*, the equator *Æ♎*, the prime vertical *ZN*, the parallel of declination *Dd*, &c. will, it is well known, be projected into right lines, and the azimuth circles into ellipſes, each interſecting *Dd* in two points, except that (*ZuN*) which touches it; and which, therefore, is the azimuth required.

For the ſun coming afterwards to the other points of interſection with *Dd*, will apparently recede back again, and be ſeen ſucceſſively on the ſame points of the compaſs as before, and ſo the azimuth decreaſe again by the ſame ſteps it before increaſed; an appearance ſurprizing enough to thoſe who are ignorant of the true cauſe thereof!

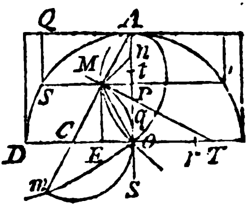
Therefore to determine the poſition (or projection) of this azimuth circle, (and conſequently to ſolve the prob.) produce NZ and Dd till they meet in I, and conceive a ſemicircle to be deſcribed upon the diameter IQ, cutting the primitive in r, and let rn be drawn $\perp NZ$, and its interſection m with Dd will be the place of the ſun at the time required, or the point through which the projected azimuth circle required muſt paſs.——For, ſuppoſing the points m, I; r, I; and Q, r to be joined, and the ſemi-periphery of an ellipſe, to the tranſverſe diameter ZN, to be deſcribed through the given point m: then will mI be a tangent to it in m, by the property of the ellipſe and its circumſcribing circle (ſee theor. 49th of the ellipſe in Steel's Conic Sections), becauſe rI is a tangent to the primitive circle in r (QrI being a right angle, by conſtruction).

The method of calculation is from hence alſo extremely natural and eaſy; for QR ($\perp Dd$) being the nat. ſine of the given declin. to rad. of the primitive; QI, and conſequently Qn ($=\frac{Qr^2}{QI}$, per prop. of the circle) are from thence readily found, the latter of them being the nat. ſine of 33° 27′, the ſun's alt. at the time required; and thence nI and nm become known: Whence, by another prop. of the ellipſe and its circumſcribing circle, $nr : nm :: QO : \frac{nm \times QO}{nr} = Qu$, the nat. coſ. of 70° 24′, the ſun's azimuth required. And hence the time from noon, when this phænomenon happens, is eaſily found = 3 h. 55 m. nearly.

Meſſ. *Robert Butler*, *Da. Kinnebrook*, and *Lycidas*, by ſpherics (from the ſpherical triangle mZP, where ZP is the complement of the given lat. Pm the complement of the ſun's declin. and mZ that of his alt. at the time required), find $\frac{\text{ſine of } \angle ZmP \times \text{ſine } Pm}{\text{ſine } PZ}$ = the ſine of the $\angle PZm$, the azimuth required; which, it is manifeſt, will be greateſt when ZmP is a right angle, becauſe the ſines of Pm and PZ are conſtant quantities; from whence the time required is directly found to be either 5 m. paſt 8 in the forenoon, or 55 m. paſt 3 in the afternoon, very near.

And Meſſ. *T. Allen*, *T. Barker*, *W. Barnes*, *W. Embleton*, *R. Gibbons*, *T. Sadler*, *P. Sharp*, *W. Spicer*, *T. Walker*, *J. Young*, and ſome others, determine the time ſought nearly the ſame as above.

XII. QUESTION 529 *anſwered by Mr.* R. Butler.



Let A repreſent the aſh, O the oak, and with any rad. Oq (not exceeding $\frac{1}{2}OA$) and center q, deſcribe the circumference of a circle cutting OA in n, and on the diam. qA (the complement of Oq) conceive the circumference of another circle to be deſcribed cutting the former in M, a point in the curve (or figure of the ditch) required; and after this manner any number of other points may be found.—— For MP being drawn $\perp OA$ and the points M, q; M, A; M, n; and M, O joined, the $\angle AMP = MqP =$ double the $\angle AOM$, as required per the queſtion.

The ditch $OMAO$ being thus conſtructed, to find the area thereof, put $a = OA$ ($=$ 10 chains), $x =$ abſciſſa OP, $y =$ ordinate PM; then, per ſim. triangles, $a - x$ (AP) : y (PM) :: y (PM) : $\frac{yy}{a-x} = Pq$; and $\frac{yy}{x} = Pn$: whence $nq = \frac{yy}{a-x} + \frac{yy}{x} = Oq = x - \frac{yy}{a-x}$, and conſequently $y = x\sqrt{\frac{a-x}{a+x}} = \frac{ax-xx}{\sqrt{aa-xx}}$, and $y\dot{x}$ (the fluxion of the area OMP) $= \frac{ax\dot{x}-x^2\dot{x}}{\sqrt{aa-xx}} = \frac{ax\dot{x}}{\sqrt{aa-xx}} + \frac{1}{2} \times \frac{aax\dot{x}-2x^3\dot{x}}{\sqrt{aaxx-x^4}} - \frac{\frac{1}{2}aa\dot{x}}{\sqrt{aa-xx}}$, and the correct fluent thereof (or area OMP) $aa - a\sqrt{aa-xx} + \frac{1}{2}x\sqrt{aa-xx} - \frac{1}{2}a \times$ circ. arc whoſe rad. $= a$ and right ſine $= x$; which, when $x = a$, becomes * $aa - \frac{1}{2}a \times \frac{3{\cdot}1416a}{2} =$ 21·46 ſquare chains $=$ the area of the ſpace $OMAO$; and doubled is 42·92 ſquare chains $=$ the area of the whole ſpace required.

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* This expreſſion reducing to $aa \times \overline{1 - {\cdot}7854}$, it may be remarked that $OMAO$, the ſemi-area, is equal to the difference between a ſquare whoſe ſide is a, and its inſcribed circle. Which alſo agrees with the other ſolution by *Plus Minus*.

Mr. *Da. Kinnebrook*'s anſwer is the ſame, exceeding near. And much after the ſame manner the anſwer is alſo given by Meſſ. *J. Barnard, W. Embleton, C. Hutton, Lycidas, W. Spicer*, and ſome others.

The ſame anſwered otherwiſe, by Plus Minus, *the Propoſer.*

Let A and O be the places of the aſh and oak, and M a point in the required curve. 'Tis plain, from the data, that the exceſs of the angle at the fence above 90° = the angle at the oak. Draw $MT \perp AM$, and then $TMO = MOA$; whence we have the method of deſcribing the required curve, viz. draw $OD \perp AO$, and AMC at pleaſure: With the center C, and radius CO deſcribe the circumference of a circle which will cut AC in two points M, m of the required curve. Call AO, a; AP, x; and PM ($\perp AO$) y; then (per ſim. triangles APM, AOC) CO, or $CM = \frac{ay}{x}$, and $CE = \frac{ay}{x} - y$; but $EC^2 + EM^2$ (or PO^2) $= CM^2$, i.e. $2ayy - xyy = x^3 - 2axx + aax$, which is an equation for the hyp^a. defectiva nodata diametrum habens, (ſee Newton's Enumeratio Lin. 3tii Ord. Species 41^a.) and $y = \frac{\sqrt{x\times\overline{a-x}^{\,2}}}{\sqrt{2a-x}} = \frac{a}{x^0\sqrt{-1+2ax^{-1}}} - \frac{1}{x^{-1}\sqrt{-1+2ax^{-1}}}$, which belongs to tab. 2, form 4 caſ. 2 and 3, of Newton's Quad. Curv. or $y = \frac{ax - xx}{\sqrt{2ax - xx}}$, which belongs to caſ. 2 and 3 of form 8; and by either of them, it appears, that if you deſcribe a quad. AD on the center O, with AO for a radius, and produce PM meeting AD in S, and draw SQ, QA parallel to OA and OD, the area APM ſhall be equal to the area ASQ; and the whole area of the field $= 2AO^2$ — a ſemicircle whoſe rad. is the ſame AO. That this is the whole of the area is plain, if we look at the equation $2ayy - xyy = x^3 - 2axx + aax$: for when $y = 0$, $x = 0$ or a; ſo that the trees ſtand in the fence: And that no other part of the curve beyond O is to be conſidered as belonging to the field, is alſo plain; becauſe, by the data, the angle at the fence muſt never be leſs than a right one. When y is infinite, $x = 2a$, which gives the aſymptote.

If you take Or, Os each $= \frac{1}{2}a$, and ſet Ar from s to t, you will have the foot of the greateſt ordinate.*

XIII. QUESTION 530 *anſwered by Mr.* E. Nott.

This gentleman conſiders the inſurer as certain of receiving 20l. per annum during the huſband's life, though the wife was to die immediately after the agreement. conſequently that (by tab. 5th on pag. 260 of Simpſon's Select Exerciſes) the preſent value of his expectation is worth 14·1 years purchaſe, or 282 l.

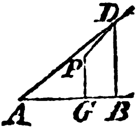
In like manner, the remainder of the wife's life after that of her huſband's being (by prob. 15 on pag. 285 of Simp. Select Exerciſes aforeſaid) worth 4·7 years purchaſe, the preſent value of her expectation will be found to be 60 × 4·7 = 282 l. which, being the ſame with the other, ſhews that neither ſide will have the advantage in this caſe.

And in this light it is conſidered by Meſſ. *R. Butler*, *W. Embleton*, *R. Gibbons*, *M. Hitchins*, *C. Hutton*, *D. Kennebrook*, *T. Sadler*, *J. Tarratt*, and ſeveral others.

But the propoſer, ſuppoſing the inſurer to be no longer intitled to receive than during the joint exiſtence of the man and his wife, makes the advantage about 22 l. on the ſide of the inſured: And if the intereſt the inſurer might make of the money he has a chance of receiving, from time to time, before he is liable to pay any, were to be taken into conſideration, the balance would then be found to turn ſomewhat in favour of him, &c.——His ſolution is from firſt principles, but our narrow limits oblige us to omit it.

XIV. QUESTION 531 *anſwered by Mr.* Rob. Butler.

Let PD repreſent the required curve, whoſe abſciſſa $GB = x$, and ordinate $BD = y$, and then $\frac{y\dot{x}}{\dot{y}}$ being = its ſubtangent AB and $\frac{y\sqrt{\dot{x}^2+\dot{y}^2}}{\dot{y}}$ = its tang. AD, $\frac{y\sqrt{\dot{x}^2+\dot{y}^2} - y\dot{x}}{\dot{y}} = d$,



per

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[*] This will eaſily appear by putting the fluxion of ($y =$) $\frac{ax - xx}{\sqrt{2ax - xx}}$ equal to 0.

per queſtion. Whence $\dot{x} = \frac{y\dot{y}}{2d} - \frac{1}{2}d \times \frac{\dot{y}}{y}$; and y being $= d$ ($= GP$, the tangent at the vertex) when $x = 0$, per the nature of the queſtion, the correct equation of the fluents will be $x = \frac{yy - dd}{4d} +$ hyp. log. of $\overline{\frac{d}{y}}\Big|^{\frac{d}{2}}$; the equation of the curve required.

Again, $y\dot{x} = \frac{y^2\dot{y}}{2d} - \frac{1}{2}d\dot{y}$, and the correct fluent, or area required, $= \frac{y^3}{6d} - \frac{1}{2}dy + \frac{1}{3}dd$.

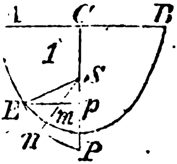
Laſtly, $\dot{x}^2 + \dot{y}^2 = \dot{y}^2 \times \frac{\overline{yy - da}\,\big|^2}{4ddyy} + \dot{y}\dot{y}$, and $\sqrt{\dot{x}^2 + \dot{y}^2} = \frac{y\dot{y}}{2d} + \frac{1}{2}d \times \frac{\dot{y}}{y}$; and the correct fluent $\frac{yy - dd}{4d} +$ h. log. of $\overline{\frac{y}{d}}\Big|^{\frac{d}{2}}$ is $=$ the length of the required curve PD.

COROLLARY. It hence appears that the curve's length always exceeds the axis by the quantity $d \times$ hyp. log. $\frac{y}{d}$. —— It is alſo anſwered in this manner by Mr. *T. Allen*, and Mr. *Da. Kinnebrook*, exceeding near.

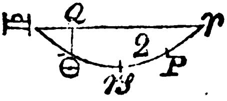
And Meſſ. *T. Barker*, *W. Barnes*, *W. Embleton*, *C. Hutton*, *P. Sharp*, *W. Spicer*, *T. Sanderſon*, *T. Walker*, and ſome others, have likewiſe anſwered it.

XV. QUESTION 532 *anſwered by Mr.* T. Allen.

Let *AEPB* (fig. 1.) be one half of the ellipſis in which the earth revolves about the ſun, in the focus *S*, *P* the place of the perihelion, and *ESm* an elliptic ſector deſcribed in an indefinitely ſmall particle of time t'. Put $CP = a$, $AC = CB = b$, $CS = c$, and let the coſine of *ESP*, the earth's diſtance from its perihelion $= x$, rad. $= 1$. Then, by the properties of the ellipſis, $ES = \frac{bb}{a + cx}$, and (by trigonom.) $Ep = \frac{bb\sqrt{1 - xx}}{a + cx}$, and $Sp = \frac{bbx}{a + cx}$. There-



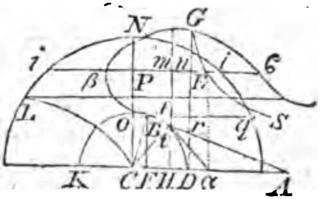
fore $\frac{bb\sqrt{1-xx}}{a+cx} : \frac{bb}{a+cx}$, or $\sqrt{1-xx} : 1 :: \frac{abb\dot{x}}{\overline{a+cx}|^{2}}$ (= flux. Sp) : $\frac{abb\dot{x}}{\sqrt{1-xx}\times\overline{a+cx}|^{2}}$, the fluxion or increaſe (En) of the true longitude in the circular arc EP, whoſe rad. is ES, in the time t'.



Now, let ♎♑♈ (fig. 2) repreſent the ſouthern ſemicircle of the ecliptic, ♑ the Tropic of Capricorn, P the place of the perihelion, ⊖ the place of the earth in the ecliptic at the time propoſed; then will ⊖$\mathcal{Q}$ = the ſun's declination. Put m and n = the ſine and coſine of ♑P, the diſtance of the perihelion from ♑, p and q = the ſine and coſine of ⊖♎$\mathcal{Q}$, and the coſine of P⊖ = x (as above), then will $pnx + pm\sqrt{1-xx}$ = the ſine of $\mathcal{Q}$⊖ (ſee art. 478 of Simpſon's Flux.) and $\frac{pn\dot{x}\sqrt{1-xx}-pmx\dot{x}}{\sqrt{1-\overline{pnx+pm\sqrt{1-xx}}|^{2}}\times\sqrt{1-xx}} =$ the fluxion or alteration of the declin. in the time t'. Conſequently $\frac{\sqrt{1-\overline{pnx+pm\sqrt{1-xx}}|^{2}}}{n\sqrt{1-xx}-mx\times\overline{a+cx}|^{2}}$ by the queſt. muſt be a min. By making the fluxion of which = 0, the value of x may be found.

Meſſ. *R. Butler*, *D. Kinnebrook*, *J. Moreland*, and *A. Rowe* find, by very eaſy, ſhort, and elegant proceſſes, that at croſſing the line the declination alters the faſteſt, ſuppoſing the earth's orbit to be a circle, and the apparent motion of the ſun in the ecliptic uniform, &c.

XVI. QUESTION 533 *anſwered by* Plus Minus.



Let AB (= 60 feet = a) repreſent the firſt poſition of the given rod, and $\alpha\beta$ any other poſition of it, touching the given hemiſphere DBK in t, and draw the rad. Ct, and CP, $P\beta$ (perpendicular and parallel to CD), DG

($= AB$) a tangent to the given circ. at D, αE ($\parallel DG$) meeting βP (produced) in E, and or, through B, perpendicular to CP and DG; and then, putting Ct ($= CD =$ 15 feet) $= r$, $CP = x$, and $P\beta = y$, x ($E\alpha$) will be to a ($\alpha\beta$) $::$ r (Ct) $: \frac{ar}{x} = C\alpha$, and $\beta E = y + \frac{ar}{x}$: but $\beta E^2 + \alpha E^2 = \alpha\beta^2$, and therefore $y = -\frac{ar}{x} \pm \sqrt{aa - xx}$; conſequently, if about C, with the rad. AB, the circumference of a circle be deſcribed cutting CP, βE and Br, produced, in N, i, and q, and alſo an hyperbola through the point G to the aſymprotes CP, CA, and meeting Bq, produced, in s, the area $G\beta Br$ will be $=$ the circular area Nqo — the hyperbolic area $Grs = 1369{\cdot}58$; and the area of any portion $\beta G\mathcal{G}$ thereof, cut off by a right line $\beta\mathcal{G} \parallel CA$ will be $=$ the circ. ſeg. iNi.—— The fluxion of y or $-\frac{ar}{x} \pm \sqrt{aa - xx}$, being equated to nothing, gives $x^3 = \pm ra\sqrt{aa - xx}$; therefore if the cubic parabola, whoſe equation is $x^3 = ray$, be deſcribed through C, it will cut the aforeſaid circ. in L, the point through which the ordinate paſſes, which is a maximum ($= 38{\cdot}02$) on one hand, and gives the point of contrary flexure on the other.

In this manner, exceeding near, the anſwer is given by Meſſ. *T. Allen* (the propoſer), *R. Butler*, *W. Embleton*, *C. Hutton*, *D. Kinnebrook*, *T. Sanderſon*, *W. Spencer*, and ſome others.

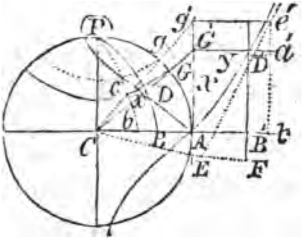
XVII. QUESTION 534 *anſwered by Mr.* Da. Kinnebrook.

The ball, in the firſt part of its motion, moves through the quadrant of a circle, and in the latter part through the involute of the given circle or cylinder; in both which caſes the thread is perpendicular to the curve, and therefore cannot affect the ball's motion, and conſequently it moves uniformly through both curves. Now the length of the quadrant, to rad. 100 feet, is $= 157{\cdot}08$; whence $\frac{157{\cdot}08}{20} = 7{\cdot}854$ ſeconds, the time of its deſcription, and $45'' - 7{\cdot}854'' = 37{\cdot}146''$ the time of the ball's moving in the involute; whence $37{\cdot}146 \times 20 = 742{\cdot}92$ the length of the involute. Put now $a = 100$, $b = 2$, $x =$ the rad. of involution, and $l =$ the length of the involute; then (per pag. 163 of

Simp. Flux. 2d edit.) $\frac{aa-xx}{2b}=l$; whence $x=\sqrt{aa-2bl}$ $=$ (when $l=742{\cdot}92$) $83{\cdot}836$ = the radius of involution, or the part of the thread or ſtring remaining unwound at the end of $45''$: from whence the poſition of the ball may be found, &c.——The tenſion of the ſtring is $=\frac{5\times 20^2}{32\frac{1}{6}\times 83{\cdot}836}$ $=\frac{3}{4}$ lb. nearly.

In this manner, very near, it is anſwered by Meſſ. *R. Butler*, *W. Embleton*, *C. Hutton*, *E. Nott*, &c.

The PRIZE QUESTION *anſwered by* Philálethes Londinenſis (*the Propoſer*).



Let $AGPDB$ be part of the ſphere, its center C, pole P, part of the equator AB, and PA, PB, Pb meridians, of which Pb is infinitely near PB. Let ADe be a loxodromic line that makes equal angles with all the meridians through which it paſſes, of which let t be the tangent to the rad. of the ſphere $AC=r$, and put AB $(=AB')=y$, $AG=l$, its tangent $AG'=x$, and then $\sqrt{rr+xx}$ (CG') : r (CA) :: $\dot{y}$ (Bb) : $\frac{r\dot{y}}{\sqrt{rr+xx}}=Da$, and $r:t::\dot{l}$ (ea) : $\frac{r\dot{y}}{\sqrt{rr+xx}}$ (Da); whence $\dot{y}=\frac{t\dot{l}\sqrt{rr+xx}}{rr}$: but $\dot{l}=\frac{rr\dot{x}}{rr+xx}$, and therefore $\dot{y}=\frac{t\dot{x}}{\sqrt{rr+xx}}$, and (putting $2{\cdot}30258=M$) $y=$ $tM\times$ log. $\frac{x+\sqrt{rr+xx}}{r}$, the equation expreſſing the relation betwixt x and y on the globe, or, which is the ſame thing, between the abſciſſa and ordinate in the propoſed chart or projection. But $x+\sqrt{rr+xx}$ is = tangent of $\frac{90^\circ+l}{2}$; therefore, laſtly, $y=tM\times$

log. cotan. $\frac{90^\circ - l}{2}$ — log. r.——The curve may be very eaſily conſtructed by the table of logarithmic tangents; and the preceding figure is exact, when the angle of the loxodromic is 45°.

CorOL. 1. Becauſe $\acute{e}\acute{a} : \acute{a}D' :: \dot{x} : \dot{y} :: \sqrt{rr+xx} : t$, therefore if through any point D' of the curve a line be drawn perpendicular to the equator AB', and the point F be taken therein, ſo that $D'F = G'C$, and a line be drawn through $F \parallel AB'$, on which let the point E be taken ſuch, that $FE =$ the tang. of the conſtant loxodromic angle, then a line drawn from E to D' will touch the curve in D'; from whence it follows that all loxodromics in this projection (except thoſe whoſe direction are north and ſouth or eaſt and weſt) are mechanical or tranſcendent curves, having a point of contrary flexure when they cut the equator.

CorOLLARY 2. Since $\dot{y} = \frac{t\dot{x}}{\sqrt{rr+xx}}$, the fluxion of the ſpace $AB'D'$ ($= x\dot{y}$) will be $= \frac{tx\dot{x}}{\sqrt{rr+xx}}$, and that ſpace itſelf $= t\sqrt{rr+xx} - tr = t \times GG'$.

CorOLLARY 3. Since $\dot{y}$ is found $= \frac{t\dot{l}\sqrt{rr+xx}}{rr}$, if $r\dot{m}$ be put $= \dot{l}\sqrt{rr+xx}$ or rm for the ſum of all the ſecants from 0 to l degrees, then will $\dot{y} = \frac{t\dot{m}}{r}$, or $ry = mt$, and therefore $r : t :: m : y$. But here m will be the meridional parts of the lat. l for the rad. r; whence the diff. of longitude found by Mercator's ſailing correſponds exactly with what is derived from the globe itſelf.

The ſolutions by Meſſ. *T. Allen*, *R. Butler*, *W. Embleton*, *C. Hutton*, *Da. Kinnebrook*, *Plus Minus*, *W. Spencer*, and *G. W.* agree with this ſo exactly, that, even if room would permit, it would be needleſs to repeat them here.

The prize of 12 *Diaries, for the ſolution of the prize queſtion, was won by Mr.* David Kinnebrook, *and that of* 8 *by Mr.* Charles Hutton.

The

The Eclipſes calculated for 1765.

There will happen ſix eclipſes this year, four of the ſun and two of the moon, according to the following order.

The firſt is an inviſible eclipſe of the ſun, on Tueſday the 19th of February, 22 m. paſt 11 at night, apparent time. ——It will be ſeen only in the unknown parts of the great South Sea.

The ſecond is a total and nearly central eclipſe of the moon, on Thurſday the 7th of March, 17 m. paſt one in the afternoon, and therefore inviſible in Europe, but will be viſible in the South Sea and Aſia.

The third is another inviſible eclipſe of the ſun, on Thurſday the 21ſt of March, 2 m. after 1 in the afternoon.—— This will appear a ſmall eclipſe in North America, and to places near the north pole.

The fourth is alſo of the ſun, on Friday the 16th of Auguſt, and will be viſible from the beginning to the end, according to the following calculations.

Calculated by	Beg.		Mid.		End		Dur.		Dig.	
	h.	m.	h.	m.	h.	m.	h.	m.	°	′
Mr. William Baylis, for Warwick	3	32	4	12	4	49	1	17	1	50
Amicus —— ——	3	39	4	19	4	57	1	18	1	46
Mr. T. Quanbrough, for London	3	29	4	5	4	44	1	15	1	35½
Mr. Stephen Hodges, for Althorp, near Northampton	3	52¼	4	24	4	57	1	4¾	1	25⅙
Mr. T. Walker, for London	3	37¼	4	16	4	54¾	1	17½	2	3
Mr. J. Metcalfe, for London	3	37¼	4	16	4	54¼	1	17½	2	3
Mr. T. Allen, fcr Spalding, by Maſkelyne's Tables and Conſtruction	3	35	4	5	4	49	1	14	1	40

The fifth is another total eclipſe of the moon, on Friday the 30th of Auguſt, 6½ m, paſt 2 in the afternoon, but inviſible in Britain, by reaſon it will be over before the moon riſes.

The ſixth and laſt is of the ſun, on Sunday the 15th of September, 35 m. after 4 in the morning, and conſequently inviſible here, and to all theſe parts of the world.

JOHN METCALFE.

New Queſtions.

I. QUESTION 535, *by Mr.* Tho. Sadler.

Declare ye wits, well known to fame,
A celebrated lady's name;
Whoſe equal's ſcarcely to be found
On this terraqueous globe around.
Mount-ſcience ſhe can ſcale with eaſe—
Aſcend its ſummit, if ſhe pleaſe.
She is the muſes' fond delight;
Who're always pleas'd when ſhe's in ſight.
Artiſts, with eaſe, her name you'll find
From th' equations here ſubjoin'd.

viz. $$\left.\begin{array}{l} w+x+y+z=31 \\ wz-x-y=58 \\ xy-w+z=22 \\ wy+xz\ \ =157 \end{array}\right\}.$$ Where w denotes the 1ſt letter in the alphabet, x the 2d, y the 3d, and z the 4th, that compoſe this lady's name; and the 5th letter is the ſame with the 2d, and the 6th the ſame with the 3d.

II. QUESTION 536, *by Mr.* W. Spicer.

To find two ſuch ſquare numbers, that their ſum may be a ſquare, and their difference a cube number, and the ſides of the ſaid ſquare and cube equal.

III. QUESTION 537, *by Mr.* W. Toft.

Given ($a =$) the ſum of 3 numbers in harmonical porportion, and alſo ($b =$) their continual product; to find the numbers themſelves.

IV. QUESTION 538, *by Mr.* Tho. Barker.

To determine a point within an equilateral plane triangle, whoſe perimeter is = 195, from which if right lines be drawn to the three angular points thereof, their ſum and the ſum of their ſquares ſhall be = 113½ and 4337¼ reſpectively.

V. QUESTION 539, *by Mr.* Paul Sharp.

In what latitude on the 21ſt of March at 9 o'clock, will the ſhadow of an object, ſtanding perpendicular to the horizon, be = to the ſhadow of the ſame when ſtanding in a poſition to the horizon, ſo as to caſt the longeſt ſhadow poſſible at 10 o'clock on the ſame forenoon.

VI. QUESTION 540, *by Mr.* Rob. Butler.

A gentleman has a triangular garden, whoſe ſides are 50, 60, and 70 poles reſpectively, in which reſpectively grow three trees *A*, *B*, and *C*, ſo poſited, that if right lines be drawn from tree to tree, a triangle will be ſo formed that the angles at *A*, *B*, and *C* will be = 50, 60, and 70 degrees reſpectively: Moreover the tree *A* (in the garden's leſſer ſide) is juſt 30 poles from the garden's greateſt corner. Hence is required the diſtance of the trees from each other (geometrically), and their ſituation in the ſides of the garden.

VII. QUESTION 541, *by* Jack Hazard.

Three perſons *A*, *B*, and *C* toſs up, in their turns, and upon equality of ſkill, a die with two equal faces only, whereof the one is white and the other black, and he that brings up the white face firſt, is to be entitled to a depoſit of 30 guineas: Required the value of each perſon's expectation.

VIII. QUESTION 542, *by* Curioſus.

An horizontal dial, brought from the coaſt of France in the late expedition (but without the gnomon), was put into my hands, and being deſirous of knowing for what latitude it was made, I found, by means of a pair of compaſſes, that the angle included between the hour lines of 12 and 3 was exactly = the angle comprehended between thoſe of 4 and 6; from which the latitude may be found, and is here required.

IX. QUESTION 543, *by Miſs* Ann Nicholls.

One night I made obſervation of an eclipſe of the moon, but clouds interpoſing, I could not ſee the beginning; however, ſome time after it cleared up, and at 9 h. 0′ that evening, I obſerved her to be exactly 6 digits eclipſed on her lower limb: At 9 h. 36 m. 22 s. the digits eclipſed were 10° 18′, and then clouds appearing again, prevented all further obſervation. By calculation I found the ſemidiameter of the moon = 16 m. her latitude at the middle = 30 m. north, and her horary motion from the ſun = 33 m.——From this curious obſervation, the beginning, middle, and end of this eclipſe are required.

X. QUESTION 544, *by Mr.* W. Toft.

A young man, at ſea, in working his day's work, forgot to allow for the variation of the compaſs, and by that means made his difference of latitude too much by 22, and the departure too little by 34 miles; and the diſtance ſailed that day was 100 miles in a direct courſe, in the north-weſt quarter: From whence is required the true difference of latitude and departure, and the nature and quantity of the variation.

XI. QUESTION 545, *by Mr.* T. Moſs.

To inveſtigate a general theorem (without finding the whole content) for determining the true ullage, in ale and wine gallons, of any ſtanding ſpheroidical caſk, whoſe bung, head diameters, length, and wet inches are given; and that by a rule fit for the practical gauger.

XII. QUESTION 546, *by* Lycidas.

In what point of the ecliptic between ♈ and ♋ does the ſun's longitude exceed his right aſcenſion by the greateſt difference poſſible?

XIII. QUESTION 547, *by Mr.* Paul Sharp.

There is a conical tub whoſe bottom diameter, depth, and top diameter are in arithmetical proportion, whoſe common difference is 12 inches, and which, when full of water, will empty itſelf through a circular hole, of one inch diameter, in its bottom, in 15·261584 minutes: Quere its content in ale gallons?

XIV. QUESTION 548, *by Mr.* T. Moſs.

Suppoſing there are two given concentric ellipſes, whoſe given difference of tranſverſe diameters is equal to that of their conjugates; 'tis required to demonſtrate, in a general manner, whether the part of any common diameter, intercepted between the peripheries of thoſe ellipſes, is greater or leſs than half the ſaid given difference of their tranſverſe or conjugate diameters.

XV. QUESTION 549, *by Mr.* Rob. Butler.

A chain's length is 140 feet, and its variable thickneſs (or law of denſity) ſuch, that if its ends are faſtened to two pins ſituated in the ſame horizontal line at the diſtance of 100 feet from each other, it will diſpoſe itſelf into a circular form; from whence is required the law of denſity and the chain's weight, ſuppoſing one foot of either end to weigh one pound.

The PRIZE QUESTION, *by Mr.* Da. Kinnebrook.

Suppoſe a given right-angled plane triangle, whoſe hypothenuſe is an uniform ſlender rod, and its baſe parallel to the horizon, to revolve about its perpendicular as an axis, whilſt a ring ſlides freely along its hypothenuſe: It is required to determine the time of the ring's deſcent along the ſaid hypothenuſe, its length being = 50 feet, the perpendicular = 40, and the time of one revolution round the axis = 3 ſeconds.

1766.

Queſtions anſwered.

I. QUESTION 535 *anſwered by Mr.* Rich. Gibbons.

FROM the addition of the two firſt equations is had the ſum of the two quantities w, z added to their product = 89; which (as they muſt be whole poſitive numbers, each not greater than 24, per queſt.), it is eaſy to perceive, will be 4 and 17, and conſequently $x + y = 10$ and $xy = 9$, from the 1ſt and 3d given equations: whence x evidently appears to be 9 and $y = 1$: and theſe are alſo equal to the 5th and 6th values required, per queſt. Conſequently the required numbers are 4, 9, 1, 17, 9, 1, and anſwer to *D, I, A, R, I, A*; the lady's name.

In this manner, exceeding near, Meſſ. *J. Bliſs*, *M. Gordon*, and *W. Rawle* have likewiſe anſwered it.

But if the four given equations are added together, &c. the following one will be derived, viz. $\overline{z+y} \times \overline{w+x} = 268 - 2z$: From which, and the firſt given equation $(z + y + \overline{w+x} = 31)$ will be found $\overline{z+y}|^2 - 2 \times \overline{z+y} \times \overline{w+x} + \overline{w+x}|^2 = 8z - 111$; and conſequently $z + y = 15\frac{1}{2} + \frac{1}{2}\sqrt{8z - 111}$.

Then, as w, x, y, and z are to be whole poſitive numbers, each not exceeding 24 (per queſt.), z muſt be ſuch a one as will bring out $\sqrt{8z-111}$ not only a whole, but alſo an odd number; which, with very little trouble, will be found to be 17: Whence $y=1$, and the 1ſt and 3d given equations become $x+w=13$ and $x-w=5$; conſequently $x=9$ and $w=4$, and the celebrated lady's name required is DIARIA.

It is alſo anſwered by Meſſ. *J. Aſhmore, T. Atkinſon, T. Barker, W. Barnes, T. Boſworth, W. Cole, J. Coulthred, Iſ. Dalby, R. Dening, H. Fry, W. Gough, R. Hale, R. Peckham, J. Potter, J. Probert, Tho. Robinſon, Alex. Rowe, W. Sewell, P. Sharp, Edw. Smith, W. Spicer,* Mrs. *Eleanor Suggett, J. Swift, J. Tarratt, J. Thorne, T. Wilkin, J. Woolley, J. Young,* and ſome others.

II. QUESTION 536 *anſwered by Mr.* William Spicer (*the Propoſer*).

Aſſume $\frac{1}{2}x^2+\frac{1}{2}x^3$ and $\frac{1}{2}x^2-\frac{1}{2}x^3$ for the two required ſquare numbers, whoſe ſum and difference are manifeſtly a ſquare and cube number, having each the ſame root x; then, finding two other ſquare numbers $4nn\times\overline{1+nn}|^{-2}$ and $\overline{1-nn}|^{2}\times\overline{1+nn}|^{-2}$, ſuch that their ſum may be $=1$, $\frac{1}{2}x^2+\frac{1}{2}x^3$ will be $=4nnxx\times\overline{1+nn}|^{-2}$ and $\frac{1}{2}x^2-\frac{1}{2}x^3=\overline{1-nn}|^{2}\times x^2\times\overline{1+nn}|^{-2}$, and conſequently, the equal root, $x=\overline{6n^2-n^4-1}\times\overline{1+nn}|^{-2}=$ (when n is $\frac{1}{2}$) $\frac{7}{25}$; which value ſubſtituted for x in the above aſſumed expreſſions, they become $\frac{784}{15625}$ and $\frac{441}{15625}$, two numbers anſwering the conditions of the queſtion.——Mr. *Wales* denotes them by $9xx$ and $16xx$ (it being evident, from 47 Euc. 1, that they will be in the ratio of 9 to 16); whence (their ſum and diff. being $=25xx$ and $7xx$ reſpectively) $5x$ will be $=\sqrt[3]{7xx}$ (per queſt.), and conſequently $x=\frac{7}{125}$, and the required numbers the ſame as before, &c.——Mr. *R. Gibbons'* anſwer is not eſſentially different from this laſt; and Mr. *C. Hutton* anſwers it, very ingeniouſly, by a different method.

III. QUESTION 537 *anſwered by Mr.* S. Woodbey.

Let x, y, and z denote the required numbers, and then, $x+y+z$ being $= a$, $xyz = b$ (per queſt.), and $x : z :: x-y : y-z$ (per the nat. of harmonic proportion), $y \times \overline{x+z}$ will be $= 2xz$, or $yy \times \overline{a-y} = 2b$; from the reſolution of which, the value of y, &c. may in any caſe be found.——If $a = 191$ and $b = 254016$, then will y, x, and z come out $=$ 72, 63, and 56 reſpectively.

And in this manner, nearly, the anſwer is given by Meſſ. *T. Barker*, *William Barnes*, *Iſaac Dalby*, *R. Gibbons*, *J. Probert*, *T. Robinſon*, *P. Sharp*, *W. Spicer*, *T. Wilkin*, and others.

But Mr. *W. Toft* (the propoſer) conceives the three numbers ſought to be repreſented by $z \times \overline{z+v}$, $\overline{z-v} \times \overline{z+v}$ and $z \times \overline{z-v}$ (viz. the numerators of three fractions) $\frac{1}{z-v}$, $\frac{1}{z}$, and $\frac{1}{z+v}$, having their denominators in arithmetic progreſſion, reduced to a common denominator, it being eaſy to prove them to be in harmonic proportion), and then, $zz \times \overline{zz-vv}|^2$ being $= b$, and $3zz - vv = a$ (per queſt.), $2z^3 - az + \sqrt{b}$ will be $= 0$; from whence the value of z, when b is a ſquare number, may be eaſily found, by the method of diviſors, and then v $(= \pm \sqrt{3zz-a})$, and the required numbers, will become known.——But when $\sqrt{b}$ is irrational, let it be divided by the greateſt ſquare number m^2 it will admit of, and the irrational quotient call n, and then the foregoing equation (ſuppoſing $x\sqrt{n} = z$) will become $2nx^3 - ax + m = 0$; from which x may be found, and conſequently the values of z and v, and the numbers required.

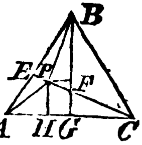
EXAMPLE 1. Let $a = 26$ and $b = 576$; then the 1ſt of the preceding equations becomes $z^3 - 13z + 12 = 0$; whence $z = 3$ and $v = 1$, and conſequently 12, 8, and 6 are the required numbers.

EXAMPLE 2. Where $a = 13$ and $b = 72$. Then $\sqrt{b} = 6\sqrt{2}$, $m = 6$, and $n = 2$, and the equation in the 2d caſe becomes $4x^3 - 13x + 6 = 0$: From which $x = \frac{3}{2}$, $z = \frac{3}{\sqrt{2}}$, $v = \frac{1}{\sqrt{2}}$, and 6, 4, 3 $=$ the three numbers required.

EXAMPLE 3. Where $a = 39$ and $b = 1944$.——Here $\sqrt{b} = 18\sqrt{6}$, $m = 18$, and $n = 6$, and the aforeſaid equation becomes $4x^3 - 13x + 6 = 0$, as in the laſt example, and, of conſequence, $x = \frac{3}{2}$ alſo: But $z = 3\sqrt{\frac{3}{2}}$; whence $v = \sqrt{\frac{3}{2}}$, and the required numbers are 18, 12, and 9.

SCHOL. The two foregoing general equations being of three dimenſions, z and x will each have three values therein, and v as many correſponding ones; as for inſtance, in example 1, where the values of z are 1, 3 and -4, and thoſe of v, correſponding, $\sqrt{-23}$, 1 and $\sqrt{22}$; from whence three ſets of terms, anſwering the conditions of the prob. may be found, if thoſe ariſing from the firſt and laſt (viz. from 1 and $\sqrt{-23}$, or from -4 and $\sqrt{22}$) of theſe correſponding values, can be called real quantities; and, what's ſomewhat odd, they, as well as the others, anſwer the conditions of the queſt. notwithſtanding they are abſolutely imaginary: But perhaps neither of them can ſo properly be called anſwers, ſeeing there is one in whole poſitive numbers, which the prob. in ſome ſort tacitly requires.

IV. QUESTION 538 *anſwered by Mr.* Iſaac Tarratt.



Let P repreſent the required point within the given equilateral plane $\triangle ABC$, and let the perpendiculars PE, PH, BG, upon AB and AC, be drawn; alſo let PF be $\perp BG$, and put $a = 32{\cdot}5 = AG$, $b = 56{\cdot}29 = BG$, $c = 113{\cdot}5$, $d = 4337{\cdot}25$, $h = -4112\frac{3}{4}$, $x = BF$, and $y = PF = HG$.

Then (by 47 Euc. 1) $\sqrt{xx + yy} + \sqrt{\overline{a-y}|^2 + \overline{b-x}|^2} + \sqrt{\overline{a+y}|^2 + \overline{b-x}|^2} = c$, and $3xx + 3yy - 4bx = h$; whence $y = \sqrt{\frac{4bx - 3xx + h}{3}}$; and this value ſubſtituted for y, an equation, containing one unknown quantity only, will emerge, from which x comes out $= 40$; and thence y is found $= 5{\cdot}6$ (fere), and the diſtances AP, BP, $CP = 31{\cdot}44$, $40{\cdot}16$, and $41{\cdot}43$, very near.

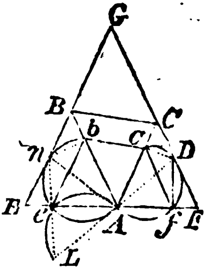
In this manner, exceedingly near, it is anſwered by *Amicus*, Meſſ. *T. Barker* (the propoſer), *R. Hale*, *J. Probert*, *Tho. Robinſon*, *Alex. Rowe*, *Edw. Smith*, *W. Spicer*, *E. Suggett*, and *Ja. Young*.

QUESTION 539 *anſwered by Mr.* Tho. Boſworth.

t $m, n =$ ſine and coſine of the declin. $p, q =$ coſines of .nd 30° (the ſun's diſtance from the merid. at 9 and 10 ck), $x, y =$ ſine and coſine of the required latitude; , per ſpherics, $\overline{npy + mx} \times \overline{1 - \overline{npy + mx}|^2}|^{-\frac{1}{2}}$ and $+ mx$ will be the tangent and ſine of the ſun's alt. at 1 10 o'clock reſpectively; which, whenever an appear- happens, at thoſe times, like what is mentioned in this tion, will (as is eaſy to demonſtrate) be equal to each r, or (when the ſun is in the line, and $m = 0$, which is caſe, very near, at the time ſpecified in the queſt.) $py \times \overline{ppyy}|^{-\frac{1}{2}} = qy$: From whence $y = \sqrt{\frac{1}{pp} - \frac{1}{qq}} =$ 6496, the coſine of 35° 15′ 52″ the lat. of the place re- ed; which is the ſame as found by Mr. *Paul Sharp* (the poſer) by a different method.

he ſolution by Mr. *W. Wales* is nearly the ſame as this; Meſſ. *W. Barnes*, Rev. Mr. *J. Croſs*, *J. Dalby*, *R. le*, *S. Hodges*, *C. Hutton*, *W. Rawle*, *Edw. Smith*, *W. cer*, *J. Tarratt*, *Joſ. Webſter*, and *Ja. Young*, have like- e anſwered it.

I. QUESTION 540 *anſwered by Mr.* Rob. Butler, (*the Propoſer*).



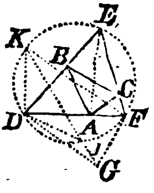
On a right line *Ab* (of any aſſumed length, at pleaſure) ıſtruct a plane triangle having its s *A*, *b*, *c* = 50°, 60°, and 70° re- ctively, and upon *Ab* and *Ac* de- ibe two ſegments of circles to con- n 78° 28′ and 57° 7′ (the garden's eateſt angles), and let the diameter *D* be drawn, producing it till *AD* *AL* in the given ratio of 2 to 3, and on *AL* deſcribe a ſemicircle cut- ıg the ſegment *Ab* in *e*; through hich point and *A* a right line being awn, meeting the circumf. *AD* in produce it both ways till *AF* = 20 id *AE* = 30 poles; join the points *b* and *f*, *c*, and parallel to thoſe lines let *EG* and *FG* neeting each other in *G*) be reſpectively drawn, producing *Ab*, *Ac* till they meet the ſame in *B* and *C*; then will *EGF* be the given triangle, and A, B, and C the places of the trees in the ſides thereof.

DEMONST. Join the points e, L; D, f; and B, C: Then (per ſim. triangles, &c.) $Af : Ae$ ($:: AD : AL :: 2 : 3$) $:: AF : AE$ (per conſtruction); alſo $\frac{Ab \times AE}{Ae} = AB$, and $\frac{Ac \times AE}{Ae} = AC$: Whence $AB : AC$ ($:: \frac{Ab \times AE}{Ae} : \frac{Ac \times AE}{Ae}$) $:: Ab : Ac$, and conſequently BC is ∥ Bc,* and the $\triangle\, ABC$ ſim. Abc; and the $\angle$s E, F being reſpectively $=$ the $\angle$s e, f (the given angles per conſtruction), and the ſide $EF =$ the given ſide (per conſtruction), EGF will be the given triangle, and A, B, C the true places of the trees in the ſides thereof.

CALCUL. Draw the diam. An, and let the points e, n; b, n; and c, D be joined; then (by the common caſes of plane trigonometry) AC will be found $= 29{\cdot}92$, $AB = 32{\cdot}46$, $BC = 26{\cdot}48$, $BE = 29{\cdot}89$, and $CF = 32{\cdot}46$ poles.

Anſwer to the ſame, by Mr. W. Wales.

CONSTRUC. Conceive the given $\triangle\, DEF$ to be circumſcribed by a circle $DKEF$, and upon its leaſt ſide DF, conſtitute a $\triangle\, DGF$ ſimilar to the required one. From the vertex E, to the given point A, draw the right line EA, and make the $\angle\, ADI = AEF$: Then, the interſection I, of the right line DI with the circumference and the point G being joined, make the $\angle$s EAC, EAB reſpectively $=$ the $\angle$s IGF, IGD, and, joining C, B (the points where the right lines AC, AB meet the given lines FE, DE), CAB will be the triangle, and A, B, C the places of the trees required.



DEMONST. Produce GI to meet the circumference in K, and let the points D, K and F, K be joined: then (per cor.

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* The paralleliſm of BC, Bc, may perhaps be proved more geometrically thus: Since, as is proved above, $Af : Ae :: AF : AE$, or $Af : AF :: Ae : AE$; but, by ſim. triangles, $Af : AF :: Ac : AC$, and $Ae : AE :: Ab : AB$, $\because Ac : AC :: Ab : AB$, and therefore BC ∥ bc.

to 9. 3. of Simp. Geom. 1ſt edit.) the $\angle$s DKF, DEF are equal; and, ſince the $\angle$s FDI, AEF are equal (per conſtruction), and FDI, FKI ſtand on the ſame arc FI, the $\angle$s FKI, AEF are equal: Whence the $\angle$s DKI, DEA are alſo equal; and, as EAC and IGF, EAB and IGD are equal (per conſtruction), the $\triangle$s ABE and GDK, ACE and GFK are ſimilar, and conſequently $AC : FG :: EA : KG :: AB : GD$; whence the $\triangle$s ABC, GDF, having their reſpective ſides proportional, muſt have their angles equal and be ſimilar.——The given point A being the point of contact of the ſide DF, with a circle inſcribed in the given $\triangle\ DEF$, a very ſimple conſtruction, in that particular caſe, may from thence be derived.

Meſſ. *R. Hale* and *C. Hutton* very eaſily reduce it to prob. 39 on pa. 244 of Simp. Geom. 2d edit.——And it may alſo be conſtructed from prob. 23 on p. 169 of his Geom. 1ſt edit.

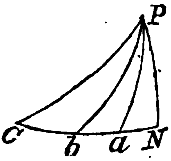
VII. QUESTION 541 *anſwered by Mr.* Ja. Hazard.

Put the given depoſit (30 guineas) $= d$; then, the probability of bringing up a white face at any one throw being conſtantly $\frac{1}{2}$, the expectation of A on the 1ſt throw will be $\frac{1}{2} \times d$, which would alſo be the expectation of B on the 2d throw, were it certain that A would miſs the 1ſt; but the brobability of that being alſo $\frac{1}{2}$, his expectation can only be $\frac{1}{2}$ of $\frac{1}{2}d$, or $\frac{d}{4}$: In like manner, the value of the expectation of C, on the 3d throw, will be found to be $\frac{d}{8}$; for, it would be $\frac{d}{2}$, if it was certain that A would miſs the 1ſt and B the 2d throw, but the probability of that being $\frac{1}{2} \times \frac{1}{2}$, his expectation thereon can only be $\frac{1}{2} \times \frac{1}{2}$ of $\frac{d}{2}$, or $\frac{d}{8}$: And by reaſoning in this manner, their expectations on the ſucceeding throws, and conſequently the values of their whole expectations (forming three infinite progreſſions) may be found, that of A's being $\frac{d}{2} \times : 1 + \frac{1}{8} + \frac{1}{8^2} + \&c. = \frac{d}{2} \times \frac{1}{1 - \frac{1}{8}} =$ 18l. that of B's $= \frac{d}{4} \times : 1 + \frac{1}{8} + \frac{1}{8^2} + \&c. = \frac{d}{4} \times \frac{1}{1 - \frac{1}{8}} =$ 9l. and that of C's $= \frac{d}{8} \times : 1 + \frac{1}{8} + \frac{1}{8^2} + \&c. = \frac{d}{8} \times \frac{1}{1 - \frac{1}{8}} =$ 4l. 10s. Or, the ſame concluſion may be derived without ſummation, by conſidering that, as their correſponding terms are every where in the conſtant ratio of 1, ½, and ¼, their ſums will alſo be in the ſame invariable ratio.

The anſwers by Meſſ. *T. Barker*, *T. Boſworth*, *C. Hutton*, *P. Sharp*, *W. Spicer*, and *W. Wales*, are not materially different.

VIII. QUESTION 542 *anſwered by Mr.* W. Wales.

In the ſpherical triangles PNa, PNb, and PNc, all right-angled at N, are given their reſpective angles at the pole $P = 45°$, $60°$, and $90°$, and the are $cb = aN$, to find PN, the lat. of the place, the ſine of which ſuppoſe $= x$: Then (per ſpherics) 1 (rad.) : x :: 1 (tang. $\angle NPa = 45°$) : x = tang. arc Na, and, putting t = tang. $\angle NPb = 60°$, $1 : x :: t : tx$ = tang. of arc Nb; whence its cotang. (or the tang. of the arc bc) is $\frac{1}{tx}$, which, per queſt. is $= x$, and conſequently $2xx = \frac{2}{t} = 1{\cdot}15475$, &c. the verſed ſine of 98° 52′, double the lat. required.



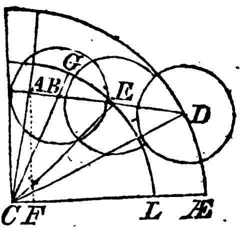
Mr. *Paul Sharp*'s anſwer is nearly the ſame as this.

Meſſ. *T. Allen*, *T. Barker*, *W. Barnes*, *T. Boſworth*, *J. Chipchaſe*, *J. Dalby*, *R. Hale*, *C. Hutton*, *J. Probert*, *W. Rawle*, *T. Robinſon*, *Alex. Rowe*, *W. Spicer*, *J. Tarratt*, *Joſ. Webſter*, *Tho. Wilkin*, and *J. Young*, by eaſy ſhort proceſſes, find $x = t^{-\frac{1}{2}} = {\cdot}7598357$ the ſine of 49° 26′ 59″, the lat. required; and Mr. *John Potter*, by analogy, without algebra, very ingeniouſly derives the ſame concluſion.

It may alſo be anſwered another way, independent of algebra, whereby the ſine of the arc ab comes out = ſine of 15° ÷ ſine of 105°; from whence the lat. the dial was made for may be readily found.

IX. QUESTION 543 *anſwered by Mr.* T. Allen.

Let $ABED$ and $CFÆ$ repreſent the portions of the lunar orb and the ecliptic, E, B the places of the ☽'s cent. at the 1ſt and 2d obſervations, and AF (= 30′) her lat. at the middle of the eclipſe. Then, the $\angle CAF$ being found =



5° 15′ (from Aſtron. Tables), CA will be $= 30·1517'$, which put $= a$, $AB = x$, BE ($= 20'$) $= c$, and $CE - CB$ ($= 11·46'$) $= n$; then will $\sqrt{aa + xx} = CB$, $\sqrt{aa + \overline{c+x}^{2}} = CE = n + \sqrt{aa + xx}$, and (putting $cc - nn = 2m$), $x = \sqrt{\frac{cc}{4} + \frac{nnaa}{2m}} - \frac{m}{2} - \frac{c}{2} = 11·84'$: Whence $AE = 31·84'$, CE (the ſemidiameter of the ⊖'s ſhad.) $= 43·85'$, and the meaſure of the ſemiduration $AD = 51·7'$; and thence are eaſily deduced 8h. 23m. 54s. 9h. 57m. 54s. and 11h. 31m. 54s. for the beginning, middle, and end of the eclipſe, and 11° 8′ 12″ = the greateſt quantity eclipſed.

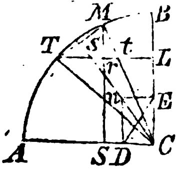
The ſame anſwered by Mr. Wm. Wales.

As 60′ : 33′ (the given hor. mot) : : 36′ 22″ (the time between the obſerv.) : 20′ $= EB = a$; alſo 6 dig. : 16′ (the ☽'s ſemidiameter) : : 4 dig. 18′ : 11·466′ &c. $= BG = b$; whence, putting $x = GE$ (the ſemidiam. of the earth's ſhad.), $a : 2x - b :: b : \frac{2bx - bb}{a}$, half of which = the diſt. of CA ($\perp DA$ and = 30′, the given lat. of the ☽ at A, the middle of the eclipſe) from the middle of the baſe BE of the triangle BCE; whence (per 47 Euc. 1) $xx - \frac{\overline{aa + 2bx - bb}^{2}}{4aa} = 900$ (CA^2), and conſequently $x = 43·6965'$, and $EA = 31·7708'$: Then 33′ (horary motion) : 60′ : : 31·7708′ : 57′ 46″, which added to the time of the 1ſt obſervation, gives 9h. 57m. 46s. for the middle. Alſo (per 47 Euc. 1) $DA =$ $= 51·6107'$ = (in time) 1h. 33m. 50s. which ſubtracted from and added to the middle, gives 8h. 23m. 56s. and 11h. 31m. 36s. for the beginning and end reſpectively.

Meſſ. *W. Barnes*, *T. Boſworth*, *R. Hale*, *C. Hutton*, *Rob. Peckham*, *W. Spicer*, and *Joſ. Webſter* have alſo anſwered it very ingeniouſly, and bring out nearly the ſame concluſion.

X: QUESTION 544 *anſwered by Mr.* Cha. Hutton.

CONSTRUC. About the center C with rad. CA ($= CB \perp CA$) = 100 (the diſt. ſailed) deſcribe the quad. AB, and take



take CD, CE thereon, reſpectively $= 22$ and 34 (the given err. in lat. and depart.), and let the points D, E be joined; thro' the center C and the middle of DE draw the right line Cs, and parallel thereto, and at the diſt of $\frac{1}{2}DE$ therefrom, conceive two other right lines to be drawn interſecting the arc AB in T and M; join the points C, T and C, M, and draw $TL \parallel AC$, meeting CB in L; then is the $\angle TCB$ the meaſure of the *true* and MCB the *erroneous* courſe, and the $\angle TCM$ the variation, which is weſterly; CL is the *true* diff. of lat. and TL the departure.——For, joining the points T, M, and drawing $MS \parallel BC$, cutting TL in r, and compleating the $\square\, DCEn$, the $\triangle$s TMr and DCE or DCn are equal and ſimilar, becauſe of the equal and perpendicular hypothenuſes TM, Cn, and the paralleliſm of the legs, *i. e.* $Tr = CE = 34$, and $Mr = DC = 22$, as by the queſt. they ought.——The calculation from this conſtruction is extremely eaſy; whereby the $\angle TCM$ ($=$ variat.) and CL, TL ($=$ *true* diff. of lat. and depart.) come out $= 23° 22'$, 71·217898 and 70·199816 miles reſpectively.

Meſſ. *T. Boſworth*, *W. Toft* (the propoſer), and *W. Wales* likewiſe conſtruct it in this manner, very near.

Meſſ. *T. Allen*, *T. Barker*, *W. Barnes*, *W. Cole*, the Rev. *J. Croſs*, *J. Dalby*, *R. Gibbons*, *R. Hale*, *J. Probert*, *T. Robinſon*, *Alex. Rowe*, *P. Sharp*, *W. Spicer*, Mrs. *Eleanor Suggett*, *Tho. Wilkin*, and *Ja. Young have likewiſe ſent neat and conciſe algebraic ſolutions.*

XI. QUESTION 545 *anſwered by Mr.* Joſeph Walker, *Superviſor in the Exciſe.*

Let $h =$ the head diameter, $b =$ the bung diameter, $l =$ the length, and $w =$ the wet-inches of the propoſed caſk, (all which are given); then will $lb \div \overline{\sqrt{bb - hh}} =$ the tranſverſe of the generating ſemiellipſe, and $\frac{1}{2} lb \div \overline{\sqrt{bb - hh}} - \frac{1}{2} l =$ the part thereof intercepted between the vertex of the ellipſe and the middle of the head diameter of the caſk; and conſequently, putting $a = {}^{\cdot}7854$ and $x =$ any variable depth, eſtimated from the center of the bottom up the axis, the fluxion of the content correſponding will be $= a\dot{x} \times \overline{hh + \frac{bb - hh}{\frac{1}{4}ll} \times \overline{l - x} \times x - \frac{1}{4}ll}$; and the fluent, viz.

$aw \times \overline{bb - \frac{bb-hh}{\frac{1}{4}ll} \times \overline{\frac{1}{4}ll - \overline{\frac{1}{2}l - \frac{1}{3}w} \times w}}$, when $x = w$, divided by 282 for ale, or 231 for wine, gives the ullage, in ale or wine gallons, required; and, expreſſed in words, furniſhes the following

GENERAL RULE.

Multiply the difference between the ſemi-length and $\frac{1}{3}$ the wet inches by the wet inches, and ſubtract the product from the ſquare of the ſemi-length, and multiply the remainder by the rectangle under the ſum and difference of the bung and head diameters divided by the ſquare of the ſemi-length; then, ſubtracting this laſt product from the ſquare of the bung diameter, and multiplying the remainder by the wet inches divided by 359·65 for ale or 294·11 for wine, the reſult will be the ullage, in ale or wine gallons, required.

Ingenious anſwers to this queſtion (with practical rules deduced therefrom) have alſo been received from Meſſ. *T. Allen*, *R. Gibbons*, *C. Hutton*, and *Barth. Sikes*; but our ſcanty room will not permit us to inſert them.

The anſwer by Mr. *T. Moſs* (the propoſer) may now be ſeen in his excellent Treatiſe of Gauging, lately publiſhed.

XII. QUESTION 546 *anſwered by Mr.* T. Allen.

Put $s =$ coſine of 23° 29′, the obliquity of the ecliptic, $x =$ tang. of the ☉'s right aſcenſion to rad. 1; then, per ſpherics, $\frac{x}{s} =$ tang. of the longitude, and $\frac{x - sx}{s + xx}$ (per pa. 55 of Simp. Trigo.) $=$ tang. of their diff. which, per queſt. is to be a max. and therefore its flux. made $= 0$, and reduced, gives $x = \sqrt{s} = {}^{\cdot}957693$ the tang. of 43° 45′ 43″: Whence their greateſt diff. = 2° 28′ 35″, and the ☉'s place will be ♉ 16° 14′ 16″.

Meſſ. *T. Barker*, *T. Boſworth*, *W. Barnes*, *J. Dalby*, *R. Hale*, *Step. Hodges*, *J. Potter*, *J. Probert*, *T. Robinſon*, *A. Rowe*, *P. Sharp*, *W. Spicer*, *J. Tarratt*, *W. Wales*, *T. Wilkin*, and *J. Young* anſwer it in this manner, and bring out the ſame concluſion, exceedingly near.

Aſtronomicus, by trigonometric reaſoning only, finds the ſine of the diff. of the ☉'s long. and his right aſcenſion = the right angle under the ſine of their ſum and the verſed

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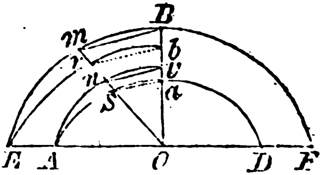
TION 548 *anſwered by Mr.* T. Moſs (*the Propoſer*).

BF be two concentric ſemi-ellipſes, in which FD. Take the given o OA, and $OB : OE$, the ellip- v, reſpec- o Aa, EB, Draw the m, cutting liptic arcs in d let the points r, b; n, v; and s, a be joined; ly) $Ob : Oa :: OE : OA$ and $OB : Ov ::$ hence, by equality and diviſion of ratios, &c. $b :: Ov - Oa : Oa$, or, alternately, Bb : $a) :: OB : Ov$, by equal. from above. Alſo, c. ariſing from the ſimilarity of the ellipſes,



$$\left.\begin{array}{l} :: OB : Ov\ (:: OE : OA, \\ \text{ſition}) \\ Ob : Oa\ (:: OE : OA,\ \text{by} \\ \text{on likewiſe}) \end{array}\right\}\ \text{and}\ Ov : Bv ::$$

hence, by equality, diviſion of ratios, and al- $r : ns$ ($:: Or : Os :: Ob : Oa$) and ſo is Bb : lity, from above): Conſequently the flux. (or de- oſing Om to be increaſing and indefinitely near r. is to the flux. of ns in the conſtant ratio of nd the flux. of Om to the flux. of On as OB : , if the fluents of two variable quantities are conſtant ratio, their fluxions muſt neceſſarily be e ſame ratio); that is, alternately,

$$\left.\begin{array}{l} \text{of}\ mr :: va : \text{flux. of}\ ns, \\ \text{of}\ Om :: Ov : \text{flux. of}\ On, \end{array}\right\}\ \text{and, alternately}$$

al. from above, &c. $Ov : va ::$ flux. of $On : \frac{va}{Ov}$ $n =$ flux. of ns; alſo, for the ſame reaſon, Ov : mn, from above) :: flux. of $On : \frac{Bv}{Ov} \times$ flux. ux. of mn; which, as Bv is always $\sqsubset va$, demon- proves the increaſe of mr to be always $\sqsubset$ the de- ns, and conſequently that ms is every where EA (becauſe, if the ſaid increaſe and decreaſe ys equal, ms would be a conſtant quantity $= Ba$, decreaſe was greateſt, ms would be $\sqsupset Ba$; as is ent).

ſine of the obliquity of the ecliptic directly, and the verſed ſine of the ſupplement of the obliquity of the ecliptic inverſely, which, when a max. will evidently become = the rectangle under rad. and the verſed ſine of the obliquity of the ecliptic directly, and the verſed ſine of the ſupplement thereof inverſely, = 0·043201, the ſine of $2° \, 28\frac{1}{4}'$ nearly; from which the ſame concluſion as above exhibited may be found, very near.

XIII. QUESTION 547 *anſwered by* Mr. P. Sharp (*the Propoſer*).

Put $a = 16\frac{1}{12}$ feet, $b = 0·005454$ (the area of the circular hole), $c = 3·8197$, and $x =$ the depth of the tub, in feet; then will $x+1 =$ top, and $x-1 =$ bottom diameter, $\frac{3x^3+x}{c} =$ its content, and $\frac{3x^3+x}{bc\sqrt{2ax}} =$ the time of evacuation with the 1ſt velocity (per pa. 173 of Emer. Mechan. 1ſt dit.): Whence $\frac{3x^3+x}{bc\sqrt{2ax}} \times \frac{30xx-20x+14}{15xx+30x+15} = 15·261584 \times 60 = 915.69504''$ (per pa. 111 of Emer. Flux. 1ſt edit.); from which x is found = 4 feet, and the top and bottom diameters = 5 and 3 reſpectively, and the content = 51·31293 ſolid feet = 314·42767 ale gallons.*

Meſſ. *T. Allen* and *C. Hutton*'s anſwers are nearly the ſame.——It is alſo anſwered by Meſſ. *W. Barnes*, *W. Rawle*, *Tho. Robinſon*, *Alex. Rowe*, *Edw. Smith*, and *W. Spicer*.

XIV. QUES-

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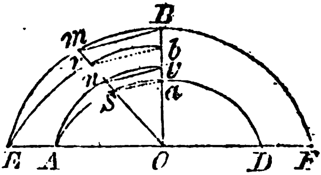
* This ſolution is falſe, by the wrong application of the theor. in Emerſ. Flux. above quoted. The concluſion will be very different when that theor. is properly applied, as any one may eaſily find. But the ſame will be more eaſily and directly deduced from the 1ſt Art. of our Miſcel. cor. 1. pa. 7, thus: Uſing the ſame notation as above, we have $\frac{2\sqrt{x}}{b\sqrt{2a}} \times ·7854 \times \frac{8.\overline{x-1}|^2 + 4.\overline{x-1}.\overline{x+1} + 3.\overline{x+1}|^2}{15} = \frac{288\sqrt{x}}{\sqrt{32\frac{1}{6}}} \times \frac{15xx-10x+7}{15} = 915·69504$; or $\sqrt{x} \times \overline{xx - \frac{2}{3}x + \frac{7}{15}} = 18·0833$; Hence the length x is $= 3·4026$ nearly; and therefore the top and bottom diameters $= 4·4026$ and $2·4026$.

XIV. QUESTION 548 *anſwered by Mr.* T. Moſs (*the Propoſer*).

Let AaD, EBF be two concentric ſemi-ellipſes, in which $Ea = Ba = FD$. Take Ob to OE in the given ratio of Oa to OA, and Ov to $OA :: OB : OE$, and conceive the elliptic arcs Eb, Av, reſpectively ſimilar to Aa, EB, to be deſcribed. Draw the ſemidiameter Om, cutting the ſimilar elliptic arcs in s, n, and r, and let the points r, b; n, v; and s, a be joined; then (alternately) $Ob : Oa :: OE : OA$ and $OB : Ov :: OE : OA$; whence, by equality and diviſion of ratios, &c. $OB - Ob : Ob :: Ov - Oa : Oa$, or, alternately, $Bb : va$ $(:: Ob : Oa) :: OB : Ov$, by equal. from above. Alſo, by ſim. △s, &c. ariſing from the ſimilarity of the ellipſes,



$$\left.\begin{array}{l} Om : On :: OB : Ov \ (:: OE : OA, \text{ by ſuppoſition}) \\ Or : Os :: Ob : Oa \ (:: OE : OA, \text{ by ſuppoſition likewiſe}) \end{array}\right\} \text{and } Ov : Bv ::$$

$On : mn$; whence, by equality, diviſion of ratios, and alternation, $mr : ns$ $(:: Or : Os :: Ob : Oa)$ and ſo is $Bb : va$ (by equality, from above): Conſequently the flux. (or decreaſe, ſuppoſing Om to be increaſing and indefinitely near to OB) of mr. is to the flux. of ns in the conſtant ratio of Bb to va, and the flux. of Om to the flux. of On as $OB : Ov$ (becauſe, if the fluents of two variable quantities are always in a conſtant ratio, their fluxions muſt neceſſarily be always in the ſame ratio); that is, alternately,

$$\left.\begin{array}{l} Bb : \text{flux. of } mr :: va : \text{flux. of } ns, \\ OB : \text{flux. of } Om :: Ov : \text{flux. of } On, \end{array}\right\} \text{and, alternately}$$

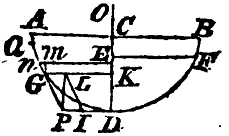
and by equal. from above, &c. $Ov : va ::$ flux. of $On : \dfrac{va}{Ov} \times$ flux. of $On =$ flux. of ns; alſo, for the ſame reaſon, $Ov : Bv$ $(On : mn$, from above$) ::$ flux. of $On : \dfrac{Bv}{Ov} \times$ flux. of $On =$ flux. of mn; which, as Bv is always $\sqsubset va$, demonſtratively proves the increaſe of mn to be always $\sqsubset$ the decreaſe of ns, and conſequently that ms is every where $\sqsubset Ba$, or EA (becauſe, if the ſaid increaſe and decreaſe were always equal, ms would be a conſtant quantity $= Ba$, and if the decreaſe was greateſt, ms would be $\sqsupset Ba$; as is very evident).

The certainty of this may alſo in ſome meaſure be evinced by conſidering the ray *Om* as revolving about the center *O*; it being eaſy to perceive that its intercepted part *sm* will continually vary, as the inclination to the axis *EF* varies, and conſequently that it muſt paſs through a max. or a min. before it can become of the ſame magnitude again, in *E*, as it was of, at firſt, in *B*.——But it is very eaſy to demonſtrate that it will begin to increaſe immediately on its departure from the poſition *OB*, and be greater than *aB* the moment before its coincidence with *AE* (= *aB*).——It therefore muſt paſs through a max. between the poſitions *OB*, *OE*, and be every where greater than its two extreme equal values *aB*, *AE*.

Mr. *T. Allen* has given an algebraic ſolution to it.

XV. QUESTION 549 *anſwered by Mr.* Cha. Hutton.

Call the abſciſſa *DE*, *x*; the ord. *EF*, *y*; the arc *DF*, *z*; and its tangent *t*: alſo call the tangent of the whole arc *DB*, *T*. ——Now, in any curve formed by a perfectly flexible line, the tenſion or force drawing in direction of the tang. at *D* (the loweſt point) is a conſtant quantity, which let be denoted by *a*. Then, the general prop. of any ſuch curve being as 1 (rad.) : tang. ∠*EFD* : : *a* : weight of the arc *DF*, or (becauſe the angle at *F* is, in the circle, meaſured by the arc *DF*) 1 : *t* : : *a* : *at* = weight of the arc *DF*, and 1 : *T* : : *a* : *aT* = weight of the arc *DB*, *aT* — *at* will be = 1 lb. (per queſt.), ſuppoſing *DF* = 69 feet; whence $a = \overline{T-t}^{-1}$, and conſquently $2aT = 2T \div \overline{T-t} =$ weight of the whole chain *ADB*.



Now, ſince the arc *DB* is given = 70, and the ſine *CB* = 50, the rad. *OD* will be found = 50·99397, and the length of the ſemi-circumference = 160·20266; whence the arcs *DB* (70) and *DF* (69) appear to contain 78° 39′ and 77° 31′ 36″, and their tangents *T*, *t* to be = 4·9818813 and 4·5210821 reſpectively, and $2T \div \overline{T-t} = 21{\cdot}6227$ lb. = weight of the whole chain required——The law of denſity, or of the ſection of the chain, is directly as the flux. of the tang. and inverſely as the flux. of the arc; for $a\dot{t}$ = flux. of the weight = weight of the length $\dot{z}$; which, divided by the length $\dot{z}$, gives $a\dot{t}\dot{z}^{-1}$ = the denſity or ſection at any point.

COR. 1. The law of denſity is as $a\dot{t}\dot{z}^{-3}$ in all curves; for the manner of finding it is general, ſuppoſing t to be the tang. of the angle in which the curve cuts the ordinate.—— In the catenary, which is formed by a line of a conſtant thickneſs, or whoſe law of denſity is a conſtant quantity, in order to have it as ſuch, in the general expreſſion, $\dot{t}$ muſt be as $\dot{z}$, and thence t as z; which is a known property of that curve.

COR. 2. The arc DB can never become a quadrant; becauſe then the denſity at B would be infinitely great.

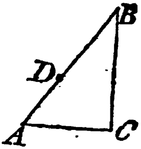
SCHOL. Hence may be eaſily deduced the requiſites for building the arches of bridges of any known curvature deſirable, either for elegance or to ſuit the ſituation of the place, &c. and yet be equally ſtrong in all their parts, as if they were catenarian arcs.

Anſwer to the ſame by Mr. Rob. Butler (*the Propoſer*).

Draw the ordinate GK, and, perpendicular thereto, PL, from P, the point of interſection of the tangents DP, GP; alſo draw nm, mG, and $LI \parallel GK$, KD, and GP reſpectively, and put the abſciſſa $DK = x$, ord. $GK = y$, $w =$ weight of the portion DG, and $a =$ the invariable tenſion at D (the loweſt point): Then, n being indefinitely near G, $Gm = \dot{x}$ and $nm = \dot{y}$, and, by mechanics, GP, GL, and PL will be as the tenſion at G and D, and weight of the part DG reſpectively: Whence $w : a\ (:: PL : GL) :: \dot{x} : \dot{y}$, and conſequently $\frac{w}{a} = \frac{\dot{x}}{\dot{y}}$; which is general, let the curve be what it will: But, in the preſent caſe, the curve being a circle (whoſe diam. ſuppoſe $= 2r$), $\frac{\dot{x}}{\dot{y}}$ will be $= \frac{y}{r - x} = \frac{w}{a}$; that is, (the weight ($w$) will be always as the tang. of its correſponding arc DG: Therefore, putting T, $t =$ the tang. of 78° 39′ and 77° 32′, the degrees, &c. contained in the arcs AD (70) and QD (69 feet) reſpectively, $T : t :: w : w - 1$ (per queſt.); whence $w = \frac{T}{T - t} = 10{\cdot}86$ lb. which doubled gives 21·72 lb. = the whole weight of the chain required.——Ingenious anſwers to it have likewiſe been received from Meſſ. *J. Chipchaſe* and *W. Spencer*, who, by different methods, find the weight, the law of denſity or thickneſs, and the diameter in the middle and at each end.

The PRIZE QUESTION *anſwered by Mr.* W. Spencer, *of* Stannington, *near* Sheffield, *in* Yorkſhire.

Let BA repreſent the given rod, revolving about the axis BC perpendicular to the horizontal line AC, D the place of the ring at the end of any time t, and v its velocity, there, in the direction DA.



Put AB (= 50 feet) $= r$, BC (= 40) $= p$, AC (= 30) $= b$, $32\frac{1}{6} = g$, $3''$ (the time of one revolution) $= s$, $3{\cdot}1416$ &c. $= \frac{1}{2}c$, and $BD = x$; then will $\frac{bccx}{rss}$ = the centrifugal force in a direction parallel to the horizon, and its effect in the direction DA (by the reſolution of forces) $= \frac{bbccx}{rrss}$; which added to $\frac{pg}{r}$, the true meaſure of the force of gravity in the ſame direction, gives $\frac{bbccx}{rrss} + \frac{pg}{r}$ = the whole force accelerating the ring along the rod, at D: Whence, by the principles of motion, $\frac{bbccx\dot{x}}{rrss} + \frac{pg\dot{x}}{r} = v\dot{v}$, and (putting $\frac{bbcc}{rrss} = m$, and $\frac{pg}{r} = n$) $v = \sqrt{mxx + 2nx}$; conſequently $\dot{t} = \frac{1}{\sqrt{m}} \times \frac{\dot{x}}{\sqrt{2qx + xx}}$, putting $q = \frac{n}{m}$; and, taking the correct fluent $t = \frac{1}{\sqrt{m}} \times$ hyp. log. of $\frac{q + x + \sqrt{2qx + xx}}{q}$ = (when x becomes $= BA = 50$) $1{\cdot}6559''$ &c. $= 1''\ 39'''\ 21''''$ &c. the time required.

COR. 1. The time of deſcent along the rod in motion, is leſs than the time of deſcent along it when at reſt, by $18'''\ 51''''$ &c.

COR. 2. When x becomes $= \frac{{}^{*}grss}{pcc}$, the ring, if at free liberty, would no longer ſlide along the rod, but fly off.

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* Found by making $\frac{bg}{p}$ (the grav. of the ring on the rod) $= \frac{bccx}{rss}$ the horizontal centrifugal force above found.

In this manner, exceedingly near, the anſwer is given by Meſſ. *T. Allen*, *R. Butler*, *J. Chipchaſe*, *Curioſus*, and *C. Hutton*; but the prize of 12 Diaries was won by Mr. *William Spencer*, and that of 8 by *Curioſus*.——Mr. *Da. Kinnebrook* has anſwered the 6th, 7th, 8th, 15th, and his own queſt. (the prize), but his letter came to hand too late to be taken any further notice of.

The Eclipſes calculated for 1766.

This year will produce four eclipſes, two of which only will be viſible in this iſland.

The firſt is of the ſun, on Sunday the 19th of February, a little after noon: It will be inviſible here, but annular and central in the ſouthern parts of the Pacific and Southern Oceans, and may be ſeen from the moſt ſouthern parts of South America, Africa, Madagaſcar, and other Indian iſlands.

The ſecond is a partial and viſible lunar eclipſe, on Monday the 24th of February, in the evening, according to the following calculations.

Calculated by	Beg.		Mid.		End		Dur.		Dig.	
	h.	m.	h.	m.	h.	m.	h.	m.	°	'
Mr. W. Chapman, for Foxton, Leiceſterſhire	6	$32\frac{3}{4}$	7	$40\frac{1}{4}$	8	$47\frac{3}{4}$	2	15	4	6
Mr. S. Hodges, for Althorp, Northamptonſhire	6	$13\frac{1}{2}$	7	$21\frac{1}{2}$	8	$29\frac{2}{3}$	2	$16\frac{1}{2}$	4	$10\frac{1}{3}$
Mr. Iſaac Tarratt, for London	6	$26\frac{5}{6}$	7	$36\frac{1}{2}$	8	43	2	$16\frac{1}{8}$	4	$6\frac{5}{6}$
Mr. Tho. Harris, for Brington	6	$13\frac{1}{2}$	7	$21\frac{2}{3}$	8	$29\frac{5}{6}$	2	$16\frac{1}{2}$	4	$8\frac{3}{4}$
Mr. J. Metcalfe, for London	6	$38\frac{1}{4}$	7	$46\frac{5}{6}$	8	$55\frac{1}{8}$	2	$16\frac{7}{8}$	4	$12\frac{1}{4}$
Mr. Allen, for Spalding, Lincoln.	6	40	7	37	8	34	1	54	4	10

The moon will be vertical at the middle of this eclipſe to the Indian Ocean, between the iſland of Ceylon and eaſt part of Abiſſinia, in Africa; it will therefore be viſible to all Europe and Aſia, and greateſt part of Africa, from beginning to end; and at the end may be ſeen in the Brazils, in South America.

The third is an eclipſe of the ſun, on Tueſday the 5th of Auguſt, in the afternoon, correſpondent to theſe computations.

Calculated by	Begin.		Mid.		End		Dur.		Dig.	
	h.	m.	h.	m.	h.	m.	h.	m.	°	′
Mr. W. Chapman, for Foxton	5	$24\frac{1}{3}$	6	$17\frac{5}{6}$	7	$7\frac{1}{2}$	1	$43\frac{1}{6}$	5	4
Mr. Steph. Hodges, for Althorp	5	$7\frac{1}{3}$	6	$20\frac{1}{4}$	7	9	1	$43\frac{1}{6}$	4	$33\frac{3}{4}$
Mr. Iſaac Tarratt, for London	5	28	6	$22\frac{1}{2}$	7	$10\frac{5}{6}$	1	$42\frac{5}{6}$	4	$35\frac{1}{8}$
Mr. Tho. Harris, for London	5	$30\frac{5}{6}$	6	$23\frac{1}{2}$	7	$12\frac{1}{2}$	1	$41\frac{2}{3}$	4	$36\frac{1}{2}$
Mr. T. Allen, by conſtruc. for Spalding	5	$25\frac{1}{2}$	6	21	7	$8\frac{1}{2}$	1	43	4	24
Mr. J. Metcalfe	5	$25\frac{1}{2}$	6	$18\frac{1}{2}$	7	$8\frac{1}{4}$	1	$42\frac{3}{4}$	4	$37\frac{1}{2}$

This ſolar eclipſe may be ſeen from all parts of North America, the Atlantic Ocean, north and weſt parts of Africa, weſtern parts of Europe, and to parts near the north pole.

The fourth and laſt is of the moon, on Wedneſday the 20th of Auguſt, in the morning, and viſible in America. This clipſe will be ended before the moon riſes at London, and conſequently will be inviſible there, and to all the weſtern parts of Europe; but to all parts of Aſia, moſt parts of Africa, and eaſtern parts of Europe, it will be viſible. The digits eclipſed will be a little more than 6 on the moon's lower limb. J. METCALFE.

We have been favoured with other calculations of the eclipſes, and particularly from Meſſ *T. Atkinſon*, *W. Baylis*, *Ed. Greenſted*, *Rob. Peckham*, *W. Swift*, and *Joſ. Webſter*; which we are ſorry our narrow bounds will not ſuffer us to publiſh.

New Queſtions.

I. QUESTION 550, *by Mr.* Tho. Sadler.

Ye fair, who can with eaſe unfold,
What puzzled Oedipus of old,
And can, from algebraic art,
Th' abſtruſeſt of all things impart;
From what you ſee appear below,*
John's age and fortune you will know;
Who courts young Suſan of the Mill,
(But ſhe is more in love with Will.)
Will's young and ſpruſe, but hath no ſtore—
No mouldy ſterling to count o'er—
John boaſts of gold, and more than that,—
Which makes Sue's heart go pit a-pat:

She begs the ladies' kind advice,
Were they to chuſe, and take their choice,
Whether 'tis beſt, to marry John,
With all his gold,—or Will, with none.

* $x+y=152$, and $\overline{x-y}^{\frac{2}{3}} \times \overline{x-y}^{\frac{3}{2}} = 8192$; where y $=$ John's age, and x his fortune.

II. QUESTION 551, *by Miſs* Ann Nicholls.

I obſerved a cloud bearing N. E. by E. and took its altitude 17° 4′; ſome time after I obſerved the ſame cloud S. E by S. at which time its altitude was 45° 23′; from whence the point the wind blew from is required.

III. QUESTION 552, *by Mr.* Iſaac Tarratt.

A pleaſant village, ladies, is defin'd
From the equation underneath ſubjoin'd,*
Whoſe pleaſing bowers and delightful ſhades
Are far ſuperior to the woodland glades.
To theſe retreats the merry nymphs will hye,
With ſudden tranſports of unbounded joy!
And meet their ſwains, the ſhepherds of the plain,
Who tune their pipes with a melodious ſtrain:
The choriſters will liſten there to hear
What Damon ſays to his beloved fair.
My reſidence, dear ladies, pray explore,
And you'll oblige your ſervant evermore.

* $$\left.\begin{array}{l} v+w+x+y+z=64, \\ v^2+w^2=5\times\overline{x+y+z}+30, \\ vwx=8yz+6, \\ x^2+yz=490, \\ v+w+z=x+y. \end{array}\right\}$$ Where v, w, x, y, and z repreſent the places of the letters in the alphabet, that compoſe the town's name.

IV. QUESTION 553, *by Mr.* Wm. Spicer.

There is a conical ſpire ſteeple, whoſe ſlant ſide is $=$ $=4$ times its baſe; from the vertex of which, two heavy bodies were let fall at the ſame time, the one down the perpendicular, and the other along the ſlant ſide: Now it was obſerved that the ſound of the body which fell down the perpendicular upon the center of the baſe, arrived at the extremity of the ſlant ſide juſt at the ſame moment of time with the other body; required the ſteeple's altitude?

V. QUESTION 554, *by Mr.* Stephen Hodges.

Near the renown'd Lord Viſcount Spencer's ſeat,
Where ſhady groves project a cool retreat,
Two children at one birth appear'd in view;
From theſe equations * pray their ages ſhew.

$$* \left.\begin{aligned} &xy^2 + z^3v^3 = 290304, \\ &x^2u + x^3y^4 = 12521472, \\ &y = u,\ z = v, \text{ and } v = 2y; \end{aligned}\right\} \text{Where } x \text{ repreſents the year,}$$

y the month, *z* the day, *v* the hour, and *u* the minute A. M. when the eldeſt was born; and the other was born 4 m. after.

VI. QUESTION 555, *by Mr.* Richard Gibbons.

A tradeſman owed his dealer 150l. who agreed to take a certain ſum yearly on being allowed lawful intereſt upon balancing each year's accompt, as the ſame became due, until the whole was paid, when the debt and intereſt amounted to 179l. 9s. 10$\frac{1}{5}$d. required the yearly payment, and the time the whole took paying off?

VII. QUESTION 556, *by Miſs* Ann Nicholls.

There is, in lat. 54° north, a town, and in lat. 64° north, a mountain, both under the ſame meridian, and on Dec. 21ſt the ſun was obſerved to riſe at the ſame moment of time at the town and on the mountain; from which data I demand the height of the mountain.

VIII. QUESTION 557, *by Mr.* Tho. Harris.

Say when, * ye aſtronomic ſpies,
Bright Sol in the leaſt time will riſe?
* Lat. 51° 32′ N.

IX. QUESTION 558, *by* Curioſus.

So to draw a right line, cutting the two ſides of a given plane triangle, that the rectangle of the two lower ſegments ſhall be equal to a given quantity, and the rectangle of the two upper ones a maximum.

X. QUESTION 559, *by Mr.* John Chipchaſe.

A ball being ſhot, at an elevation of 76°, from a cannon, whoſe greateſt horizontal range is three miles, ſtruck an object ſtanding at ſuch a diſtance that, if it had been projected againſt it in a direction perpendicular to its ſurface, its force would have been twice as great as it then was; required the object's diſtance from the cannon, ſuppoſing it to ſtand perpendicular to the horizon.

XI. QUESTION 560, *by Mr.* T. Moſs.

To determine (à priori) the form of a ſpheroidical ſtanding caſk, whoſe ullage may be truly found by means of the line of ſegments on the common ſliding rule; its whole content being 100 gallons.

XII. QUESTION 561, *by Mr.* Joſeph Fiſher.

Admit the earth and moon two perfectly ſpherical bodies, and the ſun's parallax to be 10″; and granting alſo, that the ſun and moon, when conjoined, and both at their mean diſtances from the earth, elevate the waters of the ocean 12 feet perpendicular, thereby cauſing the tides: Quære, whether a pendulum clock would gain or loſe, and how much per day, by tranſporting from the earth to the moon; and what length a pendulum ſhould be to make 60 vibrations on the moon's ſurface, ſuppoſing one 39·2 inches to make 60 in a minute on the earth, in lat. 55°.

XIII. QUESTION 562, *by Mr.* Rob. Butler.

A current of water is diſcharged by three equal arches or ſluices: The 1ſt (in ſhape) a rectangular parallelogram, the 2d a ſemicircle, and the 3d a parabola, having their altitudes equal, and their baſes (which are downwards) ſituated in the ſame horizontal line, and the water level with the tops of the arches; on this ſuppoſition let be ſhewed the proportion of the quantities diſcharged by theſe ſluices

XIV. QUESTION 563, *by Mr.* Paul Sharp.

If a veſſel, in form of a cone whoſe baſe diameter and depth are equal, ſtand on its baſe and be filled with water, it will empty itſelf, through a hole in its baſe, in the ſame time that a ſphere, whoſe internal diameter is equal to that of the cone, will empty itſelf through an equal orifice in its bottom: Quære the demonſtration?

XV. QUESTION 564, *by Mr.* W. Toft.

To find the time of ſhorteſt twilight in any given latitude, and its duration, by ſtereographic projection.

XVI. QUESTION 565, *by Mr.* Rob. Butler.

Required the content, in ale gallons, of a caſk formed by the rotation of a cycloid about its baſe or longeſt diameter, and having its head diameter 32, bung 40, and length = 48 inches.

The PRIZE QUESTION, *by Mr.* Tho. Allen, *of* Spalding *in* Lincolnſhire.

Suppoſing a heavy body to deſcend freely by the force of gravity upon a common cycloidal curve, the radius of whoſe generating ſemicircle is 18 feet; and let its diſtance from the vertex of the cycloid be 12 feet at the commencement of motion. It is required to determine where the body will quit the curve, its diſtance from the axis when it impinges upon the horizon, and time of its whole deſcent.

1767.

Queſtions anſwered.

I. QUESTION 550 *anſwered by Mr.* Edward Bayley.

PUT $x-y=z^6$, and then the 2d given equation becomes z^{13} $(=z^4\times z^9)=8192$; whence $z=2$, and $x-y=64$; from which, and the firſt given equation $(x+y=152)$, is readily found $y=44$, John's age, and $x=108$ l. his fortune.——And in this manner, exactly, the anſwer is likewiſe given by Meſſ. *J. Bennett*, *R. Gibbons*, *R. Gray*, *J. Maſon*, *W. Rawle*, *P. Sharp*, *Iſ. Tarratt*, and *S. Vince*.

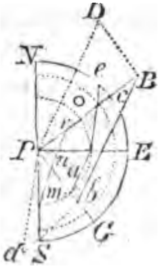
It is alſo anſwered by Meſſ. *Joſh. Adams*, *T. Adams*, *T. Atkinſon*, *T. Barker*, *W. Baylis*, *W. Baily*, *W. Cole*, *J. Coultbred*, *Iſ. Dalby*, *R. Dening*, *W. Dent*, *W. Godhe*, *J. H. R. Hale*, *W. Hubbard*, *J. Probert*, *P. De Quir*, *T. Robinſon*, *B. Rotherham*, *Alex. Rowe*, *B. Sikes*, *W. Sewell*, *R. Smetham*, *Edw. Smith*, *W. Spicer*, *W. Stoker*, Mrs. *Eleanor Suggett*, *T. Veſſey*, *Ja. Urmſtow*, *Marſhal Wroot*, and *Ja. Young*.

Mr. James Mills *anſwers it in the following Advice to fair* Suſan.

My charming fair, John's age is forty-four,
His money one hundred eight pounds, and no more.
If my advice from theſe few hints you'll take,
Oh! marry William, for your own dear ſake:
Better for John will broths and flannels be,
Than in a bridal bed with lovely thee ——
He'll claſp thy charms as miſers hug their gold,
But cannot uſe the treaſure they behold!

II. QUESTION 552 *anſwered by Mr.* Tho. Veſſey, *at Mr.* Allen's *School, in* Spalding.

Let P be the place of obſervation. Draw $Pb \perp PB$, and let BD, bd be perpendicular to the horizon in the points B and b. Suppoſe D the place of the cloud at the 1ſt, and d that at the 2d obſervation. Put $t =$ tang. of $\angle BPD = 17° 4'$, $T =$ that of $\angle dPb = 45° 23'$, rad. $= 1$; then (per trigonom.) $BD = t \times PB$, and $bd = T \times Pb$; whence, the height of the cloud above the horizon being ſuppoſed to continue the ſame, $t \times PB = T \times Pb$, or $PB : Pb :: T : t$; therefore (the points B, b being joined) the $\angle PBb = 16° 52'$, and conſequently the wind blew from N. E. ¼ N.



And in this manner, nearly, it is alſo anſwered by Meſſ. *Joſh. Adams*, *T. Barker*, *J. Bennett*, the Rev. Mr. *J. Croſs*, *J. Maſon*, *J. Probert*, *W. Rawle*, *T. Robinſon*, *Alex. Rowe*, *P. Sharp*, *B. Sikes*, *R. Smetham*, *W. Spicer*, and Mrs. *Eleanor Suggett*.

If right lines PC, PG be conceived to be drawn, making $\angle$s NPC, NPG, with the meridional line NPS, equal to 56¼° and 146¼°, the given bearings of the cloud from the north, and the diſtances Pr, Pn be taken thereon equal to 3·25729 and ·98671 &c. the cotangents of 17° 4' and 45° 23' (the cloud's altitude), at the two obſervations, to rad. 1, and the points r, n be joined, and the right line rn produced till it meets NS in m; the $\angle rmN$ will meaſure the point of the compaſs required, which is too evident to need a demonſtration, the height of the cloud above the plane of the horizon being the ſame at each obſervation; and hence, by trigonom. 4·244 $(Pr + Pn)$: 2·27058 $(Pr - Pn)$ :: 1 (tang. of 45°) : ·53500 $= \text{tang. of } \frac{\angle Pnr - \angle Prn}{2}$; ∵ the angle $rmN = 39° 24'$; the ſame as before.

According to this method, very nearly, it is anſwered by Meſſ. *T. Adams*, *Felix M'Carthy*, *R. Hale*, *B. Rotherham*, *W. Sewell*, and *W. Stoker*.

But Mr. *R. Gibbons*, ſuppoſing the direction of the cloud to form the arc of a great circle, gives the following ſolution. Lay down $NC = 5$ and $SG = 3$ points, on the horizontal projection NES, whereof N, E, and S repreſent the north, eaſt, and ſouth points, and draw the two right circles PC, PG, cutting the parallels of the given altitudes (17° 4′ and 45° 23′) in o and a; through which points the arc of a great-circle aoe being deſcribed, cutting the horizon in e, it will ſhew the direction of the cloud, and the arc NE, the point which the wind blew from, which is found, * at one operation in ſpherics, to be $= 39° 24'$, anſwering to N. E. by N. ½E.

And upon this ſuppoſition it is likewiſe anſwered by Meſſ. *W. Bayly*, *Iſ. Dalby*, *R. Gray*, and *Edward Smith*.

* Viz. Sine Po : tang. Pa :: ſine Co : tang. $Ce = 16° 51'$, which ſubtracted from $CN (= 56° 15')$ leaves $Ne = 39° 24'$.

III. QUESTION 551 *anſwered by Miſs* Anna Nicholls; *To the Propoſer.*

May you of ev'ry joy partake,
That EPSOM fair can give;
And, when you ſhall this earth forſake,
May you for ever live.

Anſwer to the ſame by Mr. B. Sikes.

From the 1ſt and 5th given equations, $x + y = 32$; and, ſince v and w are concerned exactly alike in every equation where they enter, let $s = \frac{1}{2}$ their ſum, and $r = \frac{1}{2}$ their difference; then, w being $= s + r$ and $v = s - r$, thoſe values ſubſtituted in the given equations, by reduction, &c. will found $ss = \frac{32^2 - 64z + zz}{4} = \frac{5zx + 199x + 16yz + 12}{4x}$:

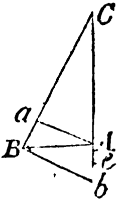
Whence, z and y being exterminated, &c. $x^5 - 53x^4 + 1034x^3 - 10928x^2 + 514320x - 8073216 = 0$; from which x comes out $= 18$, and conſequently $y = 14$, and $z = 12$. Hence $w = 15$ and $v = 5$, and the place of Mr. *Tarratt*'s reſidence is EPSOM.——Mr. *E. Bayley*'s and Mr. *W. Cole*'s anſwers are the ſame as this, exceedingly near.

Amicus, from the nature of the given equations, very readily determines $x = 18$, and conſequently $z = 12$, and $y = 14$; therefore the 1ſt, 2d, and 3d given equations become $v + w = 20$, $v^2 + w^2 = 250$, and $vw = 75$; from which $v \backsim w = 10$, &c.

Meſſ. *Joſh. Adams*, *T. Adams*, *T. Atkinſon*, *T. Barker*, *W. Bayly*, *W. Baylis*, *W. Cole*, *Iſ. Dalby*, *R. Dening*, *W. Dent*, *R. Gibbons*, *M. Gordon*, *R. Gray*, *W. Hubbard*, *R. Hale*, *J. Maſon*, *J. Mills*, *J. Probert*, *Pet. De Quir*, *W. Rawle*, *T. Robinſon*, *B. Rotherham*, *Alex. Rowe*, *Abby Sadler*, *Paul Sharp*, *W. Sewell*, *Edw. Smith*, *W. Spicer*, *W. Stoker*, *Eleanor Suggett*, *Sam. Vince*, *Ja. Urmſton*, *Marſhall Wroot*, and *J. Young*, have alſo anſwered it.

IV. QUESTION 553 *anſwered by Mr.* W. Cole.

Let AC and BC repreſent the perpendicular and ſlant heights of the ſteeple, reſpectively. Draw $Aa \perp bC$, and $Bb \parallel Aa$, meeting CA produced in b; then will A, a, and alſo B, b, be contemporary poſitions of the two falling bodies, ſuppoſing the body at A to continue in motion till the other body arrives at B (as is well known by the laws of deſcending bodies); and, BC being to BA as 8 to 1, per queſt. AB is eaſily found to be to Ab as $\sqrt{63}$ to 1. Therefore, as $\sqrt{63} : 1 :: 1142$ (the velocity of ſound per ſecond) : $1142 \div \sqrt{63} =$ the velocity of the body at e; whence the diſtance eC will be $= 321\frac{3151}{4051}$, and (per queſt.) Ab is found $= \frac{1}{64}Cb$, and $eA = \frac{1}{127}eC$; therefore, as $127 : 126 :: 321\frac{3151}{4051}$ (eC) : $319\frac{125473}{514751}$ $(AC) =$ the perpendicular height of the ſteeple required.



It is alſo anſwered by Meſſ. *Joſh. Adams*, *T. Barker*, *J. Dalby*, *W. Dent*, *R. Grey*, *R. Hale*, *Felix M'Carthy*, *J. Probert*, *W. Rawle*, *T. Robinſon*, *B. Rotherham*, *Alex. Rowe*, *W. Sewell*, *P. Sharp*, *B. Sikes*, *R. Smetham*, *E. Smith*, *W. Spicer*,

Spicer (the propoſer), *W. Stoker, J. Tarratt, T. Todd, Sam. Vince, Joſ. Webſter*, and others.*

V. QUESTION 554 *anſwered by Mr.* Iſaac Tarratt.

By ſubſtituting the values of z, v; and u, in the terms of x and y, we get $xy^2 + 64y^6 = 290304 = a$, and $yx^2 + 8y^7 = 12521472 = b$; from the 1ſt of which equations x is found $= \overline{a - 64y^6} \div y^2$, and this value ſubſtituted in the latter of them, it becomes $\overline{a - 64y^6}|^2 \div y^3 + 8y^7 = b$: Solved, $y = 4$. Conſequently the eldeſt was born anno 1760, April the 8th, at 4m. paſt 8 in the morning, and the other 4m. after that.

Much after the ſame manner it is alſo anſwered by Meſſ. *Joſh. Adams, T. Adams, T. Barker, Edw. Bayley, W. Baylis W. Bayly, W. Cole, J. Coulthred, R. Dening, J. Dalby, W. Dent, R. Gibbons, R. Gray, R Hale, Stephen Hodges* (the propoſer), *J. Maſon, James Mills, J. Probert, W. Rawle, T. Robinſon, Alex. Rowe, W. Sewell, B. Sikes, Edw. Smith, W. Spicer, W. Stokes, Ja. Urmſton, and Ja. Young.*

If it be conſidered that x and y muſt be whole poſitive numbers, and the latter of them under 12, by the queſt. the anſwer may be eaſily obtained from the given equations as they ſtand at firſt, without any further reduction, &c.

VI. QUESTION 555 *anſwered by Mr.* Richard Gibbons (*the Propoſer*).

The preſent worth 150l. $(= p)$ and the rate of intereſt 1·05l. $(= r)$ being given, put $a =$ the annuity, or yearly payment required, and $t =$ the time it was paying off; then there is further given $ta = 179{\cdot}4925$l. to find t and a ſeparately.

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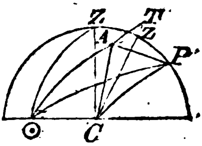
* An algebraic ſolution to this queſtion may be thus: Put $s = 16\frac{1}{12}$, $a = 1142$, and $x = AB$: Then $8x = BC$, and $x\sqrt{63} = AC$, By the laws of gravity, $\sqrt{s} : \sqrt{AC} :: 1'' : \sqrt{\frac{x\sqrt{63}}{s}} =$ time of falling through CA, and $AC : BC :: \sqrt{\frac{x\sqrt{63}}{s}} : \sqrt{\frac{64x}{s\sqrt{63}}} =$ time in CB, alſo $\frac{x}{a} =$ time of ſound's moving through AB; therefore we have $\frac{x}{a} + \sqrt{\frac{x\sqrt{63}}{s}} = \sqrt{\frac{64x}{s\sqrt{63}}}$: Hence $x\sqrt{63} = a^2 \times \frac{127 - 16\sqrt{63}}{s} = 319{\cdot}249 = AC$ required.

rately. By the doctrine of annuities and the queſtion, $atr^t - at = ptr^{t+1} - ptr^t$, or, in numbers, &c. $179{\cdot}4925r^t - 75tr^t = 179{\cdot}4925$: Whence $23{\cdot}93233r^t - tr^t = 23{\cdot}93233$: Solved, $t = 6{\cdot}527$ years, and $a = 27$l. 10s. the annual payments.

In this manner, nearly, it is likewiſe anſwered by Meſſ. *Joſh. Adams*, *T. Adams*, *T. Barker*, *J. Bennet*, *R. Gray*, *Felix M'Carthy*, *J. Maſon*, Miſs *Anna Nicholls*, *J. Probert*, *W. Rawle*, *Tho. Robinſon*, *W. Sewell*, *B. Sikes*, *Edw. Smith*, *W. Spicer*, and *T. Todd*.

VII. QUESTION 556 *anſwered by Mr.* Felix M'Carthy, *of* Aberdeen.



In the annexed projection, let P repreſent the north pole, Z the zenith of the place in lat. 54°, z that of the place in lat. 64°, and $\odot$ the ſun in the horizon in the former of them, on the day propoſed; then, in the ſpherical triangle $Z\odot P$, is given all the ſides to find the angle $ZP\odot = 53° 17' 27''$. Through $\odot$ and z deſcribe the arc of a great circle $\odot z$; alſo deſcribe the arc AP, making $\odot A = \odot Z$, and then a ſpectator at A will, it is evident, ſee the ſun riſing at the ſame time with the other two ſpectators at Z and T, the point where a tang. to the arc Az, at the point A, meets Cz (the ſemidiameter of the earth) produced, *i. e.* at the top of the mountain zT required; to determine the height of which, in the ſpherical triangle $\odot Pz$, are given the ſides $\odot P$, Pz, together with the included angle, to find $\odot z = 96° 45' 36''$, from which ſubtracting $A\odot = 90°$, there remains Az (the meaſure of the $\angle ACz) = 6° 45' 36''$: And thence CT is found $= 4009{\cdot}887$ miles (per plane trig.), from which deducting 3982 (the miles contained in Cz, the earth's rad.) leaves $zT = 27{\cdot}887$ miles, the required height of the mountain.

This queſtion we received from the ingenious propoſer in the year 1761, and now find that, through miſtake, it has been ſince ſent to, and publiſhed in, another place.

Anſwers to this queſt. have alſo been received from Meſſ. *T. Adams*, *T. Barker*, *W. Bayliss*, *J. Chipchaſe*, *J. Dalby*, *J. Probert*, *W. Rawle*, *Abby Sadler*, *W. Sewell*, *B. Sikes*, *Edw. Smith*, *W. Spicer*, *Eleanor Suggett*, *J. Webſter*, and *J. Young*.

Mr. *Joſhua Adams*, allowing 33′ for the $\odot$'s horizontal refraction, finds the height = 30·2 miles.

VIII. QUESTION 557 *anſwered by* Juvenis.

Put a and b = ſine and coſine of 51° 32′ (the given lat.), c = coſine of 90° 32′ 12″ (the diſt. of the ſun's lower limb from the zenith the moment his upper limb touches the horizon), and x = coſine of the declin. required, rad. = 1; then, per ſpherics,

$$\overline{\overline{ax+c}\times\sqrt{bb-xx}-ax\sqrt{bb\times\overline{1-xx}-\overline{ax+c}|^{2}}}\div\overline{bb\times\overline{1-xx}}=$$

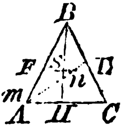
ſine of the angle meaſuring the time the ſun is in riſing, which (per queſt.) is to be a min. In fluxions, &c. and reduced, x comes out $\doteq$ ·0036441 &c. anſwering to 12½′ nearly, the declination required.

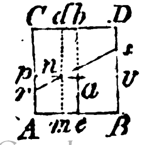
Meſſ. *T. Adams*, *T. Barker*, *R. Gibbons*, *J. Probert*, *Alex. Rowe*, and *W. Spicer*, have likewiſe anſwered it.

Mathematicus eaſily reduces this prob. to that of deſcribing the circumference of a circle to touch two circles, given in magnitude and poſition, and cut the circumference of another circle given in magnitude and poſition likewiſe, in a given angle; and from thence readily infers, nearly, the ſame concluſion as above.

Mr. *Tho. Harris* (the propoſer) and Mr. *Iſaac Tarratt* obſerve that 'the ſun will riſe quickeſt when his azimuth 'increaſes the ſloweſt, and that will be when he aſcends the horizon in a vertical direction:" and upon this principle they ſolve it independent of fluxions, and find the declination ſought to be 12′ 40″ ſouth; from whence they conclude that the ſun riſes the quickeſt the morning before the vernal, and the morning after the autumnal equinoxes.

IX. QUESTION 558 *anſwered by Mr.* T. Moſs.





Let ABH (fig. 1ſt) repreſent the given plane triangle; produce AH till the right line BC, joining its extreme and the point B, becomes = BA, and upon AB (= one of thoſe equal ſides) let a ſquare AD (fig. 2d) be conſtituted, and conceive a right line rs to be drawn through the ſame, ſo that $Ar\times Bs$ may be to the given parallelogram of the lower ſegments as BH : BA (fig. 1ſt) and the area of the part $ArsB$ a min. which, having biſected AB in m and drawn $md \parallel AC$, it is manifeſt will be when $Ar+Bs$ is a min. becauſe mn

$(=\frac{Ar+Bs}{2})\times AB$ is the meaſure of the ſpace $ArsB$, and AB is a conſtant quantity. But it is well known, and may be eaſily proved geometrically, that the ſum of two right lines, under a given rectangle, will be the leaſt poſſible when they are equal to each other: Whence, taking $Ap =$ the ſide of a ſquare, whoſe magnitude is a fourth proportional to BH, BA and the given parallelogram of the lower ſegments, and drawing $pv \parallel AB$, upon the ſide AB, of the given $\triangle\, ABH$, ſet off $AF = Ap\,(Bv)$, and draw $FE \parallel$ the baſe AC, cutting BH in s, and the thing is done.

DEMONS. Becauſe, by conſtruction, the $\square\, ApvB$ (fig. 2) is a min. when the $\square$ of the lower ſegments is a given magnitude, thence any rectangular part (Aa) thereof will alſo be a min. and the $\square$ of the ſaid ſegments remain the ſame. Let, therefore, $pa = Cp$, and draw $eab \parallel AC$; then becauſe $Apae$ is a min. the $\square pb$, which is a multiple of the $\triangle FBE$, will be a max. and conſequently FBs, being a part of FBE, will alſo be a max. when $AF\times EC\ (= Ap \times Bv)$ is a given magnitude. Now, ſeeing that EF muſt be $\parallel AC$, in order that a multiple of the $\triangle\, mBn$ may be a max. and the $\square$ under the lower ſegments of the $\triangle\, ABC$ a given magnitude (*i. e.* to the given rectangle as $BA : BH$ or $FA : sH$), it thence evidently follows, that the $\triangle\, FBs$ (being as $BF\times Bs$) will be the greateſt poſſible (and the $\square$ of the lower ſegments of the $\triangle\, ABH =$ the given rectangle) when Fs is $\parallel AH$.

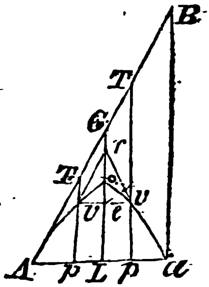
A Fluxionary Anſwer to the ſame, by Mr. Samuel Vince.

Suppoſe AB (fig. 1ſt) to be given $= a$, $HB = b$, the $\square$ of the lower ſegments, AF, Hs, $= rr$ (ſuppoſing Fs to be the poſition of the line required), and put $AF = x$, $Hs = y$; then will $xy = rr$, and $\overline{a-x}\times\overline{b-y}$ be a max. per queſt. In fluxions, $x\dot{y}+y\dot{x}=0$, and $x\dot{y}+y\dot{x}-a\dot{y}-b\dot{x}=0$, i. e. $-a\dot{y}-b\dot{x}=0$, and conſequently $a\dot{y}=b\dot{x}$; whence $x\,(AF) : y\,(Hs) :: a\,(AB) : b\,(HB)$, and $Fs \parallel AH$, by the Elem. of Geometry.

Meſſ. *T. Adams*, *T. Barker*, *W. Bayly*, *J. Bennett*, *J. Dalby*, *R. Gray*, *Felix M^c^Carthy*, *J. Maſon*, *Step. Ogle*, *J. Probert*, *W. Rawle*, *T. Robinſon*, *Alex. Rowe*, *W. Sewell*, *P. Sharp*, *W. Spicer*, *T. Todd*, and *Ja. Young* have likewiſe anſwered it.

X. QUESTION 559 *anſwered by Mr.* T. Todd.

By the nature of projectiles in a non-reſiſting med. 1 (rad.) : ·4694714, (s. 2 $\angle BAa$, the given elevation) :: 3 miles (the given, greateſt rand.) : 1·4084142 miles $= Aa$, and, by trig. $Ba = 5·64884077$ miles $= n$. Now, if $t =$ ·249328 (the nat. tang. of $\angle B = 14°$), $x = Tv$, and $y = vp$; then, by trig. $tn = Aa$, $\frac{tn}{2} = AL = La$, and therefore, by conics, $\frac{1}{4}n$ $(Go = oL)$: $\frac{1}{4}ttnn$ (Laa) :: $\frac{1}{4}n - y$ (oe) : $\frac{1}{4}ttn \times \overline{n - 4y} = ev^2$; $\because$ by 47 Euc. 1, the tang. rv at v, is $= \sqrt{\tfrac{1}{4} \times \overline{n - 4y}|^2 + \tfrac{1}{4}t^2 n \times \overline{n - 4y}}$; which (by the reſolution of forces and the queſt.) is to $\frac{1}{2}t\sqrt{nn - 4ny}$ $(= ve)$ :: 2 : 1; from whence, by multiplying extremes and means and reducing, &c. y is found $= \frac{1}{4}n \times \overline{1 - 3tt} = 1·148842$. Moreover, by Simp. Exerciſes, cor. 1 pa. 188, $Ba \times Tv = Tpp$, i.e. nx $(= \overline{x + y}|^2) = \overline{x + \tfrac{1}{4}n \times \overline{1 - 3tt}}|^2$: Solved, $x = \frac{1}{4}n \times \overline{1 + 3tt} \pm \frac{1}{2}nt\sqrt{3} = 2·89530048$, or ·45585554; and conſequently Tp $(= x + y) = 4·044142$, or 1·604697, and Ap $(= t \times \overline{x + y}) = 1·008318$, or ·4900962 miles, either of thoſe diſtances anſwering the queſtion.——The anſwer by Mr. *J. Chipchaſe* (the propoſer) is the ſame as this, exceedingly near.



It is alſo anſwered by Meſſ. *T. Barker*, *Step. Ogle*, *J. Probert*, *W. Rawle*, *Alex. Rowe*, *W. Sewell*, and *P. Sharp*.

XI. QUESTION 560 *anſwered by Mr.* B. Sikes.

Let $b =$ the bung diameter, $l =$ the ſemi-length, $x =$ the wet inches, $y =$ the diameter at x diſt. from the head on which the caſk ſtands, $c =$ the circular diviſior for gallons, and the ratio of the bung to the head diam. as $1 : n$. Then, by the nature of the ellipſis, $yy = nnbb + \frac{2 \cdot \overline{1 - nn} \cdot bbx}{l} - \frac{\overline{1 - nn} \cdot bbxx}{ll}$, which multiplied by $\frac{\dot{x}}{c}$, and the fluent taken, &c. the ullage (or quantity of liquor in the caſk, putting $2lv = x$) will be found to be $2lbb \times \frac{2 + nn}{3c} \times \frac{3nnv + 6 \cdot \overline{1 - nn} \cdot vv - 4 \cdot \overline{1 - nn} \cdot v^3}{2 + nn}$ where the factor

$2lbb\times\frac{2+nn}{3.c}$ is known to be = the whole content of the caſk; therefore $\frac{3nnv+6.\overline{1-nn}.vv-4.\overline{1\quad nn}.v^3}{2+nn}$ will expreſs the fractional part of the ullage, the whole content being the integral quantity, or 1.——If now the whole length of the caſk be ſuppoſed = 1, and to be divided into 100 equal parts, v will truly repreſent the number of parts contained in the wet inches; and, to make the foregoing expreſſion agree with the multipliers, as found, on the ſliding-rule, ſet 100 on the ſlider to 100 on the line of ſegments S. S. on the rule, then againſt any given number v of parts on the ſlider, is the proper multiplier, or fractional part of the ullage on the rule, or line S. S; which, being put = m, and made = the laſt expreſſion, the value of n will be determined = $\sqrt{\frac{2m-2vv.\overline{3-2v}}{3v-m-2vv.\overline{3-2v}}}$, which will give the form of the caſk required.

Let the wet inches divided by the length, or v, be = ·28; then m will be found = ·26, nearly, on the ſegments S. S. and conſequently n = ·8343: But if v be taken = 72, m will come out = ·743 on the line S. S. and n = ·8097; which ſhews that the line of ſegments will not give the ullage truly, in any part of the ſpheroid except the middle: But if n be taken = ·822 (= the mean between theſe two ratios), the line of ſegments will give the ullage of a ſpheroid, formed by this ratio, nearer the truth than any other.

Mr. *T. Moſs* (the propoſer) founds his ſolution on p. 207, 208, &c. of his Treatiſe of Gauging, where it is proved, in a general manner, that the ullages of two upright ſpheroidical caſks, having their bung and head diameters in the ſame ratio, and their wet inches in the ratio of their lengths, will be to each other in the ratio of their whole contents, let their lengths be what they will: Whence it follows that the ullages of every ſtanding ſpheroidical caſk, having ſome one certain proportion of bung and head diameters, may be truly determined by the ſliding-rule, let the lengths of ſuch caſks be what they will. Thus, if the ratio of the head and bung diameters of the required ſpheroidical caſk be ſuppoſed to be as x to 1 (that being all that, in the preſent caſe, is required), 1 = its content, y = its length, $\frac{1}{3}y$ = the wet inches, and p = ·7854; then, by pa. 206 and 207 of the Theory of Gauging, will be found $\overline{1-1-xx\times{}·48}\div\overline{2+xx}$ = ·316 (the ullage, in this caſe, by the ſliding-rule; whence x = ·82, very nearly.——If the content of the caſk be denoted by a, and every thing elſe as before, then will $\overline{a-\overline{a-axx}\times{\cdot}48} \div 2+xx = 316$, 31·6, 3·16, or ·316, according as a (or the content of the caſk on the ſegment line) is ſuppoſed to be divided into 1000, 100, 10, or 1 equal parts; from each of which x comes out $=$ ·82, the ſame as before.

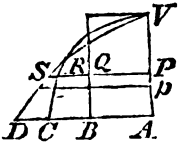
XII. QUESTION 561 *anſwered by Mr.* Joſ. Fiſher, (*the Propoſer*).

The whole ſolar force to move the waters of the ocean may be found, by a proper calculus of the earth's centripetal force towards the ſun, to be to the force of gravity with us as 1 is to 12868200, and Sir Iſaac Newton, in his Principia, has ſhewn that the earth's centrifugal force, ariſing from its vertiginous motion, is to the force of gravity at the equat. as 1 is to 289; which force, being calculated, ſhould make the waters under the equat. exceed thoſe under the poles in alt. 90640 feet: And hence the elevation of the waters by the ſun appears to be $2\frac{1}{18}$ feet; conſequently the moon raiſes them the other $9\frac{17}{18}$ feet, and the ſolar force to elevate the waters is to the lunar force as 1: 4·88.——The moon's force is to the force of gravity on the earth's ſurface as 1 is to 2634837, and the moon's denſity is to the ſun's as 1: 0·191, and the ſun's is to the earth's as ·191 is to ·75, both computed from the triplicate proportion of their true diameters, and the ſimp. propor. of gravity towards each at equal diſtances, conjunctly; whence the earth's denſity is to the moon's as ·756 is to 1. Now, the moon's diameter being to the earth's as 1 : 3·65, and the denſity as 1 is to ·756, the accelerating gravity or force with which a pendulum would be actuated on the moon, is to the ſame on the earth, under the poles, as 1 is to 2·8; whence the gravity in lat. 55° appears to be 2·7968136, and conſequently 34736″, or 9 h. 38′ 56″, the time a clock would loſe per diem on the moon's ſurface; and hence the length of the pendulum required may be eaſily known.

NOTE, The moon's centrifugal force, ariſing from its rotation, is here omitted, being ſo ſmall as could cauſe little variation.

XIII. QUESTION 562 *anſwered by Mr.* Cha. Hutton.

Let VB repreſent half the parallelogram, AVC half the ſemicircle, and AVD half the parabola, *i. e.* the halves of the reſpective ſluices or arches, $a = AV$ (the common alt.) $p =$



$p = \cdot 7854$ &c. then paa is $=$ the area of each of them, $pa = AB$, $a = AC$, and $AD = \frac{3}{2}pa$. Put $x = VP$, and $\dot{x} = Pp$; then, the water diſcharged, at any depth x, being as the velocity and aperture, and the velocity as $\sqrt{x}$, we ſhall have $\dot{x}\sqrt{x} \times PQ$, $\dot{x}\sqrt{x} \times PR$, and $\dot{x}\sqrt{x} \times PS$, or $pax^{\frac{1}{2}}\dot{x}$, $x\dot{x}\sqrt{2a-x}$ and $\frac{3}{2}p\sqrt{a} \times x\dot{x}$ as the fluxions of the quantity of water diſcharged by the ſaid □, ſemicircle, and parabola, reſpectively; the correct fluents of which (when $x = a$) are $\frac{2}{3}paa\sqrt{a}$, $\frac{2}{15}aa\sqrt{a} \times 8\sqrt{2} - 7$ and $\frac{3}{4}paa\sqrt{a}$, and the quantity of water diſcharged by the □, the ſemicircle, and parabola, as 1, 1·09847 &c. and $1\frac{1}{8}$ reſpectively.

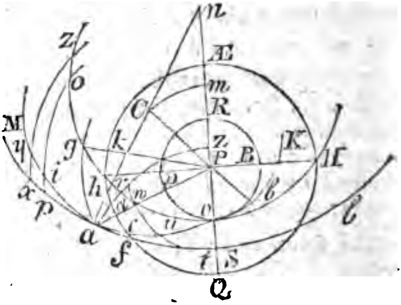
The ſolutions by Meſſ. *T. Allen*, *Rob. Butler* (the propoſer), *J. Chipchaſe*, and *Οἱ Φίλοι*, are the ſame, exceedingly near.

XIV. QUESTION 563 *anſwered by Mr.* Paul Sharp, (*the Propoſer*).

Put $a =$ the diam. of the ſphere (which is alſo $=$ the diam. and alt. of the cone, per queſt.), $e = \cdot 7854$, $t =$ the time of a cylinder's, of equal baſe diam. and altitude, emptying itſelf, at an equal orifice, when full, with its firſt or greateſt velocity, and $x =$ any variable alt. of the ſurface of the water above the baſe of the cone, &c. at the end of any time y, then will $\dot{y} = -t\dot{x} \times \frac{\overline{a-x}|^2}{aa\sqrt{ax}}$ in the cone, and $= -4t\dot{x} \times \frac{ax - xx}{aa\sqrt{ax}}$ in the ſphere; the correct fluents of which (when $x = a$) will, in each caſe, be $y = \frac{16t}{15}$, *i. e.* they will be the ſame, &c.——Mr. *T. Allen* (ſubſtituting, nearly, the ſame as above) derives the very ſame concluſion.

Much after the ſame manner the anſwer is given by Meſſ. *C. Hutton*, *Edw. Smith*, and *T. Todd*.

XV. QUESTION 564 *anſwered by* Mr. W. Toft, (*the Propoſer*).



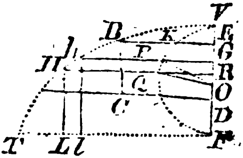
PROJECT. Having deſcribed, about the centers P, m, and n, the primitive circle $\AE HQ$, repreſenting the equator, zOH the horizon, and pSl a parallel thereto, at 18° diſt. below it, deſcribe, from the centers P, n, with the radii Pm and $nS-mO$, two arcs interſecting each other in C; then, producing the right line nC, joining thoſe points it will meet the parallel of depreſſ. pSl in a, the point through which the parallel of the ☉'s declin. will paſs on the day the propoſed phænom. happens, and aP, the right line joining thoſe points, will be the diſtance of the parallel from the viſible pole.

DEMONST. Deſcribe the circ. of perpetual apparition, $RBOD$, and the arcs tw, ag, po, xz, &c. of the parallels of declin. intercepted between pSl (the parallel of depreſſ.) and zOH (the horiz.); alſo about the center C, with the rad. Ca $(=mO)$, let the circ. KM be deſcribed, interſecting the parallels of declin. &c. in u, e, a, i, y, &c. and it will touch the parallel circ. pl in a (by 11 E 3): It will alſo touch the circ. $RBOD$ (by the ſame), becauſe $CP=mP$ (by conſtruc.), and make an angle (aeh) with the equator $=Obe$, the angle which the horizon makes with the ſame; and hence the parallel arcs uw, eh, ag, io, yz, &c. intercepted by them, (by prop. 6th on pa. 121 of Emerſon's Trigonom. 1ſt edit.) will be ſimilar, and conſequently paſſed over by the ☉ in the ſame time, which will, manifeſtly, be leſs than the time in which the correſponding twilight arcs tw, fh, po, xz, &c. will be paſſed over in, except the twilight arc ag, which will be the ſame, and therefore will will be the ſhorteſt, &c.

CALCULA. Deſcribe the vertical circle Za, cutting the horizon and equator in r and d reſpectively, and the meridional arc Pg, interſecting the equator in k; then the right-angled ſpherical triangles *ead*, *brd*, being equiangular, will be mutually equilateral, and conſequently $ad = dr = 9°$; whence (br being $= ea = hg$, by prop. 6. on pa. 121 of Emer. Trigonom. aforeſaid) 1 (rad.) : tang dr (½ the depreſſ.) : : coſ. $\angle gbk =$ ſine of lat. of the place) : ſine of gk (the required declin.)——Alſo, in the ſpheric Δaed, ſine of $\angle aed$ (or coſ. of lat.) : ſine of ad (= 9°, ½ the depreſſ.) : : 1 (rad.): ſine of ed, the arch of the equat. meaſuring ½ the duration of the ſhorteſt twilight required.*

Meſſ. *T. Allen*, *Felix M'Carthy*, *C. Hutton*, *R. Gibbons*, *Step. Ogle*, *T. Robinſon*, *Alex. Rowe*, and *B. Sikes* have alſo projected it.

XVI. QUESTION 565 *anſwered by Mr.* Cha. Hutton.



Let VKF be the generat. ſemicircle, and ſuppoſe half the caſk to be generated by the revolution of the cycloidal arc VB about DC (= the ord. EB). Put $VF = D$, $VD = 20 = m$, DF $(= d - m) = n$, $p = 3{\cdot}14159$ &c. $DC = x$, and $BC = y$; then, by the prop. of the cycloid, &c. $\dot{x}$ will be found $= -\dot{y}\sqrt{\overline{n+y} \div \overline{m-y}}$ and $py y\dot{x}$ (the flux. of the ſolid.) $= -py^2\dot{y}\sqrt{\overline{n+y} \div \overline{m-y}}$, and the correct fluent (when $y = 16 = a$) is

$$\overline{15m^2 + 4mn - 3n^2 + 2a \times \overline{5m+n} + 8a^2} \times \tfrac{1}{24}p \times \overline{m-a}\sqrt{\overline{n+a} \div \overline{m-a}} + \overline{5m^2 - 2mn + n^2} \times \tfrac{1}{8}p \times \overline{m+n} \times$$ arc, whoſe rad. is 1 and cotang. $\sqrt{\overline{n+a} \div \overline{m-a}}$, = the ſolidity of ½ the caſk required.

But to find VF, let it be now called z, and VE $(= m - a = 4) = c$; then, by the prop. of the cycloid, &c. is found

$$24\ (= BE) = 2\sqrt{cz} \times : 1 - \frac{c}{3z} - \frac{c^2}{2.4.5z^2} - \frac{3c^3}{2.4.6.7z^3} - \frac{3.5c^4}{2.4.6.8.9z^4} - \text{\&c.}$$ and hence $z = 37{\cdot}34394 = d$, and $n = 17{\cdot}34394$. Hence the above fluent comes out 26434·3196 inches = ½ the content of the caſk, and conſequently the whole content is 187·4774 ale gallons.

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* Other ſolutions may be ſeen at Queſt. 371.

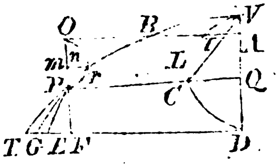
Anſwer to the ſame by Mr. Rob. Butler (*the Propoſer*).

Put $2b = VF$ (½ the bung diameter) $= 20$, $VR = 4$ (½ the given difference of the bung and head); then, by prop. of the circle, &c. $RQ = 8$, and $QH = 16 = d$ (Gh and RH being $\perp VF$, and cutting the ſemicircle VKF in P and Q). Alſo the $\angle QOV$ (at the center of the generating circle) will be found $= 53{\cdot}13^\circ$, and the length of its correſponding arc $QV = 9{\cdot}273$ inches, which put $= n$, and let $p = 3{\cdot}1416$, $C =$ the content, in inches, of the ſolid generated by the revolution of the ſpace $lhVF$, about LF, as an axis (hl and HL being $\perp TF$), the variable quantities x, y, and $z = OG$, GP and PV reſpectively: Then, per the nature of cycloids, $n : d :: z : \frac{dz}{n} = Ph = mz$ (putting $\frac{d}{n} = 1{\cdot}72544 = m$), and $mz + y =$ the ord. Gh; the flux. of which drawn into $\overline{b+x}^2 = hl^2 = GF^2$), viz. $\overline{m\dot{z}+\dot{y}} \times \overline{b+x}^2$, will be $= \frac{\dot{C}}{p}$. But, by the prop. of the circle, $b\dot{y} = x\dot{z}$, $bb - yy = xx$ and $-\dot{x}y = \dot{s}$ (s being the area of the circular ſeg. PVG); from which, and the preceding equation, (when the fluents are taken, &c.) will be had $mbbz + 2bb \times \overline{m+1} \times y + b \times \overline{m+2} \times xy - \frac{2}{3}y^3 + b \times \overline{m+2} \times s = \frac{C}{p}$: Whence the whole content required appears to be 187½ ale gallons.—— It is obſervable, that though the caſk in the 1ſt of theſe ſolutions is ſuppoſed to be generated from the revolution of part of a common cycloid about an ord. ∥ TF, and in the laſt from the revolution of part of an inflected cycloid around the longeſt ſemidiam. TF itſelf, yet they exactly agree in their contents.

In this manner, nearly, it is alſo anſwered by Meſſ. *Wm. Sewell* and *T. Todd*.

But if the caſk be ſuppoſed to be generated from the revolution of part of a common cycloid round the greateſt ſemidiam. TF, then its length need not be given, being determinable from the nature and other given dimenſions of the generating cycloid, as Meſſ. *T. Allen*, *Edw. Smith*, and *T. Todd juſtly* obſerve, and find, by elegant proceſſes, the content, in this caſe, to be 135·2 ale gallons.

he PRIZE QUESTION *anſwered by Mr.* Tho. Allen (*the Propoſer*).



It is évident that the body will quit the curve when its locity in direction of the ord. comes the greateſt poſſible. Let *B* be the place of the dy at the commencement of otion. Put *VD* the diam. of generating ſemicir. = 36 t) $= 2a$, and $VQ = x$; then, ce *BV* is given = 12 feet, have, by the properties of cycloid, $VA = 1$, and there re $AQ = x - 1$. Now, ſince the velocity of the body ong the curve, is to the velocity in direction of the ord. the fluxion of the former is to that of the latter, $\frac{\sqrt{\overline{2a-x}\times\overline{x-1}}}{2a}$ will be = the velocity in direction of e ordinate; by making the fluxion of which = 0, will be d $x = a + \frac{1}{2} = 18{\cdot}5$ feet.

Put $s = 16\frac{1}{12}$ feet, and t = the time of deſcribing BP; en, per the laws of falling bodies, $\dot{t} = \frac{1}{2}\sqrt{2as^{-1}} \times x^{-\frac{1}{2}} \dot{x}$ $\overline{x-1}|^{-\frac{1}{2}}$, whoſe fluents give $t = \sqrt{\frac{a}{2s}} \times$ hyp. log. f $\overline{\sqrt{x} + \sqrt{x-1}}|^{2}$; which, when $x = 18{\cdot}5$, will be $3{\cdot}1989''$.

Laſtly, when the body ceaſes to touch the cycloid, as at ', it will deſcribe a parabolic curve, ſuppoſe *PG*. Now, aving given the velocity at $P = 34{\cdot}5$ feet per ſecond, very early (from what is found above), and likewiſe the poſition f the tang. *TP*, or the $\angle PTF = 45^\circ\ 47'$, the diſt. *GF* ſuppoſing $FP \perp TD$) will be determined $= 12{\cdot}22$ feet, and he time of the body's deſcribing $PG = {\cdot}508''$. Whence *GD* the diſt. the body will impinge upon the horizontal line *D* from the axis *VD*) will be $= 58{\cdot}62$ feet, and the time f the whole deſcent $= 3{\cdot}7069''$.

The ſame anſwered by Mr. Cha. Hutton.

Let *VBPE* be the cycloid, and *VLD* the generating ſemicir. Suppoſe the body to commence motion at *B*, and quit the cycloid at *P*, going off, there, in the curve of a parab. *PG*. Draw $ED \perp VD$, and *PQ* and *BA* ∥ thereto, meeting the ſemi-circumf. *VLD* in *C* and *I*, producing the latter

of

of them till it meets FP, $\perp DE$, produced in O, and upon OP deſcribe the ſemi-cir. OrP. Draw nT a tang. to both the given cycloid VBE, and parab. PG, at P, meeting the ſemi-circumf. OrP in n, and DE, produced, in T, and let nm be $\perp OP$, and the points V, I and V, C be joined.—— Put $VA = a$, $VD\ (= 36) = d$, $16\frac{1}{12} = s$, and $AQ = x$. Then, by the laws of deſcending bodies, $2\sqrt{sx}$ = the velocity per ſecond, at Q or P, in the direction PT or VC, and conſequently $2\sqrt{d^{-1}sx \times \overline{d-a-x}}$ = the velocity at P in the horizontal direction. Now, ſince the body goes off at P in a parab. and the horizontal velocity in a parab. is a conſtant quantity, the laſt expreſſion will be conſtant, and conſequently its fluxion $= 0$; whence x comes out $= \frac{1}{2} \times \overline{d-a}$ ($\frac{1}{2}AD$). Alſo, $a = VI^2 \div d = \frac{1}{4}BV^2 \div d = 1$, and $VP\ (= 2VC) = \sqrt{2d \times \overline{d+a}}\ (= 2\sqrt{d \times \overline{a+x}}) = 51{\cdot}613951$. Again, per ſim. triangles (putting $\frac{1}{2} \times \overline{d+a} = \frac{1}{2}c$, and $\frac{1}{2} \times \overline{d-a} = \frac{1}{2}g$) will be found $FT = \frac{1}{2}g\sqrt{c^{-1}g}$, and $mn = \frac{1}{4}d^{-1}g\sqrt{cg}$: Then, per conics, $d^{-1}gg\ (= 4mO = 4QD \times PO \div VD$ = the parameter of the parabola's axis, and $\frac{1}{2}d^{-1}g\sqrt{g \times \overline{2d+c}}\ (= \sqrt{Fm \times d^{-1}gg})$ = an ordinate of the axe terminated at the point G of the curve; alſo, $FG = g\sqrt{g} \div \overline{\sqrt{c} + \sqrt{2d+c}}\ (= \overline{2mn + \text{ord. to point } G}^{-1} \times 4mn \times FT)$: Whence, (putting $p = 3{\cdot}14159$ &c.) DG $(= FG + QC + PC = FG + QC +$ the circ. arc $VC) =$ $g\sqrt{g} \div \sqrt{c} + \sqrt{2d+c} + \frac{1}{2}\sqrt{cg} + \frac{1}{180}dp \times$ degr. in $\angle VCQ = 59{\cdot}299337$, the diſtance from the axis when the body impinges on the horizon.——Laſtly, to find the whole time of deſcent to the horizon. Suppoſe t = the time of deſcending through the cycloidal arc BP, then will $\dot{t} =$ $\frac{1}{2}\dot{x}\sqrt{ds^{-1}x^{-1} \times \overline{a+x}^{-1}}$, and, taking the correct fluent when $x = \frac{1}{2}g$), $t = \frac{1}{2}\sqrt{ds^{-1}} \times$ hyp. log. of $\overline{d + \sqrt{cg}} = 3{\cdot}199035''$: But, for the time in the parab. the horizontal velocity being conſtant, and $= g\sqrt{sd^{-1}}$ feet per ſecond (as found above), $\sqrt{gds^{-1}} \div \overline{\sqrt{c} + \sqrt{2d+c}}\ (= FG \div g\sqrt{sd^{-1}}) = \sqrt{\frac{1}{2}dgs^{-1} \times \overline{d + c + \sqrt{c \times \overline{2d+c}}}^{-1}} =$ $\cdot 535681''$ = the time in the ſaid parab. and the ſum of theſe two, or $3{\cdot}734716''$, is = the whole time of deſcent required.

Ingenious anſwers have likewiſe been received from Meſſ. *J. Chipchaſe*, *Phil. George*, *R. Hale*, Miſs *Anna Nicholls*, *T. Robſon*, and the Rev. Mr. *Wm. Smith*; but the 1ſt prize of 12 Diaries, for the ſolution thereof, is fallen to the lot of Mr. *Cha. Hutton*, and that of 8 to Mr. *Tho. Allen*.

The Eclipſes calculated for 1767.

There will happen only two eclipſes this year, and both of the greater luminary, the ſun; and both likewiſe inviſible in Great Britain.

The firſt will be on Friday the 30th of January, near four o'clock in the morning; it will be central and total in the Indian Ocean, and will be a very great eclipſe in moſt of the Eaſt Indian iſlands.

The ſecond will happen on Saturday the 25th of July, about ſeven o'clock in the afternoon; it will be annular, and may be ſeen in moſt of the Britiſh colonies in North America, Gulf of Mexico, Weſt-Indian iſlands, Terra-Firma, in South America, and from part of both the Atlantic and Pacific Oceans.

JOHN METCALFE.

New Queſtions.

I. QUESTION 566, *by Mr.* Tho. Atkinſon.

Near Lincoln's city a worthy knight doth dwell,
For wit and valour few can him excel:
In the late war his honour there did ſhew,
How he was to his king and country true.
The poor to him do cry in their diſtreſs—
He rights the indigent and fatherleſs;
A friend to arts and ſciences is he;
An ornament to all ſociety.
Ladies, from hence * declare his worthy name,
And it record in Diary of fame.

$$\text{* viz.}\left\{\begin{array}{r} w+x+yz=61, \\ x+y+zw=501, \\ y+z+wx=381, \\ w-4=x; \end{array}\right\}$$

Where the values of w, x, y, and z ſhew the places, in the alphabet, of the letters compoſing this gentleman's name.

II. QUESTION 567, *by Miſs* Anna Nicholls.

Walking on ſhore, I was ſurprized by the flaſh of a gun, at ſea, bearing S. E. by E. ſeven ſeconds after the flaſh I heard the report, and 4 ſeconds after that I heard the echo from a caſtle bearing from me S. W. by W: the diſtance of the gun and caſtle are required.

III. QUESTION 568, *by Mr.* John Chipchaſe.

There is a town in lat. 50°, and another in lat. 30°, north; and, when it was noon, on the 4th of May, at the 1ſt of of them, the ſun was juſt riſing at the ſecond; 'tis required to find the lat. of a port that is equally diſtant from them both, and 40° W. of the firſt.

IV. QUESTION 569, *by Mr.* Rd. Gibbons.

An annuity being forborn five years, at compound intereſt, amounted to 500l. and now the ſame hath been forborn ten years it amounts to 1100l. Required the annuity, and the rate of intereſt per cent. per annum.

V. QUESTION 570, *by Mr.* Rob. Butler.

Given the radii of two circles = 6 and 8, and the diſtance of their centers = 10 inches; 'tis required to draw a right line 19 inches long, through their point of interſection, ſo as to terminate in their peripheries, and to determine the length of each ſegment thereof, intercepted between the ſaid point and peripheries.

VI. QUESTION 571, *by the late Mr.* W. Toft.

Admit a ſhip can make her way good, when cloſe hauled, within 6½ points of the wind, being then at N. N. W. and, when ſhe had ſailed on the larboard tack 62, and on the ſtarboard tack 75 miles, ſhe is found, by obſervation, to have altered her latitude 25 miles, northing; required the variation of the compaſs.

VII. QUESTION 572, *by Mr.* Paul Sharp.

There are two places, *A* and *B*, in north latitude, whoſe leaſt diſtance from each other is = 3405$\frac{1}{10}$ miles; *A* the weſtermoſt's latitude is more than *B*'s by 10°, and the ſum of both their latitudes is = their difference of longitude; required the latitude of each place, with the inveſtigation.

VIII. QUESTION 573, *by Mr.* Wm. Spicer.

Given $\overline{\frac{0\cdot 2}{x^{\frac{1}{3}}}}\Big|^{\frac{x^{\frac{1}{4}}}{0\cdot 5}} = y$, where x multiplied by 10000l. is the fortune of an agreeable young lady; required her fortune when y is a maximum.

IX. QUESTION 574, *by Mr.* Samuel Vince.

Given $ax^7y^3 = bx^8y^2 + cx^4y^6 - x^{11} + dy^{10}$, the equation of a curve; to find the area and ſubtangent.

X. QUESTION 575, *by Mr.* Tho. Barker, *Teacher of Mathematics, and Land-Surveyor, at* Wiſſett *in* Suffolk.

Given $ax^n = y^9$, the equation of a curve; 'tis required to find the exponent n, and the length of the curve, when the diſtance of the center of gravity of a ſolid formed thereby, from the vertex, is $= \frac{25x}{34}$.

XI. QUESTION 576, *by Mr.* Wm. Sewell.

Required the dimenſions of a cone, which, being ſuſpended by its vertex, ſhall vibrate as many times in a minute, as it is inches in altitude, its content being 1728 ſolid inches.

XII. QUESTION 577, *by Mr.* T. Moſs.

To inveſtigate a general expreſſion that will exhibit all the different factors, whereby any quantity of ſpirits, of any ſtrength above hydrometer proof (* as 1 to 1, 1 to 2, 1 to 3, &c. to 1 to 20) being multiplied, the reſpective products will ſhew what quantity of water will be neceſſary to reduce ſuch ſpirits to any required ſtrength under that proof († as 1 in 2, 1 in 3, &c. to 1 in 20).

* One gallon of water and one of ſpirits make 2 gallons of hydrometer proof.

† One gallon of water and one of hydrometer proof make 2 gallons of 1 in 2 under that proof, &c.

XIII. QUESTION 578, *by Mr.* Joſeph Walker.

To determine the ratio of the axes of a ſpheroid, to which if the common diagonal rod be applied, in the ſame manner as in the gauging of a caſk, it ſhall exhibit its true content.

XIV. QUESTION 579, *by Mr.* T. Moſs.

To determine the number of fifteens that can be made out of a pack of cards, with the inveſtigation.

XV. QUESTION 580, *by Mr.* Cha. Hutton.

Let *AIB*, *RISO* [*ſee the fig. to the ſolution*] be two right lines, any-how interſecting each other within the circle *ARBWO*, whoſe diameter is *ASW*; then, putting $AW = d$, and $p, q, r =$ the coſines of the ∠s *I*, *S*, *A* reſpectively, the general equation of the circle, defined by *AI* (x) and *RI* (y), conſidered as an abſciſſa and an ordinate, is $\left\{ y^2 \begin{matrix} +dq \\ +2px \end{matrix} \right\} y \left. \begin{matrix} -drx \\ +x^2 \end{matrix} \right\} = 0$: Required the demonſtration or inveſtigation.

XVI. QUESTION 581, *by Mr.* Tho. Allen, *of* Spalding.

Let *DGB* be a ſemi-circle [*ſee the fig. to the ſolution*] *P* a given point in its diameter; to find the nature of the curve *DEA*, whoſe ordinate *EC* ſhall be always proportional to the area *DGP*, bounded by the right lines *DP*, *GP*, and the circular arc *DG*.

The PRIZE QUESTION, *by Mr.* Tho. Moſs.

The longeſt ſide of a trapezium (being the diameter of a circle in which it may be inſcribed), the diſtance intercepted between the extremity thereof and the point of interſection made by that ſide and that which is oppoſite, both produced, being given; alſo the angle formed at the interſection of the diagonal: To conſtruct the trapezium.

1768.

Queſtions anſwered.

I. QUESTION 566 *anſwered by Mr.* James Mills.

LONG may you dwell on earth, immortal WRAY,*
And, when departed, ſhine in endleſs day.

Pamphagus obſerves that this gent. * is a deſcendant of Sir *Cecil Wray*, who was created a baronet Nov. 25th, 1612.

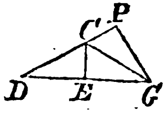
Anſwer to the ſame, by Mr. Paul Sharp.

By ſubſtituting for w its equal $x + 4$ in the 1ſt given equation, the value of x will, from thence, be found $= \frac{1}{2} \times \overline{57 - yz}$, and conſequently $w = \frac{1}{2} \times \overline{65 - yz}$; and theſe values ſubſtituted in the 2d and 3d given equations, they become $\overline{57 - yz} + \overline{65z - yz^2} + 2y = 2 \times 501$, and $y + z + \frac{1}{4} \times \overline{57 - yz} \times \overline{65 - yz} = 381$; from which, by pa. 82 of Simp. Eſſays, $y = 1$ and $z = 23$; and thence $x = 17$ and $w = 21$, and the honourable gentleman's name is *Wray*.

Meſſ. *G. Lodge, E. Jones, L. Ker, J. Probert*, and *J. Roper* find the values of w, x, y, and z the ſame as above, by means of equations of the 5th power with all the terms (to which dimenſion the queſt. ſeems naturally to riſe).—— It is alſo anſwered by Meſſ. *J. B. Aſhton, T. Atkinſon* (the propoſer), *T. Barker, E. Bayley, N. Brownell, J. Buddle, W. Cave*, jun. *W. Dent*, Miſs *Ann Nicholls, J. Nordon, T. Robinſon, W. Spicer, W. Stoker*, Mrs. *E. Suggett, E. Smith, W. Smith, S. Vince*, and *J. Young*.

Meſſ. *T. Bennett, R. Dening, W. Dixon, H. Fry, R. Gibbons, T. Hague, J. Oſborn, Pamphagus, J. Paty, W. Rawle, E. Reed, A. Rowe, W. Sewell*, and *Watſon* of *Crawcrook*, have likewiſe anſwered it.

II. QUESTION 567 *anſwered by Mr.* W. Spicer.



CONSTRUC. From the $\triangle GPD$, having its $\angle P = 112^\circ\ 30'$ (the ſum of the given bearings of the gun and caſtle), and the containing ſides PG, PD equal to 7×1142 and $\overline{7+4} \times 1142$ reſpectively (1142 being the number of feet ſound paſſes over in 1″), and draw the right line GC, making the $\angle CGD = \angle CDG$; then, P being the place of obſervation, and G the gun, C will be the place of the caſtle required: For, CG being $= CD$ (per conſtruc.), $GC + CP$ will be $= PD$ (the given diſtance, by conſtruction).

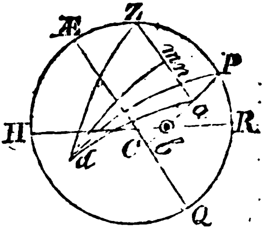
CALCULA. In the $\triangle GPD$, the two ſides PG, PD, and included angle being given, the $\angle D$ is found $= 25^\circ\ 18'\ 15''$, and its double, viz. the $\angle PCG$, is $= 50^\circ\ 36'\ 30''$; then, in the $\triangle PCG$, all the angles, and the ſide PG, are given, to find $GC = 9556{\cdot}4844$ and $PC = 3005{\cdot}5156$ feet.—And in this manner, nearly, it is alſo conſtructed by *Nugo Dàrgnas*; but Meſſ. *J. Chipchaſe*, *W. Cole*, *J. Oſborne*, *Pamphagus*, *J. Paty*, *E. Reed*, and *W. Wales*, after deſcribing the $\triangle DPG$ nearly as before, biſect DG with the perpendicular EC meeting PD in the point C, the place of the caſtle required.

Mr. *W. Sewell* obſerves that the point C aforeſaid, will be in the periphery of an ellipſe, having its foci in P, G, and tranſverſe $= PC + GC$, as from the well known property of that curve is manifeſt.

It is likewiſe anſwered by Meſſ. *J. Addiſon*, *J. Ainſworth*, *Tho. Barker*, *W. Cave*, *W. Dent*, *W. Dixon*, *R. Gibbons*, *T. Hague*, *E. Jones*, *G. Lodge*, Miſs *A. Nicholls* (the propoſer), *J. Probert*, *W. Rawle*, *T. Robinſon*, *J. Roper*, *P. Sharp*, *C. Smith*, *E. Smith*, *W. Stoker*, Mrs. *E. Suggett*, *T. Todd*, *S. Since*, *H. Watkins*, jun. *Watſon* of *Crawcrook*, and *J. Young*.

III. QUESTION 568 *anſwered by Mr.* John Roper.

Let P repreſent the north pole, Z the zenith of the place in lat. 30°, a that of the place in lat. 50°, ☉ the ſun in the horiz. HR of the former, and the merid. Pb of the latter of them, on the day propoſed, d the port required, $ÆQ$ the equat. interſecting HR in the cent. of projec. C, Za, Pd two arcs of great circles interſecting each other in n, and



dm another arc $\perp Za$, which will biſect it, as Zd is $= ad$, per queſt. Then, in the right-angled ſpherical $\triangle Cb\odot$, are given $b\odot$ (the ☉'s declin.) $= 15^\circ\ 58'$, and the $\angle bC\odot$ (the comp of lat. of the place Z) $= 60^\circ$, to find $Cb = 9^\circ\ 30\frac{1}{2}'$; whence $Æb$ (or the $\angle ÆPb$ or ZPa) is $= 99^\circ\ 30\frac{1}{2}'$: And then, in the ſpherical $\triangle ZPa$, the ſides ZP, aP with their included angle being given, Za will be found $= 73^\circ\ 4'\ 39''$, and the $\angle PaZ = 63^\circ\ 13\frac{3}{4}'$. Now, the $\angle aPn$ (the weſtward bearing of the merid. of the required port d from that of the place a) being given $= 40^\circ$ (per queſt.), in the $\triangle aPn$ are given two angles with the interjacent ſide aP; whence the ſides nP, na, and the $\angle Pna$ ($= \angle dnm$) are found $= 35^\circ\ 12'\ 4''$, $24^\circ\ 31'\ 18''$, and $84^\circ\ 34'\ 25''$ reſpectively; and conſequently in the right-angled ſpherical $\triangle dmn$, mn ($= \frac{1}{2}Za - na$), and the $\angle mnd$ being now known, dn is, from thence, found $= 66^\circ\ 2'\ 49''$; to which adding np, and ſubtracting 90° from their ſum, the remainder $11^\circ\ 14'\ 53''$ ſouth, will be the lat. of the port required.

And in this manner, nearly, the anſwer is alſo given by Meſſ. *T. Barker*, *J. Chipchaſe* (the propoſer, *W. Dixon*, *J. Dymond*, *R. Gibbons*, *E. Jones*, Miſs *Ann Nicholls*, *J. Nordon*, *Pamphagus*, *J. Probert*, *R. Pulman*, *T. Robinſon*, *A. Rowe*, *C. Smith*, and *W. Wales*.

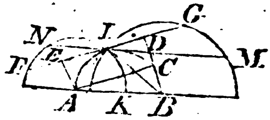
Meſſ. *J. Ainſworth*, *J. Buddle*, *J. Dalby*, *W. Rawle*, *P. Sharp*, *W. Spicer*, and *J. Young* anſwer it algebraically.

IV. QUESTION 569 *anſwered by Maſters* J. Paty *and* J. Oſborne, *Youths of about* 13 *Years of Age, at the Mathematical Academy*, Briſtol.

Let $x =$ the annuity, $y =$ the amount of 1 l. in one year, $a = 500$, $m = 1100$; then, by the doctrine of compound intereſt and annuities (ſee Note, p. 350 of Donn's Arithmetic) $\frac{ay - a}{y^5 - 1} = \frac{my - m}{y^{10} - 1}$ $(= x)$, and conſequently (putting $\frac{m}{a} - 1 = c$, and $\frac{m}{a} = 2n$) $y^{10} - 2ny^5 = -c$; whence (completing pleating the ſquare, &c.) * $y = \sqrt[5]{n \pm \sqrt{nn - c}} =$ 1·037137 &c. and the rate of intereſt = 3 l. 14 s. $3\frac{1}{4}$d. per cent. per annum, and the annuity required = 92 l. 16 s. $10\frac{1}{2}$d. ——And thus it is alſo anſwered by Meſſ. *J. Addiſon, T. Barker, J. Bennett, T. Boſworth, N. Brownell, J. Buddle, J. Dalby, Nugo Dàrgnas, W. Dent, W. Dixon, J. Dymond, T. Hague, E. Jones, G. Lodge, J. Nordon, J. Probert, W. Rawle, T. Robinſon, J. Roper, A. Rowe, W. Sewell, P. Sharp, C. Smith, W. Spicer, W. Stoker, T. Todd, S. Vince,* and *J. Young*, very nearly.

Pamphagus obſerves that ' ſince the annuity forborn 5 ' years amounts to 500l. and when forborn 10 years to ' 1100l. it is manifeſt that the intereſt of 500l. for 5 years ' muſt be = 100 l. Therefore the rate per cent. will be eaſily ' found = 3 l. 14 s. 3 d. $\frac{1}{4}$·19; and thence, by a well known ' theorem, the annuity = 92 l. 16 s. 10 d. $\frac{1}{4}$·76.'——And, from this conſideration of the matter, Meſſ. *Jer. Ainſworth* and *W. Wales* have likewiſe anſwered it.

V. QUESTION 570 *anſwered by Mr.* Wm. Cole.



CONSTRUC. Upon *AB* (the right line joining the centers of the given circles interſecting each other in *I*) conceive a ſemi-circle to be deſcribed, and apply therein, from either *A* or *B*, the chord *AC* = 9·5 (half the given line), and parallel thereto let *FG* be drawn through *I*, terminating in their peripheries, and it will be the line required.

DEMONST. Through *C* draw *BD*, and parallel to it, *AE*, meeting *FG* at right angles (by 31. III, and 29. I. Euc.) and biſecting the chords IG, IF (by 3. III. Euc.), and then $FE + DG$ will be $= ED = AC$ (= 9·5, per conſtruction), and conſequently $FG = 2\,AC = 19$.

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* The value of *y* is here brought out by means of a quadratic equa. without any neceſſity, for it naturally becomes a ſimple one, thus: Divide the numerators of the terms of the given equation $\left(\frac{ay - a}{y^5 - 1} = \frac{my - m}{y^{10} - 1}\right)$ both by $y - 1$, and the denominators by $y^5 - 1$, &c. and you have $y^5 + 1 = \frac{m}{a}$; hence $y = \sqrt[5]{\frac{m}{a} - 1}$ $\sqrt[5]{\frac{m - a}{a}} = \sqrt[5]{\frac{6}{5}} = 1{\cdot}037137.$

CALCULA. Join the points A, I and B, I: Then, in the $\triangle\, AIB$ the ſides being given, the $\angle\, ABI$ is found = 36° 52′ 12″; which taken from the $\angle\, ABC$ (found = 71° 48′ 18″, from the right-angled $\triangle\, ACB$) leaves $34^\circ\, 56'\, 6'' = \angle\, IBD$: Whence, from the right-angled $\triangle\, IDB$, is found $ID =$ 4·5811; and therefore the ſegment in the greater circle is = 9·1622 &c. and that in the leſſer = 9·8377 &c. inches.——If the chord AC be inſcribed the other way from B, the conſtruction and method of calculation will be the ſame, and the ſegment in the greater circle will, in that caſe, be found = 15·1681 &c. and that in the leſſer = 3·8318 &c.

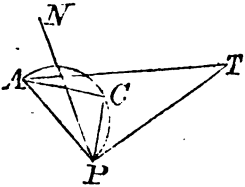
Meſſ. *J. Ainſworth*, *R. Butler* (the propoſer), *T. Boſworth*, *J. Buddle*, *J. Dalby*, *Nujo Dargnas*, *J. Dymond*, *R. Gibbons*, *C. Hutton*, *E. Jones*, Miſs *Ann Nicholls*, *Pamphagus*, *J. Paty*, *R. Pulman*, *A. Rowe*, *W. Sewell*, and *W. Wales* conſtruct it in this manner likewiſe, very nearly.

Algebraic Solution to the ſame, by Mr. John Addiſon.

Put $FG\,(=19) = a$, $AI\,(=6) = \frac{1}{2}b$, $BI\,(=8) = \frac{1}{2}c$, and $x = FI$; then will $IG = a - x$, and (per 3 Euc. III.) $EI = \frac{1}{2}x$, and $ID = \frac{1}{2} \times \overline{a - x}$. Then, the ſides of the triangle AIB being in the ratio of 3, 4, and 5 (per queſt.), the angle AIB, oppoſite the longeſt of them, will therefore be a right angle, and conſequently the right-angled $\triangle$s AEI, BDI ſimilar; whence $\frac{1}{2}b\,(AI) : \frac{1}{2}x\,(EI) :: \frac{1}{2}c\,(BI) : \frac{cx}{2b} = BD$, and (per 47 Euc. I.) $\overline{\frac{cx}{2b}}\big|^2 + \frac{1}{4} \times \overline{a - x}^{\,2} = \overline{\tfrac{1}{2}c}\big|^2$, or (in numbers), $x^2 - 13{\cdot}68\,x = -37{\cdot}8$; ſolved, $x\,(FI) =$ 9·8375, or 3·8423; and conſequently $IG =$ 9·1625, or 15·1577, &c.

In this manner, nearly, it is alſo anſwered by Meſſ. *T. Barker*, *W. Dixon*, *J. Probert*, *W. Rawle*, *T. Robinſon*, *J. Roper*, *S. Vince*, and *J. Young*.

VI. QUESTION 571 *anſwered by Mr.* J. Chipchaſe.



CONSTRUCTION. From P, the port ſailed from, let the right line PN be drawn to repreſent the magnetical or erroneous meridian. Make the $\angle NPT =$ 4½ and the $\angle PTA = 3$ points, taking $PT = 62$ miles (the diſt. run on the larboard tack), and $TA = 75$ miles, the diſtance on the ſtarboard tack), and on the right line joining the points A, P deſcribe a ſemicircle, and inſcribe therein the chord $PC = 25$ miles (the given difference of lat.), and PC will be the direction of the true meridian, AC the departure, and the $\angle NPC$ the variation required; the demonſtration of which is evident from the nature of turning to windward, &c.

CALCULATION. In the $\triangle APT$ are given PT, TA together with their included angle, whence AP is found $= 41{\cdot}6694$, and the $\angle APT = 90^\circ\ 29\tfrac{3}{4}'$, nearly; from which take the $\angle APC$ (found $= 53^\circ\ 8'$, from the right-angled $\triangle ACP$) and the remainder $37^\circ\ 21\tfrac{3}{4}'$ $(= \angle CPT)$ taken from the $\angle NPT$ $(= 50^\circ\ 37\tfrac{1}{2}'$, per conſtruc.) leaves the $\angle NPC$ $= 13^\circ\ 15_{+}'$ the variation required, which is weſterly.

In this manner, nearly, it is alſo conſtructed by Meſſ. *J. Addiſon*, *T. Barker*, *T. Boſworth*, *J. Buddle*, *J. Dalby*, *W. Dixon*, *R. Gibbons*, *C. Hutton*, *E. Jones*, Miſs *Ann Nicholls*, *Pamphagus*, *J. Probert*, *W. Rawle*, *T. Robinſon*, *J. Roper*, *A. Rowe*, Mrs. *E. Suggett*, *H. Watkins*, jun. *J. Young*, and *W. Wales*.

Meſſ. *W. Spicer* and *W. Stoker* have given neat algebraic ſolutions to it.

VII. QUESTION 572 *anſwered by Mr.* W. Wales.

Put $c =$ coſine of 10° (the given diff. of lat. of the two places); $a =$ coſine of $56^\circ\ 45'\ 2''$ (their given neareſt diſt.) and $x =$ coſine of their difference of longitude required (rad. $= 1$); then, the ſine or coſine of the ſum of two arcs being every where equal to the ſine or coſine of the ſum of their complements, the coſine of the ſum of the co-latitudes of the ſaid places will be $= x$ (per queſt.); and (per Simp. Trig. prop. 27 p. 74) $\frac{2c - 2a}{c + x} = 1 - x$ (the verſed ſine of their diff. of long.): Whence, by reduction and putting $1 - c = 2s$, there results $x = \sqrt{ss + 2a - c} + s = \cdot 34199$, the nat. coſ. of 70°; and ſo the latitudes are 40° and 30°. ——But x may be either affirmative or negative, and conſequently the ſum of the lat. of the two places either 70°, or its ſupplement to 180°, &c.

This queſtion is alſo ingeniouſly anſwered by Meſſ. *J. Ainſworth*, *T. Barker*, *W. Dixon*, *R. Gibbons*, Miſs *Ann Nicholls*, *Pamphagus*, *W. Rawle*, *T. Robinſon*, *P. Sharp* (the propoſer), *W. Spicer*, Mrs. *E. Suggett*, *S. Vince*, and *J. Young*.

VIII. QUESTION 573 *anſwered by* Pamphagus.

Would you the lady's fortune know,
Look down, and you will ſee't below.*

* 937l. 13s. 1d.

The ſame anſwered by Mr. N. Brownell.

The given equation being $\overline{\frac{0\cdot8}{x^{\frac{1}{3}}}}\Big|^{2x^{\frac{1}{4}}} = y$, put $n = \frac{1}{4}$, $z =$ hyp. log. of $\frac{0\cdot8}{x^{\frac{1}{3}}}$, and $m = 0\cdot43429448$, and then $2zx^n$ (= log. of y) is a max. (per queſtion). In fluxions, $2\dot{z}x^n + 2nzx^{n-1}\dot{x} = 0$, or $-\frac{2}{3} \times x^{n-1}\dot{x} + 2nzx^{n-1}\dot{x} = 0$ (becauſe $\dot{z} = \frac{-\dot{x}}{3x}$, by the nature of logs.); whence z (= hyp. log. of $\frac{0\cdot8}{x^{\frac{1}{3}}}$) $= \frac{4}{3}$, and $0\cdot5790593$ $(\frac{4m}{3})$ is = tabular log. of $\frac{0\cdot8}{x^{\frac{1}{3}}}$, or the log. of $x = \overline{\text{log. of } 0\cdot8 - \cdot5790593} \times 3 = 7\cdot9720921$, and the natural number anſwering thereto is $0\cdot009377608$; which multiplied by 100000, gives 937l. 15s. 2½d. = the lady's fortune.

Meſſ. *J. Ainſwarth*, *T. Barker*, *J. Bennett*, *J. Buddle*, *J. Dalby*, *W. Rawle*, *T. Robinſon*, *J. Roper*, *W. Sewell*, *P. Sharp*, *E. Smith*, *W. Spicer* (the propoſer), and *S. Vince* have alſo anſwered it in this manner nearly, and bring out the ſame concluſion.

IX. QUESTION 574 *anſwered by Mr.* J. Leader.

Subſtitute vx for y in the given equation, and it becomes $av^3 \times x^{10} = bv^2x^{10} + cv^6x^{10} - x^{11} + dv^{10}x^{10}$; and thence $x = dv^{10} + cv^6 - av^3 + bv^2$, $y\ (= vx) = dv^{11} + cv^7 - av^4 + bv^3$, and conſequently $y\dot{x}$ (the fluxion of the area $= \overline{dv^{11} + cv^7 - av^4 + bv^3} \times \overline{10dv^9\dot{v} + 6cv^5\dot{v} - 3av^2\dot{v} + 2bv\dot{v}}$; and the fluent $\frac{10}{21}d^2v^{21} + \frac{16}{17}cdv^{17} - \frac{13}{14}adv^{14} + \frac{1}{13} \times \overline{12bd + 6c^2} \times v^{13} - \frac{9}{10}acv^{10} + \frac{8}{9}bcv^9 + \frac{3}{7}a^2v^7 - \frac{5}{6}abv^6 + \frac{2}{5}b^2v^5 - f$ is $=$ the area required (f being the ſum of the coefficients of the powers v).——Then, from the given equation in fluxions, (by tranſpoſition, diviſion, and multiplying by y) is had $\frac{y\dot{x}}{\dot{y}} = \frac{2bx^8y^2 - 3ax^7y^3 + 6cx^4y^6 + 10dy^{10}}{11x^{10} - 8bx^7y^2 + 7ax^6y^3 - 4cx^3y^6}$, the ſubtangent required.

Anſwers to this queſtion have alſo been received from Meſſ. *J. Ainſworth*, *T. Allen*, *T. Barker*, *Pamphagus*, *T. Robinſon*, *W. Sewell*, *P. Sharp*, and *S. Vince* (the propoſer).

X. QUESTION 575 *anſwered by Mr.* Pamphagus.

Since $y^9 = ax^n$, y^2 will be $= a^{\frac{2}{9}}x^{\frac{2n}{9}}$, and (by page 204 of Simp. Flux.) the $\frac{\text{flu. of } x^{\frac{2n}{9}+1}\dot{x}}{\text{flu. of } x^{\frac{2n}{9}}\dot{x}} = \frac{2n+9}{2n+18}x =$ the diſtance of the center of gravity of the ſaid ſolid from its vertex, which (per queſt.) is $= \frac{25}{34}x$; and conſequently $n = 8$.——Having thus obtained the numerical value of n, we ſhall, in the next place, have (putting $\frac{81}{64a^{\frac{1}{4}}} = c$) $\sqrt{1 + cy^{\frac{1}{4}}} =$ the flux. of the curve's length; and the correct fluent * thereof $\overline{\frac{8\times\overline{1+cy^{\frac{1}{4}}}}{9c}}\Big|^{\frac{3}{2}} \times y^{\frac{3}{4}} - \overline{\frac{6y^{\frac{1}{2}}}{7c} + \frac{24y^{\frac{1}{4}}}{35c^2} - \frac{16}{35c^3}}$ $\frac{128}{315c^4}$ will be the length of the curve required.

Meſſ. *J. Ainſworth*, *T. Allen*, *T. Barker* (the propoſer) *Bennett*, *T. Boſworth*, *N. Brownell*, *T. Todd*, *W. Rawle*, *Sewell*, *W. Spicer*, and *S. Vince* anſwer it in the ſame nner, very nearly.

* See Simpſon's Flux. art. 84.

XI. QUESTION 576 *anſwered by Mr.* T. Todd.

ʼut $c = 3{\cdot}1416$ (the circumference of the circle whoſe meter is 1), $s = 1728$ (the given ſolidity of the required ne), $x =$ its altitude, $y =$ the rad. of its baſe, and $p =$ 2 inches (the length of a pendulum vibrating ſeconds); n (per p. 225 of Simp. Flux.) $\frac{4xx+yy}{5x} =$ the diſt. of the nter of oſcillation from its vertex (or point of ſuſpenſion), d (by the doctrine of pendulums, their lengths being inſely as the ſquares of the number of vibrations made by m in the ſame time) $\frac{60^2\times 5px}{4xx+yy}$ (the ſquare of the number of vibrations performed by the cone in 1′) $= xx$ (per eſt.); from which, and the equation $\frac{1}{3}cxyy = s$, is found x $\overline{4500p - \frac{3s}{4c}}\Big|^{\frac{1}{3}}) = 56{\cdot}039$ &c. and y $(=\sqrt{\frac{3s}{cx}}) = 5{\cdot}426$ inches.

Mr. *Ja. Young* puts $a = 39{\cdot}2$ inches, $s = 1728$, $p = 3{\cdot}1416$, $=$ the height, and $y =$ rad. of the baſe of the required ne, in inches; then $\frac{1}{3}pxyy = s$, and (per pa. 239 of Emer. ux. 1ſt edit. $\frac{4xx+yy}{5x} =$ diſt. of the cent. of oſcillation m its vertex, or the length of a pendulum iſochronal to e cone: Whence, $\sqrt{\frac{4xx+yy}{5ax}}$ being $=$ the time of one ration, $\sqrt{\frac{4xx+yy}{5ax}} : 1 :: 60'' : x$, or $x\sqrt{\frac{4xx+yy}{5ax}} =$ (per queſt.); from whence, and the preceding equation, comes out $= 56{\cdot}03904$ &c. and $y = 5{\cdot}4264$ &c. inches.

In the ſame manner as above, nearly, the anſwer is alſo given by Meſſ. *J. Addiſon*, *J. Ainſworth*, *T. Allen*, *T. Barker*, *T. Boſworth*, *N. Brownell*, *J. Buddle*, *W. Dawes*, *W. Dent*, *W. Dixon*, *E. Jones*, *Pamphagus*, *W. Rawle*, *T. Robinſon*, *W. Sewell* (the propoſer), *C. Smith*, *E. Smith*, *S. Vince*, and *W. Wales*.

XII. QUESTION 577 *anſwered by* Mr. T. Moſs, (*the Propoſer*).

Let ſuch a quantity of the ſtrong ſpirits (*i. e.* 1 to 1, 1 to 2, 1 to 3, &c.) as one gallon of water will reduce to hydrometer proof be $= a$, and the quantity when ſo reduced $= b$ (viz. $b = a + 1$): Moreover, let ſuch a quantity of hydrom. proof as one gall. of water will reduce to the lower ſtrength (*i. e.* 1 in 2, 1 in 3, &c.) $= c$, and the quantity when ſo reduced $= d$; alſo, let the required quantity of water neceſſary to reduce one gallon of the ſtrong to any given ſtrength under hydrom. proof $= x$:——Then, $d : c :: 1 + x : \frac{c + cx}{d}$ = the quantity of dydrom. proof ſpirits in $1 + x$ quantity (or in one gallon of ſtrong); but $a : b :: 1$ (*i. e.* one gall. of ſtrong ſpirits): $\frac{b}{a}$; whence $\frac{c + cx}{d} = \frac{b}{a}$, and $x = \frac{bd}{ac} - 1$.

GEN. RULE. Let ſuch a quantity of the ſtrong as one gallon of water will reduce to hydrometer proof, be multiplied by ſuch a quantity of the ſtrong as one gallon of water will reduce to the given ſtrength under hydrometer proof: Then let the product of the two quantities ſo reduced be divided by the former product; from the quotient ſubtract unity, and the remainder will be the required factor for multiplying any given quantity of ſpirits above hydrometer proof, in order to ſhew the quantity of water neceſſary to be added to make them of any given ſtrength below that proof.

Suppoſe, for example, ſpirits of 1 to 3 were to be reduced to 1 in 7 (*i. e.* to export ſtrength): Here $a = 3$, $b = 4$, $c = 6$, and $d = 7$; therefore, by the above rule, $\frac{bd}{ac} - 1 = \frac{4 \times 7}{3 \times 6} - 1$ $(= 1{\cdot}5555 \text{ \&c.} - 1) = {\cdot}555$ &c. the required factor.

NOTE, This prob. will be of great utility to perſons concerned in trying the ſtrength of ſpirits by the hydrometer.

Meſſ. *J. Ainſworth*, *T. Barker*, and *E. Jones* have likewiſe ſent anſwers to this queſtion.

XIII. QUESTION 578 *anſwered by Mr.* Rd Gibbons.

As this queſt. refers to the common diagonal rod, I ſhall take my example therefrom, where, againſt 22 inches, is 29 wine gallons or 6699 cubic inches. Then, putting $2x =$ the diameter and $2z =$ the axis of the required ſpheroid, $xx + zz$ will be $= 22^2$ (per 47 Euc. I.), and $4{\cdot}1888\,xxz$ (the ſolid content of the ſpheroid, in inches) $= 6699$ (per queſt.). Whence (xx being exterminated) $484z - z^3 = 1599{\cdot}2647$: Reduced, $z = 20{\cdot}112$; and thence $x = 8{\cdot}917$, and their ratio (or the ratio required) is as 2·25 to 1, or as 9 to 4.

Meſſ. *J. Addiſon* and *J. Young* have alſo anſwered this queſt. very ingeniouſly, and bring out the ſame concluſion.

The ſame anſwered by Mr. Joſ. Walker (*the Propoſer*).

Let the content of a ſpheroid (in inches) be denoted by a, its correſponding diagonal (on the gauging rod) by d, and the ſemi-axis, on which the ſpheroid is ſuppoſed to be generated, by x ($p = 3{\cdot}14159$ &c. Then, by the known theorem, $\overline{dd - xx} \times \frac{4px}{3} = a$; which is a general equation. Now, let a, for inſtance, be interpreted by 60 ale gallons, or 16920 cubic inches, and d the correſponding diagonal (nearly on the rod), by 30 inches; then, the above equation becomes $900x - x^3$ $(= 16920 \div \frac{1}{3} \times 4p) = 4040{\cdot}1165$: Which having 2 affirmative roots, we therefore have $x = 27{\cdot}425$ and 4·61, nearly; being reſpectively the ſemi-lengths of the oblong and prolate ſpheroid: Whence ($\sqrt{dd - xx}$) the ſemi-diamenters thereof will be 12·16 and 29·64. Hence it appears that the axes of every oblong ſpheroid, whoſe content may be truly obtained by the common diagonal rod, muſt be in the ratio of 12·16 to 27·425, but thoſe of the prolate one, as 4·61 to 29·64, very nearly: Which ratios agree with thoſe in Moſs's Gauging, pa. 197, and are a confirmation of the truth of the author's principles, with reſpect to the vaſt comprehenſiveneſs and utility of the diagonal rod.

Pamphagus thinks this queſtion is ſolved in a manner ſufficiently ſcientifical in Moſs's Gauging, pa. 197, he ſays, 'excepting to ſuch as are reſolved to join in opinion with 'the doughty reviewer of that excellent treatiſe, and pronounce ſuch a thing impoſſible.

XIV. QUES-

XIV. QUESTION 579 *anſwered by Mr.* H. Brown (*of the* Tower), *Mr.* T. Moſs (*the Propoſer*), *and Mr.* J. Wore.

Cards.	*Combinations.*		*Cards.*	*Combinations.*	
10,5	16.4	64	7,3,2,2,1	4.4.6.4	384
10,4,1	16.4.4	256	7,3,2,1,1,1	4.4.4.4	256
10,3,2	16.4.4	256	7,2,2,2,2		4
10,3,1,1	16.4.6	384	7,2,2,2,1,1	4.4.6	96
10,2,2,1	16.6.4	384	7,2,2,1,1,1,1	4.6	24
10,2,1,1,1	16.4.4	256	6,6,3	6.4	24
9,6	4 4	16	6,6,2,1	6.4.4	96
9,5,1	4.4.4	64	6,6,1,1,1	6.4	24
9,4,2	4 4.4	64	6,5,4	4.4.4	64
9,4,1,1	4 4.6	96	6,5,3,1	4.4.4.4	256
9,3,3	4.6	24	6,5,2,2	4.4.6	96
9,3,2,1	4.4.4.4	256	6,5,2,1,1	4 4.4 6	384
9,3,1,1,1	4.4.4	64	6,5,1,1,1,1	4.4	16
9,2,2,2	4 4	16	6,4,4,1	4.6 4	96
9,2,2,1,1	4.6.6	144	6,4,3,2	4.4.4.4	256
9,2,1,1,1,1	4.4	16	6,4,3,1,1	4.4.4.6	384
8,7	4 4	16	6,4,2,2,1	4.4.6.4	384
8,6,1	4.4.4	64	6,4,2,1,1,1	4.4.4.4	256
8,5,2	4.4.4	64	6,3,3,3	4.4	16
8,5,1,1	4.4.6	96	6,3,3,2,1	4.6.4.4	384
8,4,3	4.4.4	64	6,3,3,1,1,1	4.6.4	96
8,4,2,1	4.4.4.4	256	6,3,2,2,2	4 4.4	64
8,4,1,1,1	4.4.4	64	6,3,2,2,1,1	4.4.6.6	576
8,3,3,1	4.6.4	96	6,3,2,1,1,1,1	4.4.4	64
8,3,2,2	4.4.6	96	6,2,2,2,2,1	4.4	16
8,3,2,1,1	4.4.4.6	384	6,2,2,2,1,1,1	4.4.4	64
8,3,1,1,1,1	4.4	16	5,5,5		4
8,2,2,2,1	4.4.4	64	5,5,4,1	6.4 4	96
8,2,2,1,1,1	4.6 4	96	5,5,3,2	6.4.4	96
7,7,1	6.4	24	5,5,3,1,1	6.4.6	144
7,6,2	4.4.4	64	5,5,2,2,1	6 6.4	144
7,6,1,1	4.4.6	96	5,5,2,1,1,1	6 4.4	96
7,5,3	4.4.4	64	5,4,4,2	4.6.4	96
7,5,2,1	4.4 4.4	256	5,4,4,1,1	4.6.6	144
7,5,1,1,1	4.4.4	64	5,4,3,3	4.4.6	96
7,4,4	4.6	24	5,4,3,2,1	4.4.4.4.4	1024
7,4,3,1	4.4.4.4	256	5,4,3,1,1,1	4.4.4.4	256
7,4,2,2	4.4.6	96	5,4,2,2,2	4.4.4	64
7,4,2,1,1	4.4.4.6	384	5,4,2,2,1,1	4.4.6.6	576
7,4,1,1,1,1	4.4	16	5,4,2,1,1,1,1	4.4 4	64
7,3,3,2	4.6 4	96	5,3,3,3,1	4 4.4	64
7,3,3,1,1	4.6.6	144	5,3,3,2,2	4.6.6	144

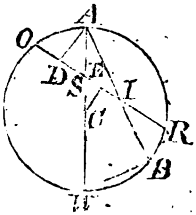
Cards.	Combinations.	
5,3,3,2,1,1	4.6.4.6	576
5,3,3,1,1,1,1	4.6	24
5,3,2,2,2,1	4.4.4.4	256
5,3,2,2,1,1,1	4.4.6.4	384
5,2,2,2,2,1,1	4.6	24
5,2,2,2,1,1,1,1	4.4	16
4,4,4,3	4 4	16
4,4,4,2,1	4.4.4	64
4,4,4,1,1,1	4.4	16
4,4,3,3,1	6.6.4	144
4,4,3,2,2	6.4.6	144
4,4,3,2,1,1	6.4.4.6	576
4,4,3,1,1,1,1	6.4	24
4,4,2,2,2,1	6.4.4	96
4,4,2,2,1,1,1	6.6.4	144
4,3,3,3,2	4.4.4	64
4,3,3,3,1,1	4.4.6	96
4,3,3,2,2,1	4.6.6.4	576
4,3,3,2,1,1,1	4.6.4.4	384
4,3,2,2,2,2	4.4	16
4,3,2,2,2,1,1	4.4.4.6	384
4,3,2,2,1,1,1,1	4.4.6	96
4,2,2,2,2,1,1,1	4.4	16
3,3,3,3,2,1	4.4	16
3,3,3,3,1,1,1		4
3,3,3,2,2,2	4.4	16
3,3,3,2,2,1,1	4.6.6	144
3,3,3,2,1,1,1,1	4.4	16
3,3,2,2,2,2,1	6.4	24
3,3,2,2,2,1,1,1	6.4.4	96
3,2,2,2,2,1,1,1,1		4

The number ſought 17264.

Mr. *J. Pike* has alſo anſwered it very ingeniouſly by a different method.——Mr. *T. Barker* and *Pamphagus* anſwer it from p. 520, 521, of Birks' Arithmetic, where, they inform us, it is calculated by Major Watſon, and the number found to be 17264, or 33528 holes.

XV. QUESTION 580 *anſwered by Mr.* W. Wales.

Draw AD, $CE \perp OR$ (C being the cent. of the circ.), and let the points W, B be joined.



Then, d being $=$ the diameter AW, p, q, $r =$ the coſines of the angles I, S, A (to rad. 1), $x = AI$, and $y = IR$ (per queſt.), $1 : d :: r : dr = AB$; $\because BI = dr - x$. Alſo, $1 : x :: p : px = DI$, and $y \pm px = DR$. Laſtly, (becauſe of the parallel lines AD, CE, and by compoſition of ratios) $1 : \frac{1}{2}d :: q : \frac{1}{2}dq = DE$, and $DR \mp DE = ER = EO = \frac{1}{2}RO$ (cor. 2, 3. Simp. Geom. 2d edit.) $= y \mp \frac{1}{2}dq \pm px$; $\because IO = y \mp dq \pm 2px$, and (21.3) $y \times \overline{y \mp dq \pm 2px} = x \times \overline{dr - x}$; which, properly ordered, gives $yy \mp dqy \pm 2pxy - drx + xx = 0$, a general equation; the upper ſigns of the 2d and 3d terms obtaining when the $\angle S$ is obtuſe and the lower ones when the $\angle I$ is obtuſe, and when they are both acute, the two affirmative ſigns take place. But, if AB be drawn on the other ſide of the diam. AW, the contrary muſt be obſerved in every caſe.

COR. 1. If AB coincides with AW, $p=q$, and $r=$ radius; ∵ the equation, in this caſe, is $yy \mp dqy \pm 2qxy - dx + xx = 0$.

COR. 2. When OR is ⊥ AW, q vaniſhes, and the equation becomes $yy \pm 2pxy - drx + xx = 0$.

COR. 3. Laſtly, when both theſe ſuppoſitions take place, the expreſſion becomes $yy - dx + xx = 0$; the common equation.——And in this manner, nearly, the anſwer is alſo given by Meſſ. *J. Ainſworth*, *Pamphagus*, *W. Sewell*, and *T. Todd.*

Anſwer to the ſame by Mr. Cha. Hutton (*the Propoſer*).

Put $a=AB$, and s, n, and $c=$ the ſines of the ∠s I, S, and A (the reſt the ſame as above); then (by trigonom.) $cxn^{-1}=IS$ and $sxn^{-1}=AS$; and (by the prop. of the circle, OS being $=AS\times SW\times RS^{-1}=sxn^{-1}\times\overline{d-sxn^{-1}}\times\overline{y+cxn^{-1}}|^{-1}$ and $OI\times IR=AI\times IB$) $\overline{snx^{-1}\times\overline{d-sxn^{-1}}\times\overline{y+cxn^{-1}}|^{-1}+cxn^{-1}}\times y=x\times a-x$: But $a=dr$, $s=cq+nr$, and $p=cn-qr$; whence, by ſubſtitution, $\left.\begin{matrix}yy+dq\\+2px\end{matrix}\right\}y\left.\begin{matrix}-drx\\+xx\end{matrix}\right\}=0.$

COR. 1. Hence, $y=-px-\frac{1}{2}dq\pm\sqrt{\overline{px+\frac{1}{2}dq}|^{2}+drx-xx}$.

COR. 2. If $y=0$, then $x=0$, or $=dr(=AB)$ as it ought.

COR. 3. If $x=0$, then $y=0$, or $=-dq$.

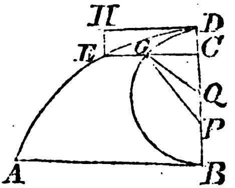
COR. 4. If $x=AB=dr$, then $y=0$, or $=-d\times\overline{2pr+q}$.

COR. 5. If either S or I be an obtuſe angle, the ſign of q or p muſt be changed wherever it is found. A can never be an obtuſe angle, and therefore r is always affirmative.

COR. 6. If AB be on the other ſide of AW, the ſigns of p and q will be changed.

COR. 7. If AB coincides with AW, then $r=1$, $p=q$, and the general equation is $yy+dqy+2qxy-dx+xx=0$; which, when $S=$ a right angle (q being then $=0$) becomes barely $yy=dx-xx$, the common equation to the circle.

XVI. QUESTION 581 *anſwered by Mr.* T. Allen (*the Propoſer*).



Let *DGB* be any curve meeting the axis *DB* in *B*, and let *P* be a given point in that axis. Then, putting the abſciſſa $DC = x$, ord. $CG = y$, and $DP = a$, $\frac{y\dot{x} + a\dot{y} - x\dot{y}}{2}$ will truly exhibit the fluxion of the ſpace *DGP*. Wherefore, in the preſent caſe, putting *DQ* (rad.) $= r$, $QP = q$, and the arc $DG = z$, we ſhall have $\frac{y\dot{x} + a\dot{y} - x\dot{y}}{2} = \frac{\dot{x}\sqrt{2rx - xx}}{2} - \frac{rx\dot{x} - x^2\dot{x}}{2\sqrt{2rx - xx}} + \frac{a\dot{y}}{2} = \frac{rx\dot{x}}{2y} + \frac{a\dot{y}}{2}$; whoſe fluent, $\frac{rz + qy}{2}$, * is the area of the ſpace *DGP*. If therefore this area be to the ordinate *EC* in the conſtant ratio of 1 to m, the ordinate itſelf will be $= \frac{m}{2} \times \overline{rz + qy}$.

CORROL. Suppoſe $m = 2$, and draw *DH* parallel and $EH \perp CE$, then will $rx\dot{z} + qx\dot{y} = \frac{arx\dot{x} - qx^2\dot{x}}{y}$ be the flux. of the ſpace *EDH*; whoſe fluent is $ar \times \overline{z - y} + \frac{3qry + qyx - 3qrz}{2}$: And therefore the area of *EDC* will be $= rzx + \frac{1}{2}qyx + ar \times \overline{y - z} + \frac{3}{2}qr \times \overline{z - y}$; which, when $x = 2r$, becomes $= \frac{3{\cdot}1416rr}{2} \times \overline{4r - 2a + 3q}$, the true area of the whole curve *DAB*.

Anſwer

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* This expreſſion is no more than the ſum of the two values of the ſector $DGQ = \frac{1}{2}rz$, and $\triangle QPG = \frac{1}{2}qy$, as given by the common known rules; and therefore all the fluxionary proceſs from the beginning of the ſolution to this expreſſion, is quite needleſs, and may be omitted.

Anſwer to the ſame by Mr. Cha. Hutton.

Let $\mathcal{Q}$ be the center of the ſemicircle DGB, and draw $\mathcal{Q}G$. Put $r = D\mathcal{Q}\ (= \mathcal{Q}G = \mathcal{Q}B)$, $d = \mathcal{Q}P$, and $n = \dfrac{\text{the ſemicir. } DGBD}{AB} = \dfrac{\text{the area } DGPD}{EC}$. Then, it is evident that $EC = \left(\dfrac{DGPD}{n} =\right) \frac{1}{2} rn^{-1} \times$ the arc $DG \pm \frac{1}{2} dn^{-1} \times GC$, according as P is below or above $\mathcal{Q}$.

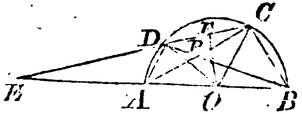
COR. 1. If P coincides with $\mathcal{Q}$, $d = 0$, and then $EC = \frac{1}{2} rn^{-1} \times DG$.

COR. 2. If P coincides with B, then $d = r$, and $EC = \frac{1}{2} rn^{-1} \times \overline{DG + GC}$.

COR. 3. If P coincides with D, $d = -r$, and then $EC = \frac{1}{2} rn^{-1} \times \overline{DG - GC}$.

This queſtion is likewiſe anſwered by Meſſ. *T. Barker*, *J. Bennett*, *J. Buddle*, *Pamphagus*, *Plus Minus*, *W. Sewell*, *T. Todd*, and *S. Vince*.

The PRIZE QUESTION *anſwered by Mr.* W. Wales.



CONSTRUC. From O, the center of the circumſcribing ſemicircle, draw the radii OC, OD, making with each other an angle = twice the complement of that formed by the diagonals at the point of interſection; and through O draw BE, meeting the ſemi-circumference in B, A and the right line, joining the points C, D, produced in E, ſo that AE may = the given intercepted diſtance; then, join the points B, C and A, D, and the thing is done.

DEMONST. Let the diagonals AC, DB be drawn, interſecting each other in P. The $\angle ACB$ (PCB), being in a ſemicircle, is a right angle; whence the $\angle PBC$ is the complement of the $\angle P$, = half the $\angle DOC$ (Simp. Geom. 10. 3. 2d edit.).——The method of calculation, from this conſtruc. is extremely obvious and eaſy.

CONSTRUCTION *to the ſame by Mr.* Tho. Boſworth.

Deſcribe the $\triangle ECO$, having the $\angle C =$ the given angle to be formed at the interſection of the diagonals, the ſide $OC = \frac{1}{2}$ the given ſide of the trapezium, and the ſide $OE = OC +$ the given diſtance to be intercepted, and with the radius OC deſcribe a ſemicircle meeting EO, produced in B, and cutting EC in D; join A, D and B, C, and then will be formed the trapezium required.——For, letting fall $OF \perp DC$ and drawing the diagonals, interſecting each other in P, it is evident (from the properties of the circle, &c.) that the $\angle DBC = \angle FOC$, and the $\angle ACB = \angle OFC$ (being both right angles); wherefore the $\angle CPB = \angle ECO$, the given angle, by conſtruction.

Exceedingly neat and conciſe conſtructions to this queſtion have alſo been received from Meſſ. *T. Adams*, *J. Ainſworth*, *T. Barker*, *J. Bennett*, *D. Bolton*, *J. T. Berſham*, *J. Buddle*, *R. Butler*, *J. Chipchaſe*, *W. Cole*, *J. Dalby*, *Nujo Dargnas*, *J. Davidſon*, *W. Dixon*, *J. Dymond*, *P. George*, *R. Gibbons*, *C. Hutton*, *E. Jones*, *L. Ker*, *Pamphagus*, *Plus Minus*, *J. Powle*, *R. Pulman*, *Don Quixote*, *W. Rawle*, *J. Roper*, *W. Sewell*, *E. Smith*, *W. Smith*, and *S. Vince*.——The conſtruction by Mr. *T. Moſs* (the propoſer) is very ingenious, and depends on a new prop. of the circle not yet publicly known.

The two prizes of 12 *and* 8 *Diaries, for the ſolution of the prize queſtion, are fallen to the reſpective lots of Mr.* William Wales *and Mr.* Tho. Boſworth.

The Eclipſes calculated for 1768.

There will happen ſix eclipſes this year; three of the ſun, inviſible, and three of the moon, viſible in Great Britain.

The firſt is of the moon, on Monday the 4th of January, in the morning, according to the following calculations.

Calculated by	Begin.		Mid.		End		Dur.		Digits	
	h.	m.	h.	m.	h.	m.	h.	m.	°	′
Mr. Chapman, for Foxton	3	$12\frac{1}{2}$	4	23	5	$33\frac{1}{2}$	2	21	4	54
Mr. T. Allen, for Spalding	3	17	4	24	5	32			4	34
Mr. Metcalfe, for London	3	$13\frac{1}{3}$	4	$24\frac{1}{2}$	5	$35\frac{1}{2}$	2	$22\frac{1}{4}$	4	$57\frac{5}{6}$

This eclipſe will be viſible to all Europe, America, and great part of Africa.

The ſecond is of the ſun, on Tueſday the 19th of January, about 6 o'clock in the afternoon, viſible in North America.

The third is a total lunar eclipſe, on Thurſday the 30th of June, in the morning, correſponding to the following calculations.

Calculated by	Beg. of Eclipſ. h. m.	Beg. of to. Da. h. m.	Mid. h. m.	End to. D. h. m.	End Eclip. h. m.	Dur. Eclip. h. m.	Digits eclipſ. ° ′
W. Chapman	2 4¾	3 17½	3 51 1/15	4 24½	5 37½	3 32⅔	14 27
T. Allen	2 7	3 22⅓	3 50⅔	4 19	5 34½		
J. Metcalfe	2 17 1/10	3 23 1/15	3 54	4 24⅚	5 30⅚	3 13⅚	14 26⅙

This eclipſe may be ſeen from the beginning to the end in moſt known parts of America, and the weſtern parts of Africa: But in the weſtern parts of Europe, only part of it can be ſeen; for the moon ſets before the eclipſe ends.

The fourth is a ſolar eclipſe, on Thurſday the 14th of July, near 2 o'clock in the morning. It will be viſible in the remote ſouthern parts of the earth, beyond New Holland.

The fifth eclipſe is alſo of the ſun, upon Friday the 9th of December, near 8 in the morning, but utterly inviſible to all known parts of the globe.

The ſixth, and laſt, this year, is another total eclipſe of the moon, and will happen upon Friday the 23d of December, in the afternoon. The moon riſes totally eclipſed a little before the ſun ſets; and, if the air proves clear, they may both be ſeen above the horizon, at the ſame time.

Calculated by	Beg. Eclip. h. m.	Beg. to. D. h. m.	Mid. h. m.	End to. D. h. m.	End Eclip. h. m.	Dura. Eclip. h. m.	Dig. eclip. ° ′
Mr. W. Chapman	1 18⅕	2 16⅕	3 5⅕	3 54⅕	4 52⅕	3 34	20 45
Mr. T. Allen	1 21⅙	2 18⅓	3 6½	3 54¾	4 51⅚		
Mr. J. Metcalfe	1 18⅔	2 16⅚	3 6½	3 56¼	4 54½	3 35⅚	20 50

At the beginning of this eclipſe the moon will be vertical to the eaſtern ocean, at the middle of the Chineſe ſea, and at the end, to the ſouth coaſt of China; it will therefore be ſeen, from the beginning to the end, in all parts of Aſia and the Eaſt Indies, the eaſtern parts of Europe and Africa, and to parts near the pole; but only viſible in part to the weſtern kingdoms of Europe, becauſe the moon riſes not in thoſe countries till after the eclipſe begins.

JOHN METCALFE.

Meſſ. *H. Andrews*, *T. Atkinſon*, and *J. Denham* have alſo obliged us with calculations of theſe eclipſes, but we could not find room for their inſertion.

Mr. *W. Chapman*, of *Foxton*, in *Leiceſterſhire*, has favoured us with a curious table of all the ſolar eclipſes that will be viſible from the year 1769 to the year 2000; which we are ſorry our narrow limits will not this year permit us to publiſh.

New Queſtions.

I. QUESTION 582, *by Mr.* Geo. Lodge.

Diarian ſages, you will find,
From the * equation here ſubjoin'd,
The age and fortune of a fair,
Whoſe life is blameleſs, heart ſincere.

* viz. $\left\{\begin{array}{l} xxy + \dfrac{\sqrt{x}}{y} = 16386, \\ xyy + \dfrac{x}{\sqrt{y}} = 1056; \end{array}\right\}$ where x repreſents the number of 10l. notes that make up the fortune, and y the ſquare root of the age of this amiable fair one.

II. QUESTION 583, *by Mr.* Wm. Spicer.

A heavy body deſcending freely by the force of gravity on an inclined plane, whoſe length is 400 feet, deſcends 112 feet of the ſaid length in the laſt ſecond; required the altitude of the ſaid plane and the time of deſcent.

III. QUESTION 584, *by Maſter* J. Paty, *at the Mathematical Academy*, Briſtol.

Suppoſing a cow to bring forth a ſhe calf at the age of two years, and then to continue yearly to do the ſame, and every one of her brood to bring forth a ſhe calf at the age of two years, and afterwards yearly likewiſe; how many may ſpring from the old cow and her brood in 40 years?

IV. QUESTION 585, *by Mr.* Jer. Ainſworth.

Required to find a fraction ſuch, that being taken from its reciprocal the remainder ſhall be a ſquare.

V. QUESTION 586, *by Miſs* Ann Nicholls.

I made obſervation, laſt ſpring, at two places under the ſame meridian, differing in latitude 4°, and found the ſun to riſe exactly at 5 o'clock in the one and at 4 in the other, and that the difference of his meridian altitude at the ſaid places, at the times of obſervation, was 2°; the latitude of each place and the days of obſervation are required.

VI. QUESTION 587, *by Mr.* C. Smith.

A deſigner, having occaſion to delineate in perſpective a longeſt half of an ellipſe coinciding with the horizontal plane, deſires to know the true breadth of its projection, when projected on a perſpective plane touching (both it and its vertical) the neareſt end of the tranſverſe diameter, and making an angle with the ſaid diameter of 50°; the diſtance of the eye from the perſpective plane being 100, the tranſverſe diameter 32·92, and the ſemi-conjugate 11 feet.

VII. QUESTION 588, *by Mr.* Joſ. Dymond.

Given the right line biſecting the vertical angle of a plane triangle, and terminating in its baſe, and the perpendiculars falling thereon (produced) from the extremities of its baſe; to determine and conſtruct the triangle.

VIII. QUESTION 589, *by* Pamphagus.

Given the baſe, one of its adjacent angles, and the line biſecting the vertical angle of any plane triangle; to determine the ſaid triangle.

IX. QUESTION 590, *by* Nujo Dargnas.

To determine a point, from which if right lines be drawn to the three angular points of a given plane triangle, the ſum of their ſquares ſhall equal a given ſpace (aa), and if from the ſaid point lines be drawn to the three ſides of the triangle (produced if neceſſary) making given angles therewith, the rectangle contained under the line drawn to one of the ſides and a given line (b) ſhall be equal to the ſum of the rectangles under the other drawn lines and given lines (c, d).

X. QUESTION 591, *by Mr.* Paul Sharp.

Required the area of the common parabola, whoſe abſciſſa is $=10$, and radius of curvature, at the bounding ordinate, equal to three times a tangent drawn from thence and terminating in its axis produced.

XI. QUESTION 592, *by Mr.* J. Chipchaſe.

There is a baſon, in form of ſemi-globe, filled with ſpring water and placed on horizontal ground, and a perſon can ſee a piece of money laid at the bottom of it to the diſtance of nine feet from its axis, when quite full; but, when filled to only half the whole depth, he can ſee it no further than to the diſtance of 6 feet therefrom: Required the height of the perſon's eye and the diameter of the baſon.

XII. QUESTION 593, *by Mr.* J. Bennett.

To determine the nature of the curve, whoſe tangent terminated every where by it and an indefinite right line CD, is a conſtant quantity $= 100$; alſo, to determine the length of the part thereof intercepted between its higheſt point and that point whoſe height, above the ſaid line CD, is $= 20$.

XIII. QUESTION 594, *by Mr.* S. Vince.

To find the ſum of the infinite ſeries $\frac{1}{1.5.v^5} + \frac{1}{1.2.3.7.v^7} + \frac{1.3.3}{1.2.3.4.5.9.v^9} + \frac{1.3.3.5.5}{1.2.3.4.5.6.7.11.v^{11}} +$ &c.

XIV. QUESTION 595, *by Mr.* T. Moſs.

To draw a right line from the given point R, [*ſee the fig. to the ſolution*] in the baſe FA, produced, of the given triangle ACF, interſecting AC in n, and meeting CF in E; ſo that, drawing the right line AE, the triangle AnE may be the greateſt poſſible.

XV. QUESTION 596, *by Mr.* Cha. Hutton.

Let NO be the fixed and MP the ſemi-revolving axe of any ſpheroid [*ſee the fig. to the ſolution*] $AGBQA$ any ſection of it by a plane inclined to the axes, and NPO another ſection through its axes and perpendicular to the former ſection: It is required to find the figure of the ſection AGB and the ſolidity of the part cut off the ſpheroid by it, both in the caſe of the oblong and oblate ſpheroid, ſuppoſing the diſtances DM, ME, of the perpendiculars BD, AE, from the center M, to be 5 and 12, and the axes 50 and 30.

The PRIZE QUESTION, *by Mr.* Tho. Allen.

If a chain 40 feet long, conſiſting of exceedingly ſmall links of equal denſity, be put over a pulley (void of friction) with the two ends *A* and *B* hanging down, to the reſpective diſtances of 21 and 19 feet from the pulley, and in theſe circumſtances, ſuppoſe the chain to put itſelf in motion. It is required to determine in what time the end *A* will arrive at an horizontal plane 200 feet below the pulley.

1769.

Queſtions anſwered.

I. QUESTION 582 *anſwered by Mr.* Rd Gibbons.

IT is manifeſt (from the data) that x and y (and conſequently xxy and xyy) are whole ſquare numbers; whence $xxy + \frac{\sqrt{x}}{y}$ being $=$ (16386 $=$) $128^2 + 2$ and $xyy + \frac{x}{\sqrt{y}}$ $=$ (1056 $=$) $32^2 + 32$ (per queſt.), $\frac{\sqrt{x}}{y}$ will be $= 2$, and $\frac{x}{\sqrt{y}}$ $= 32$, and conſequently $x = 64$ and $y = 4$. Whence the age of this amiable fair one is 16 years, and her fortune is 640l.——All vaſtly pretty!

In this manner, nearly, it is alſo anſwered by Meſſ. *J. B. Aſhton*, *T. Atkinſon*, *T. Barker*, *J. Coulthred*, *Curioſus*, *H. Curtis*, *R. Dening*, *W. Dent*, *G. Lodge* (the propoſer), Miſs *Ann Nicholls*, *J. Nordon*, Maſter *Jer. Oſborne*, *Pamphagus*, Maſter *J. Paty*, *E. Reed*, *W. Reynolds*, *Reub. Robbins*, *T. Robinſon*, *Alex. Rowe*, *P. Sharp*, *R. Snowball*, *W. Spicer*, *W. Stoker*, *T. Walker*, and *J. Young*.

Mr. *Ja. Mills*, of *Brixworth*, anſwers it in the following manner.——To the propoſer.

The age, kind ſir, of your moſt charming fair,
Is juſt SIXTEEN——th' equations make appear;
Her fortune too, more charming to behold,
Exact SIX HUNDRED FORTY POUNDS in gold.

At Hymen's altar I with joy could ſtand,
And gladly take this maiden by the hand;
Haſte then, dear Lodge, and crown the nuptial day,
Fatal may prove one moment's ſhort delay!

II. QUESTION 583 *anſwered by Mr.* Paul Sharp.

By the laws of deſcending bodies and the diviſion of ratios, $\sqrt{400}-\sqrt{289}:\sqrt{400}::1'':\sqrt{400}\div\sqrt{400}-\sqrt{289}$ $=(\frac{20}{3}=6\frac{2}{3}''$, the whole time of deſcent; and the diſtance that would be perpendicularly deſcended in the ſame time is $714\frac{22}{27}$ feet: Whence the altitude of the plane required is readily found $=(400^2\div714\frac{22}{27}=)$ $223\frac{161}{193}$ feet.

Much after the ſame manner the anſwer is alſo given by Meſſ. *J. Addiſon*, *T. Barker*, *J. Bartlett*, *J. Chipchaſe*, *J. Clough*, *W. Dent*, *J. Edwards*, *R. Gibbons*, Maſter *Jer. Oſborne*, *Pamphagus*, *E. Parnal*, Maſter *J. Paty*, *R. Reed*, *W. Reynolds*, *T. Robinſon*, *Alex. Rowe*, *R. Snowball*, *W. Spicer* (the propoſer), *W. Stoker*, *S. Vince*, and *J. Young*.

III. QUESTION 584 *anſwered by Mr.* W. Spicer.

From the nature of the queſtion, it appears that the increaſe in the firſt year will be 0, in the 2d year 1, in the 3d year 1, in the 4th year 2, in the 5th year 3, in the 6th year 5, and ſo on to 40 years or terms (each term being = the ſum of the two next preceding ones); whence, the two laſt terms are 39088169 and 63245986, and the ſum of them all (or the whole ſeries) $=2\times63245986+39088169-1=)$ 165580140, the increaſe required.

And thus it is alſo anſwered by Meſſ. *J. Brown Aſhton*, *T. Barker*, *H. Brown*, *Curioſus*, *R. Dening*, *W. Dent*, *J. Nordon*, Maſter *Jer. Oſborne*, *Pamphagus*, Maſter *J. Paty* (the propoſer), *E. Reed*, *Alex. Rowe*, *P. Sharp*, *R. Snowball*, *W. Stoker*, and *J. Young*, very nearly.

IV. QUESTION 585 *anſwered by Mr.* Tho. Barker, *of* Wiſſett.

Let $\frac{1}{x}$ be the required fraction; then will $x-\frac{1}{x}=aa$, a ſquare number (per queſt.), and conſequently $x=\frac{1}{2}\sqrt{a^4+4}$ $+\frac{1}{2}aa$; whence it appears that if ſuch a fraction *a* be found as that its biquadrate added to 4 may be a ſquare number, the conditions of the queſtion will be anſwered, &c.

The ſolutions by *Pamphagus* and Meſſ. *T. Robinſon*, *Alex. Rowe*, *R. Snowball*, and *S. Vince*, are nearly the ſame.

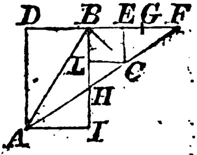
V. QUESTION 586 *anſwered by Mr.* Wm. Gawith.

Since the difference in latitude of two places ſituate under the ſame meridian is univerſally equal to the difference of the ſun's meridional altitudes at thoſe places for the ſame day, it is manifeſt that during the interval of the two obſervations, the ſun had changed his declination 2°. This being premiſed, let a and b repreſent the ſines of the two given aſcenſional differences, c and d the tangents of 2° and 4° reſpectively, and x the tangent of the ſun's declination at the firſt time of obſervation: Then (by Crakelt's tranſlation of Mauduit's Trigonometry, p. 31) $\frac{x+c}{1-cx}$ will be the tangent of the declination at the ſecond time of obſervation; and (by ſpherics) $\frac{a}{x}$ and $\frac{b-bcx}{c+x}$ the tangents of the leſs and greater latitudes reſpectively; the difference of which being 4° (per queſt.), $\frac{a+dx}{x-da}$ will be $=\frac{b-bcx}{c+x}$, or $xx \times \overline{d+bc} + x \times \overline{a+cd-b-dabc} = -ca-dab$, and $x =$ tang. 7° 54′ 50″, correſponding to April 10th: Whence the other day of obſervation will be found to be April 15th; and the two latitudes 48° 48′ 19″ and 52° 48′ 19″.

This queſtion is alſo ingeniouſly and conciſely anſwered by Meſſ. *T. Barker*, *J. Chipchaſe*, *W. Crakelt*, *E. Parnel*, *T. Robinſon*, *W. Spicer*, *W. Stoker*, and *S. Vince*.

The direction of the viſual line from the eye to the perſpective plane not being given in propoſing QUESTION 587, has prevented any of our ingenious correſpondents anſwering it; but were it ſufficiently limited, nothing more would be required than To find the breadth of the ſection of a given elliptical cone, made by a plane paſſing through the extremity of the greater diameter of its baſe in a given direction; which may be eaſily affected, as *Pamphagus* juſtly obſerves.

VII. QUESTION 588 *anſwered by* Mr. J. Chipchaſe.



CONSTRUC. On the right line DG take DB and BE reſpectively equal to the greater and the leſs given perpendiculars, and erect thereon, at B, the perp. $BH =$ the given biſecting line: Produce DG till $DF : EF :: DB : BE$ (*i. e.* in the ratio of the given perpendiculars), and through the points F, H let a right line be drawn, meeting EC, DA (perpendicular to DF) in C and A; join the points A, B and C, B, and ABC will be the triangle required.

DEMONSTRA. Draw AI and $CL \perp BH$ (produced) in I and L; then, DF being to $EF :: DA : EC$ (per ſim. △s) $:: DB : BE$ (per conſtruc.) and the ∠s D and E equal (being both right), the △s BDA, BEC will be ſimilar (Euc. 6. 6.), and conſequently, taking the equal ∠s DBA, EBC from the right ∠s DBH, EBH, their remainders, viz. the ∠s ABH and CBH will be equal; whence BH manifeſtly biſects the $\angle ABC$, and AI, CL are reſpectively $= DB$, BE (the given perpendiculars, per conſtruction).

The method of calculation from this conſtruction is exceedingly eaſy.

Neat and elegant conſtructions to this queſtion have alſo been received from Meſſ. *J. Addiſon*, *T. Barker*, *W. Crakelt*, *H. Curtis*, *J. Dymond* (the propoſer), *T. Moſs*, Maſter *Jer. Oſborne*, Maſter *J. Paty*, *T. Robinſon*, *Alex. Rowe*, *R. Snowball*, *W. Spicer*, and *S. Vince*.

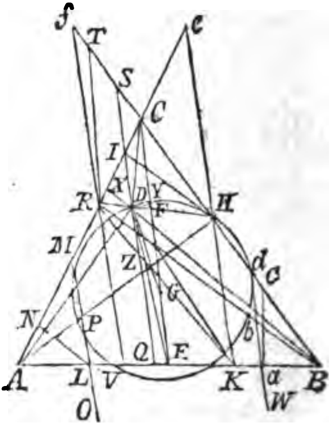
VIII. QUESTION 589 *anſwered by Maſter* Jer. Oſborne, *at the Mathematical Academy*, Briſtol.

Suppoſe the baſe given $= a$, the biſecting line $= b$, the ſine and coſine of the given angle at the baſe $= s$ and c, and the ſine of half the vertical angle $= x$ (rad. $= 1$) Then, (per trigonom.) the ſine of the angle the line biſecting the vertical angle makes with the baſe will be $= s\sqrt{1-xx} + cx$, and that of the vertical angle itſelf $= 2x\sqrt{1-xx}$; alſo $\frac{bs\sqrt{1-xx}+bcx}{s}$ will be found $=$ the ſide adjacent to the given angle, $\frac{bx}{s}$ = the ſegment of the baſe (made by the ſaid biſecting line, and) adjacent to the ſame, $\frac{sa}{2x\sqrt{1-xx}}$ = the other ſide, and $\frac{as-bx}{s}$ = the other ſegment. Whence (Euc. 3. 6.) $abss\sqrt{\overline{1-xx}} + abscx - bbsx\sqrt{\overline{1-xx}} - bbcxx = \frac{abss}{2\sqrt{1-xx}}$, or (when properly reduced) $x^6 \times 4b^2 - x^5 \times 8abs + x^4 \times \overline{4aass - 4bbss - 4bb} + x^3 \times \overline{4abs^3 + 8abs} + xx \times \overline{4bbss - 4aass} - 4abs^3x + a^2s^4 = 0$; from whence x may be determined.——Maſter *J. Paty*'s anſwer is not greatly different.

It is alſo anſwered by Meſſ. *J. Addiſon*, *T. Barker*, *Pamphagus* (the propoſer), *T. Robinſon*, *Alex. Rowe*, *R. Snowball*, *W. Spicer*, *S. Vince*, and *T. Walker*.

IX. QUEST. 590 *anſwered by* Nujo Dargnas (*the Propoſer*).

CONSTRUC. Let ACB be the given triangle; the given lines b, c, d are referred to AB, AC, and BC reſpectively. From any point L in AB drawn LM, LN making the angles ALM, ANL equal to the given angles to be made with the ſides AB, AC reſpectively: take $OL : LN :: c : b$ and $LP : PM :: OL : LM$, and draw AP meeting BC in H. In like manner from any point a in AB draw ad, ac making the angles Bad, Bca = the given angles to be made with the ſides AB, BC reſpectively: Take $Wa : ac :: d : b$ and $ab : bd :: Wa : ad$; join B, b, meeting AC in R, and join R, H. Again, biſect AB in E; join E, C and take $EG = \frac{1}{2}GC$, and deſcribe a circle with a radius GY ſuch that $3GY^2 = aa - AB \times AE - 2EC \times EG$, meeting RH (produced if neceſſary) in D, the point required.



DEMONS. From R and H draw RV, $HK \parallel DQ$ or ML; and RT, $HI \parallel DS$ or ac; and DX or LN reſpectively: Join D, E; D, G and R, K meeting DQ (produced if neceſſary) in Z, and draw $DF \perp CE$; produce, if neceſſary, HK, RV to meet AC, CB produced in e and f.——Then $AD^2 + BD^2 = AB \times AE + 2ED^2$ (Simp. Geom. II. 3.) $= AB \times AE + 2EF^2 + 2DF^2$, and adding to each $CD^2 = CF^2 + FD^2$, $CD^2 + AD^2 + BD^2 = AB \times AE + 2EF^2 + CF^2 + 3FD^2$. But $CF^2 + 2EF^2 = CD^2 - 3DF^2 + 2ED^2 = DG^2 + CG^2 - 2CG \times GF$ (CD^2, Euc. 13. 2.) $+ 2DG^2 + 2EG^2 + 2CG$ ($4EG$) $\times GF$ ($2ED^2$ (Euc. 12. 2.)$- 3DF^2 = 3FG^2$ ($3DG^2 - 3DF^2$) $+ 2EC \times EG$ ($CG^2 + 2EG^2$); whence $AD^2 + BD^2 + CD^2 = AB \times AE + 3FG^2 + 2EC \times EG + 3FD^2 = AB \times AE + 2EC \times EG + 3GY^2$ ($3DG^2$) $= aa$, the given ſpace (per conſtruc.). Again, $KH : He :: LP : PM :: OL : LM$, and $IH : He :: LN : LM$; whence, by equality, $KH : IH :: OL : LN :: c : b$ (per conſtruction), and conſequently $KH \times b = IH \times c$. Alſo, $VR : Rf :: ab : bd :: Wa : ad$, and $RT : Rf :: ac : ad$; by equality, $VR : RT :: Wa : ac :: d : b$ (per conſtruction); whence $VR \times b = RT \times d$. Again, $DZ \times b : KH \times b :: ZR : RK :: DR : RH :: DX \times c : IH \times c$: But $KH \times b = IH \times c$; whence $DZ \times b = DX \times c$. Alſo, $QZ \times b : RV \times b :: KZ : RK :: DH : RH :: DS \times d : RT \times d$: But $RV \times b = RT \times d$; whence $QZ \times b = DS \times d$, and therefore $DS \times d + DX \times c = (QZ \times b + DZ \times b =) DQ \times b$. Q.E.D.

If the point D (required) falls without the $\triangle ACB$, the preceding concluſion will become $DS \times d - Dx \times c = DQ \times b$, or $DX \times c - DS \times d = DQ \times b$, according as the ſaid point is ſituate on the left or right hand ſide of the line EC; the demonſtration whereof is exactly the ſame as above.——*Pamphagus* has ſolved this queſtion algebraically.

X. QUESTION 591 *anſwered by Mr.* J. Addiſon.

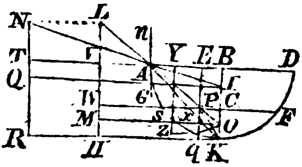
Put the given abſciſſa $(= 10) = a$, and the correſponding ordinate $= y$; then will $2a$ be the ſubtangent, $\sqrt{4aa + yy}$ the tangent and $\frac{yy}{a}$ the latus-rectum of the parabola required, and (per Simp. Flux. pa. 75, 2d edit.). $\frac{\overline{4aa + yy}|^{\frac{3}{2}}}{2ay}$ = the radius of curvature at its bounding ordinate;

whence $\frac{\overline{4aa+yy}|^{\frac{3}{2}}}{2ay}+y=3\sqrt{4aa+yy}$ (per queſtion); ſolved, $y=9·5$, and the area required $=126·66$, &c.

And according to this method, nearly, the anſwer is alſo given by Meſſ. *T. Barker*, *Pamphagus*, *T. Robinſon*, *Alex. Rowe*, and *S. Vince*; but a preſs error having unluckily happened in propoſing this queſtion, viz. *and* inſtead of *at* before "the bounding ordinate, &c." it is become neither ſo neat nor ſimple as was intended; for, according to the propoſer's ſubſtitution and meaning, a is = the given abſciſſa, $4x$ = the parameter, $2\sqrt{aa+ax}$ the tangent $\frac{2a+2x}{x}\times\sqrt{ax+xx}$ the radius of curvature at the ordinate, and $\frac{2a+2x}{x}\times\sqrt{ax+xx}=3\times2\sqrt{aa+ax}$, or $a+x=3\sqrt{ax}$; whence $x=1·459$, the parameter $=5·836$, the bounding ordinate $=15·27874$, and the area $=101·85826$.

XI. QUEST. 592 *anſwered by the Rev. Mr.* C. Wildbore.

Let ABK repreſent half the baſon, and join A, K; then ſhall the $\angle GAK=BAK$ ($45°$) = the angle of refraction at the 1ſt ſtation: Make $AI=\frac{3}{4}AK$, ſo ſhall GAI = the correſponding angle of incidence, their ſines being to each other as 3 : 4; whence $BI=\frac{1}{2\sqrt{2}}AB=·35355333\,AB$.



Now, let CS be the ſurface of the water, $BY=\frac{\sqrt{7}}{4}\times AB$, $xz=\frac{3}{8}AB$, and from the center Y, on the line Cx, and with the line (or diſtance) xz, deſcribe the conchoid of Nicomedes zO, and from the point O, where it cuts BK, draw OL through A; join K, P; ſo ſhall OPq be the angle of incidence and KPq that of refraction, at the 2d ſtation. For the common equation of the conchoid is $\frac{CO\times BY}{BO}$ $\left(=\frac{\sqrt{7}}{4}CP\right)=\sqrt{xz-CO^2}$, or $28CP^2=9AB^2-64CO^2$,

or

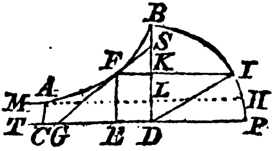
$4OP^2$ ($= 64CP^2 + 64CO^2$) $= 9AB^2 + 36CP^2$
$6CK^2 + 36CP^2 = 36KP^2$, or $8OP = 6KP$:
ence, as 8 : 6, or 4 : 3 :: $KP : OP$, and conſequently
ines of the angles OPq, KPq are in the ſame ratio,
muſt be the required angles.——The equation of the
hoid gives $CO = {\cdot}287087953 \times AB$. Alſo, by ſim. △s,
$+ {\cdot}287087953 :: 6\ (OM) : LM = 4{\cdot}7225274$, and AB
I, or $1 : \frac{1}{2\sqrt{2}} :: QI\ (9) : NQ = 3{\cdot}1819800$; whence
$- NQ = 1{\cdot}5405474 = IO = \overline{{\cdot}287087953 + \frac{1}{2} - \frac{1}{2\sqrt{2}}}$
B, and conſequently $AB = 3{\cdot}553456$ (the radius of the
n), and $IO - CO = IC = {\cdot}5203914$; which give NQ
$C + CK = 5{\cdot}4790994$ feet, the height of the ſpectator's

eſſ. *J. Addiſon*, of *Hexham*, *J. Chipchaſe* (the propoſer),
W. Crakelt have likewiſe anſwered this queſtion very
nioufly.

r. *T. Barker*, of *Wiſſett*, and Mr. *S. Vince*, of *Harleſton*,
alſo ſent anſwers to it; which are all the ſolutions we
e received.

XII. QUESTION 593 *anſwered by Mr.* T. Allen.

et the abſciſſa $CE = x$, ordinate $EF = y$, curve AF
; and let $GF = DB =$
$= a$, $AC = 20 = c$. Then
l the ſubtangent $GE = \frac{y\dot{x}}{\dot{y}}$,
l $\frac{y^2\dot{x}^2}{\dot{y}^2} + yy = aa$; whence



$\overline{\dot{x}^2 + \dot{y}^2}$ ($= \dot{z}$) $= \frac{a\dot{y}}{y}$, whoſe fluent, corrected, gives z
$a \times$ hyp. log. $\frac{y}{c}$; which when $y = a = 100$, becomes $=$
$\times$ hyp. log. of $5 = 160{\cdot}94379$, the length of the curve
FB.——From the equation $\frac{y^2\dot{x}^2}{\dot{y}^2} + yy = aa$, we alſo
ve $\dot{x} = \sqrt{aa - yy} \times \frac{\dot{y}}{y} = \frac{aa\dot{y}}{y\sqrt{aa - yy}} - \frac{y\dot{y}}{\sqrt{aa - yy}}$;

where taking the fluents, properly correcting them, and putting $d = \sqrt{aa - cc}$, will be had $x = \frac{a}{2} \times$ hyp. log. of $\overline{\frac{a - \sqrt{aa - yy}}{a + \sqrt{aa - yy}} \times \frac{a + d}{a - d}} + \sqrt{aa - yy} - d$, the equation of the curve required.

COROLLARY. Upon the center D, with the radius DB, deſcribe the circular arc BIH. Make $DL = AC$, and draw $LH \perp BD$. Then will the area $LBIHL =$ the area of the whole curve $AFBDCA$. For, putting $DK = y$, $KI = \sqrt{aa - yy}$, and the flux. of $LHIK$ will be $\sqrt{aa - yy} \times \dot{y} = y\dot{x}$, from what is given above; therefore, &c.

Mr. *W. Crakelt*'s and Mr. *S. Vince*'s ſolutions differ very little from the above.

Anſwer to the ſame by the Rev. Mr. Cha. Wildbore.

Let TD be the indefinite right line, MFB the required curve, and GF the given tangent at F; then at B the vertex (from the nature of the curve) $BD = FG = 100 = a$. Produce GF till it meets BD in S, and let fall $FK \perp BD$; then, putting $BK = x$ and $FK = y$, SK will be $= \frac{y\dot{x}}{\dot{y}}$, and $SK : FS :: FE\ (KD) : GF\ (a)$: Whence $\dot{y} = \frac{\dot{x}\sqrt{2ax - xx}}{a - x}$ $\left(= \frac{u^2\dot{u}}{aa - uu} = \frac{aa\dot{u}}{aa - uu} - \dot{u}\right.$, by putting $GE = \sqrt{2ax - xx}$ $= u$); and, reſtoring x, $y = \frac{1}{2}a \times$ hyp. log. of $\frac{a + \sqrt{2ax - xx}}{a - \sqrt{2ax - xx}}$ $- \sqrt{2ax - xx} = 131{\cdot}263155$ when $FE = 20$. Now, let z $=$ the curve FB; then $\dot{z} = (\sqrt{\dot{x}^2 + \dot{y}^2} =)\ \frac{a\dot{x}}{a - x}$, and z correct $= a \times$ hyp. log. $\frac{a}{a - x} = 160{\cdot}94379$, as required. ——Whence theſe conſectaries.

1. Becauſe, when x is taken $= a$, the expreſſions for y and z become infinite, therefore the curve will continually approach nearer to the line TD, but can never meet with it.

value which this infinite area can never exceed, may
d from the above expreſſion for $\dot{y}$, thus; *TE*'s flux.
$\dot{y}$, which multiplied by *FE* $(= KD = a - x)$ gives
$\overline{2ax - xx}$ for the flux. of the infinite area *ETMF*.
F (*DB*) radius deſcribe the quadrant *BIP*, and
e *FK* to meet it in *I*; then the area of the circular
it *DKIP* will be the fluent of this expreſſion, or $=$
a *ETMF*; and, when *FE* becomes $= BD$, the infinite
FBDT $=$ the whole quadrant *BDP*; which is alſo
from other principles, by conſidering the invariable
F, as moving from an horizontal poſition to a vertical

ecauſe $\dot{y}x$ is the fluxion of the area *FBK*, twice the
f the quadrant *BDP* will be $=$ the fluent of $\frac{1}{2}a\dot{x} \times$
og. of $\frac{a + \sqrt{2ax - xx}}{a - \sqrt{2ax - xx}}$, when $x = a$; a concluſion
ry eaſy to be derived another way.

s queſtion is alſo ingeniouſly anſwered by Meſſ. *T.*
r, *J. Bennett* (the propoſer), *J. Chipchaſe*, and *T.*

QUEST. 594 *anſwered by the Rev. Mr.* C. Wildbore.

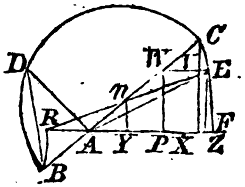
the given ſeries ſubſtitute x for $\frac{1}{v}$ and take the fluxion;
h will give $x^3\dot{x} \times : x + \frac{x^3}{1.2.3} + \frac{1.3.3.x^5}{1.2.3.4.5} + \frac{1.3.3.5.5.x^7}{1.2.3.4.5.6.7}$
c. $= x^3\dot{x}z$ (z being the arc of the circle whoſe ſine
$(\frac{1}{v})$ and rad. 1): Whence, taking the fluents and re-
ng $\frac{1}{v}$, $\frac{1}{1.5.v^5} + \frac{1}{1.2.3.7.v^7} + \frac{1.3.3}{1.2.3.4.5.9.v^9} +$
$\frac{1.3.3.5.5}{3.4\ 5.6.7.11.v^{11}} + \&c. = \frac{8z + 2 + 3vv \times \sqrt{vv - 1} - 3zv^4}{32v^4}$,
ſum of the ſeries required.

OROLLARY. This method of ſolution may be extended
ny ſeries of the form $\frac{Ax^n}{n} + \frac{Bx^{n+m}}{m+n} + \frac{Cx^{n+2m}}{2m+n} +$
by means of artifices ſimilar to thoſe made uſe of in
pſon's Fluxions, part 2d, ſec. 3d and 6th, where $A + B$
C + &c. expreſſes any known ſeries; and whereof the

ſummation of the extremely difficult ſeries $x + \frac{x^2}{4} + \frac{x^3}{9} + \frac{x^4}{16} +$ &c. propoſed by the very ſagacious Mr. *L*—— in the Ladies' Diary for 1760, is given as an example; but, for want of room, we are obliged to omit it.

Ingenious ſolutions to this queſtion have alſo been received from Meſſ. *T. Allen*, *T. Barker*, *Wᵐ. Crakelt*, *C. Hutton*, and the propoſer *S. Vince*.

XIV. Queſtion 595 *anſwered by Mr.* Cha. Hutton.



Construction. Parallel to CF draw RB, meeting CA, produced, in B; $\perp CA$ draw AD, meeting the circ. on CB in D; make $Bn = BD$, and n will be the point through which RnE muſt paſs.

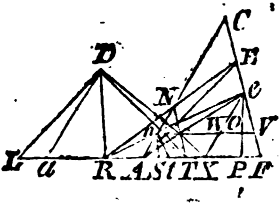
Demons. Draw $EW \parallel$, and EZ, WP, $nY \perp AF$.——Then, by conſtruction, $BA \times AC = (DA^2 = DB^2 - BA = Bn^2 - BA^2 = \overline{Bn + BA} \times \overline{Bn - BA} =) \overline{nB + BA} \times An$, or $An : AB :: AC : nB + BA :: nC\ (AC - An) : nB\ (\overline{nB + BA} - AB) ::$ (by ſim. Δs) $CE : BR$, or $An : CE :: AB : BR ::$ (by ſim. Δs) $WC : CE$; whence $An = WC$.

Again, by ſim. Δs, $WC\ (An) : Cn\ (AW) :: nY : ZE\ (WP)$, or $WC : CE :: Cn \times nY : CE \times EZ$. But when RnE moves about the center R, and cuts the lines AC, CF, it is known that the flux. of EC : the flux. of $Cn :: CE \times ER : Cn \times nR ::$ (by ſim. Δs) $CE \times EZ : Cn \times nY$; whence the flux. of CE : flux. of $Cn :: EC : CW$ (by equality of ratios), and conſequently the flux. of EZ = the flux. of nY; in which caſe, it is well known, that their diff. $(EZ - nY)$ is a max. But this difference drawn into $\frac{1}{2}$ the given line AR is = the area of the ΔAnE; and hence the ſaid triangle is a maximum.

Corollary. It appears from the demonſtration that $An = WC$; whence $nY = CV$, and $nY + EZ$ = the whole perpendicular CX.

RUCTION *to the ſame by the Rev. Mr.* C. Wildbore.

e given point R erect the perpend. DR (p. 13 Euc. 6)
proportional to RA,
d take LR and $RT=$
raw $NT \parallel CF$, and
N draw RE; join
nd the thing is done.
let Re be any other
of the required line,
ting AC in n. Draw
, eX to CA, nV to
d let fall the perpen-
s nS, eP: Then, the $\triangle$s RnA, ReA, on the com-
ſe AR, are in the ratio of their perpendiculars nS,
nſequently the $\triangle\, Ane$ (their diff.) will be as $eP-$
O, which muſt therefore be a maximum. But the
V being ſimilar to the given one ACF, the ratio of
VV is given; therefore $WV=nV\,(tF)-nW\,(AX)$
$-Rt-RX+RA$ (per fig.) muſt be a maximum:
$F+RA$ is invariable; therefore $Rt+RX$ muſt be
num. By ſim. $\triangle$s, $RA:Rt::RX:RF$; whence
uſt be likewiſe a mean proportional to Rt, RX. Take
Rt, and join D, a; then, the $\angle$s aDL, TDX be-
ual, DT greater than Da, and the $\angle DTX$ greater
aL, TX is evidently greater than aL, and conſe-
y aX than LT; wherefore $Rt+RX$ is a minimum
each of them becomes equal to RT, as per con-
on. *Q.E.D.*



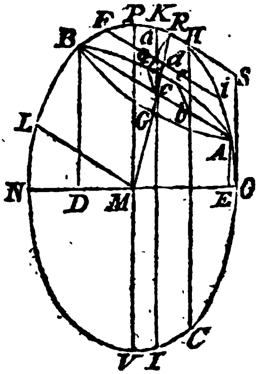
OLLARY. If $Rt+RX$ inſtead of a minimum, or the
e inſtead of a maximum, had been ſuppoſed = a given
ty, from what is ſhewn above the prob. is reduced to
iz. In any right-angled $\triangle\, aDX$, are given the per-
ular DR and the baſe aX, to conſtruct the triangle;
ethod of doing which is well known: And the con-
on will be equally eaſy, when, inſtead thereof, the
f the $\triangle\, RnA$ to the $\triangle\, ReA$ or of nS to eP or of
eF is given; for, in all theſe caſes, the ratio of aR
is given.

have alſo received a very curious geometrical con-
on to this queſtion from Mr. *Moſs*, the propoſer,
we are extremely ſorry our narrow limits will not
t us, this year, to inſert.

ſſ. *T. Allen*, *T. Barker*, *W. Crakelt*, *Steph. Ogle*, *T.*
ſon, and *S. Vince* have given ſimple, eaſy, and conciſe
nary ſolutions to it.

XV. QUESTION 596 *anſwered by Mr.* W. Crakelt.

If IK, CH be two ordinates to the fixed axis ON; MR a ſemidiameter to the ordinate BA, and OS, RS two tangents at the points O, R. Then (by Deidier's Géométrie, p. 401) will be had $Bc \times cA : Ic \times cK :: RS^2 : OS^2 :: Bb \times bA : Cb \times bH$: But (per prop. of the circle) $Ic \times cK = cQ^2$, and $Cb \times bH = be^2$; conſequently $Bc \times cA : Bb \times bA :: cQ^2 : be^2$, the property of the ellipſe.



Having thus diſcovered the figure of the ſection $AGBQ$, let 3·14159 &c. be put $= p$, the ſemi-fixed axe $NM = m$, the ſemi-revolving axe $PM = n$, the ſemi-diameter $MR = r$, its ſemi-conjugate $LM = s$, the ſemi-ordinate $Bc = t$, its correſponding abſciſſa $Rc = u$ (all which may, it is evident, be looked upon as given quantities), and any variable part Rd of $Rc = x$. Then (per prop. of the ellipſe) rr $(MR^2) : ss$ $(LM^2) :: 2rx - xx$ $(\overline{2MR - dR} \times dR) : \frac{ss}{rr} \times \overline{2rx - xx} = dF^2$, and thence $dF = \frac{s}{r}\sqrt{2rx - xx}$. Alſo (by trig.) rad. (1) : ſin. $\angle Rda$ $(= \frac{mn}{rs}) :: x$ $(Rd) : Ra = \frac{mnx}{rs}$ (Ra being perpendicular to Fv); whence the fluxion of the ſolidity (as the ſemi-conjugate of the ſection made by the plane Fv is $= \frac{n}{r}\sqrt{2rx - xx}$) will be $\frac{pmnn}{r^3} \times \overline{2rx\dot{x} - x^2\dot{x}}$: the fluent whereof $(\frac{pmnn}{r^3} \times \overline{rxx - \frac{1}{3}x^3})$, when $x = u$, gives $\frac{pmnn}{r^3} \times \overline{ru^2 - \frac{1}{3}u^3}$ for the ſolidity required, equal in the oblong ſpheroid (when $m = 25$, $n = 15$, $r = 15·0028901$, and $u = ·6418766$) 31·88508, and equal in the oblate ſpheroid (when $m = 15$, $n = 25$, $r = 24·9938494$, and $u = 5·3937069$) 1272·97097 &c.

COR. When $x = 2r$, the ſolid BRA becomes $= \frac{4pmnn}{3}$ = the whole ſpheroid.

The ſolution by Mr. *Cha. Hutton* (the propoſer) is exceedingly elegant and conciſe; who, after his demonſtration of the figure of the ſection, refers to his Treatiſe on Menſuration, now publiſhing in monthly numbers, for other properties of ſections, &c.

Ingenious anſwers to this queſtion have likewiſe been received from Meſſ. *T. Allen*, *T. Barker*, *T. Todd*, *S. Vince*, and *C. Wildbore*.

The PRIZE QUESTION *anſwered by Mr.* Tho. Allen (*the Propoſer*).

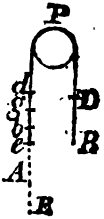
Let $c = 40$ feet, the whole length of the chain, $a = 21$, $x =$ any variable part paſſed over the pulley in the time t, $v =$ the velocity of the chain at the end of that time, and $s = 32\frac{1}{6}$ feet.——Then will $a + x$ be as the gravity of the deſcending, and $\overline{c - a - x}$ that of the aſcending part; therefore $a + x - \overline{c - a - x} = d + 2x$ (by putting $d = 2a - c$) will be as the force acting on the chain, and $c : s :: d + 2x : s \times \frac{d + 2x}{c} =$ the velocity that would be generated in 1″, by that force. Therefore 1″ (time) : $s \times \frac{d + 2x}{c}$ (velocity) $:: \frac{\dot{x}}{v} (\dot{t}) : \dot{v}$, the velocity generated in the time $\dot{t}$; $\therefore s \times \frac{d\dot{x} + 2x\dot{x}}{c} = v\dot{v}$; where, taking the fluents, &c. $v = \sqrt{\frac{2s \times \overline{dx + xx}}{c}}$. Therefore $\dot{t} = \sqrt{\frac{c}{2s}} \times \frac{x^{-\frac{1}{2}}\dot{x}}{\sqrt{d + x}}$; whoſe corrected fluent gives $t = \sqrt{\frac{2c}{s}} \times$ hyp. log. of $\frac{\sqrt{x} + \sqrt{d + x}}{\sqrt{d}}$; which, when $x = 19$ (or the chain quits the pulley) will be $= 2{\cdot}908137$ ſeconds.

To find the remaining part of the time, put $p = 25{\cdot}33$ (the value of v $(= \sqrt{\frac{2s \times \overline{dx + xx}}{c}})$ when $x = 19$), $z =$ the ſpace deſcended, after quitting the pulley, in the time T, and $V =$ the velocity at the end of that time. Then, if the chain deſcended from reſt, V^2 would be $= 2sz$; therefore, in the preſent caſe, $V^2 = 2sz + pp$: $\therefore V = \sqrt{pp + 2sz}$, and $\dot{T}\ (\frac{\dot{z}}{V}) = \frac{\dot{z}}{\sqrt{pp+2sz}}$; whoſe corrected fluent gives $T = \frac{\sqrt{pp+2sz}-p}{s}$; which, when $z = 160$, becomes 2·46342″. And hence the whole time required will be = 5″ 22·3‴, very nearly.

The ſame anſwered by Mr. W. Crakelt.

If 40 feet (the length of the whole chain) $= l$, 2 feet (the diff. of the two ends *A* and *B* hanging down from the pulley) at firſt) $= d$, $32\frac{1}{6}$ feet (the velocity generated at the earth's ſurface in 1″, by gravity) $= s$, $x =$ a ſmall part of the chain aſcended or deſcended in any variable time t, and $v =$ the correſponding velocity of the point *A* at the end of that time (in ſeconds). Then, let the weight of the chain be what it may, $d+2x$ will repreſent the motive force acting thereon; and therefore (ſince $l : d+2x :: s : \frac{s}{l} \times \overline{d+2x}$, and $1'' : \frac{\dot{x}}{v}\ (=\dot{t}) :: \frac{s}{l} \times \overline{d+2x} : \frac{s\dot{x}}{l} \times \frac{d+2x}{v} = \dot{v}$) $v\dot{v} = \frac{s}{l} \times \overline{d+2x} \times \dot{x}$; or, by taking the fluents, &c. $v = \sqrt{\frac{2s}{l} \times \overline{dx+xx}}$ = (when $x = 19$) 25·3322916 &c. and conſequently $\dot{t}\ (= \frac{\dot{x}}{v}) = \overline{\frac{2s}{l} \times \overline{dx+xx}}\,^{-\frac{1}{2}} \times \dot{x}$; and, by taking the correct fluents, $t = \sqrt{\frac{l}{2s}} \times$ hyp. log. of $\frac{\frac{1}{2}d+x+\sqrt{dx+xx}}{\frac{1}{2}d}$ = (when $x = 19$) 2·90825683 &c. ſeconds, the time elapſed when the chain quits the pulley. Now, the height from which a heavy body muſt fall freely from reſt, to acquire the foregoing velocity, will (by Simpſon's Select Exer. pa. 184) be $= \frac{vv}{2s} = 9{\cdot}975$ feet, and the correſponding time $= \frac{v}{s} = {\cdot}78753238''$: Then $200 - 40 + 9{\cdot}975 = 169{\cdot}975$ feet, and the time in which a heavy body would deſcend through that diſt. = 3·25090661″; whence 2·90825683″ + 3·25090661″ − 0·78753238″ = 5·37163106″, the time in which the end *A* will reach the horizontal diſtance given; which was required.

er to the ſame by the Rev. Mr. Cha. Wildbore.



PB, at its commencement of motion, and EPD e end A has deſcended to E, be poſitions iain; then it is manifeſt that the motive E will be as $PE - PD = dE = Ab +$ which multiplied into $A\dot{E}$, is known to lf the fluxion of the ſquare of the velo- E; wherefore that velocity will be as $\overline{Ab \times AE + AE^2}$, and $A\dot{E}$ divided will be as the fluxion of the time, and itſelf (biſecting Ab in e, and taking eg $\overline{b \times AE + AE^2}$) a mean proportional and AE) will be as $\frac{1}{\sqrt{2}} \times$ hyp. log.

But the real motive force at E, meaſured by the

that might be uniformly gone over in 1″ with the at E, is to dE, as the force of gravity to $AP +$ as $32\frac{1}{6} : 40$, or $193 : 240$; therefore the velocity nd A per ſecond, when B arrives at P, $= \sqrt{\frac{193 \times 399}{120}}$ 32287 feet, and the time $\doteq 3{\cdot}6882542\sqrt{\frac{120}{193}} =$ 66″; and hence, by the laws of deſcending bodies, without the help of fluxions) the remaining part of e is eaſily found $= 2{\cdot}463375''$, and conſequently the ime of deſcent required $= 5{\cdot}371631''$.

OLLARY. If B be ſuppoſed to aſcend along an inclined then, the ſine of the plane's inclination to the ho- to rad. 1, being called s, the motive force at E will $PE - s \times PD = PA + AE - s \times PB + s \times AE$ $s \times \frac{PA}{1+s} - \frac{s \times PB}{1+s} + AE$; whence it is manifeſt e velocity and time may be found with the ſame eaſe caſe, as before. Or, if A deſcends, or A and B d and aſcend along two differently inclined planes, thod of ſolution will be ſtill the ſame.

nious and neat ſolutions to this queſtion have alſo eceived from Meſſ. *T. Barker*, *J. Bennett*, *R. Butler*, *pchaſe*, *J. Clough*, *J. Dymond*, *P. George*, *R. Holden*, *ton*, *S. Ogle*, *Alex. Rowe*, *T. Sanderſon*, *W. Sewell*.

E.

E. Smith, Stanningtonienſis, T. Todd, S. Vince, and *W. Wales*; but the two prizes of 12 and 8 Diaries for the ſolution thereof, are fallen to the reſpective lots of Meſſ. *Wm. Crakelt* and *J. Chipchaſe.*

Eclipſes calculated for 1769.

Five eclipſes are expected this year; three of the ſun and two of the moon, as follow.

The firſt is of the ſun, on Sunday the 8th of January, in the morning, inviſible at London.

The ſecond is a partial and viſible ſolar eclipſe, on Sunday the 4th of June, in the morning, according to the following calculations.

Calculated by	Beg.		Mid.		End		Dur.		Dig.	
	h.	m.	h.	m.	h.	m.	h.	m.	°	′
Mr. R. Robbins, for Greenwich	6	38¾	7	33⅔	8	23½			6	9¼
Mr. W. Chapman, for Foxton	6	35⅓	7	27¼	8	22⅚	1	47⅐	6	46
Mr. J. Edwards, for Cambridge	6	40½	7	36½	8	28¼	1	47¾	6	18
Mr. J. Coates, for London	6	42	7	32⅔	8	23⅚	1	41⅚	6	5
Mr. J. Metcalfe, for London	6	38	7	30	8	25			5	58

This eclipſe will be very formidable in the north-eaſtern parts of North America, Greenland, Hyperborean ocean, and beyond the pole; for the center of the lunar ſhade paſſes in its traverſe, very little to the eaſt of it.

The third eclipſe is a lunar one, on Monday the 19th of June, about 8 o'clock in the morning, and therefore inviſible to thoſe parts of the world.—It may be ſeen in South America and the unknown lands and ſeas near the ſouth pole. It will be total above an hour, and the whole duration above 3 and ¼.

The fourth is of the ſun, on Tueſday the 28th of November, near 8 of the clock in the morning; but becauſe of the moon's great ſouthern latitude, increaſed by its vertical parallax, it cannot be ſeen in theſe parts of the globe.—It will be ſeen a partial eclipſe in the unexplored Indian ſouth ſeas, and an annular and central one in the terra incognita, near the ſouth pole.

The fifth, and laſt, this year, is of the moon, and will happen on Wedneſday the 13th of December, in the morning, and be viſible in Great Britain and Ireland, from the beginning to the end, according to the following computations.

Calculated by	Beg. h. m.	Mid. h. m.	End h. m	Dur. h. m.	Dig. ° ′
Mr. W. Chapman, for Foxton	4 53$\frac{5}{6}$	6 14$\frac{5}{6}$	7 35$\frac{5}{6}$	2 42	8 0
Mr. J. Edwards, for Cambridge	5 6	6 25$\frac{1}{4}$	7 51	2 45	8 30
Mr. J. Coates, for London	4 58	6 22$\frac{1}{12}$	7 46$\frac{1}{4}$	2 48$\frac{1}{4}$	8 58$\frac{1}{2}$
Mr. J. Metcalfe, for London	4 53$\frac{1}{2}$	6 18$\frac{7}{10}$	7 42$\frac{2}{3}$	2 49$\frac{1}{5}$	9 2$\frac{1}{2}$

The moon at the beginning of this eclipſe will be vertical to the Weſt Indian ſea near Cuba, and at the end to the Pacific ocean, near the ſouth coaſt of California. This eclipſe therefore will be viſible to the weſtern parts of Europe and Africa, the Atlantic and Pacific oceans, and to all parts of America.

This year both the inferior planets Venus and Mercury will tranſit the ſun's diſk, but only part of the firſt can be ſeen here.—The times that theſe unuſual phænomena happen are as follow.

Beg. Tran. h. m.	Cent. Ingreſs h. m.	Total Immer h. m.	Mid. Tranſit h. m.	Cent. Egreſs h. m.	End of Tranſit h. m.	Near ap. of Cent. ′ ″	Diam. of ☉ & ♀ ′ ″
	7 27$\frac{5}{6}$		10 35$\frac{3}{4}$	13 44$\frac{1}{3}$			☉ 31 31
7 13$\frac{2}{3}$	7 25$\frac{2}{3}$	7 37$\frac{5}{6}$	10 34$\frac{3}{4}$	1 43	1 55$\frac{5}{6}$	9 51	♀ 0 58
7 3$\frac{2}{3}$	7 14$\frac{1}{4}$	7 26$\frac{2}{3}$	10 21$\frac{1}{3}$	1 27$\frac{3}{4}$	1 39$\frac{1}{20}$		

On Thurſday the 9th of November, Mercury will paſs over the ſun at night, and therefore will be inviſible in Europe. JOHN METCALFE.

*** The above calculation of the tranſit of Venus is as follows, viz.

The 1ſt. By Mr. William Chapman, for London.
2d. By Mr. J. Coates, for London.
3d. By Mr. J. Metcalfe, for London.

New Queſtions.

I. QUESTION 597, *by Mr.* Tho. Sadler.

Dear ladies, you with eaſe may find*
A matchleſs hero's name,
Who was beloved by mankind,
And mounted up to fame:

To ſerve his country boldly dar'd
Hot ſulphur, ſmoke, and fire,
And long campaigns fatigue he ſhar'd,
To conquer proud Monſieur.

* viz. From the equations $\left\{\begin{array}{ll} w+x+y+z & =52 \\ wx+yz & =360 \\ wz+xy & =280 \\ wy+xz & =315 \end{array}\right\}$: Where w, x, y, and z denote the places of the letters in the alphabet compoſing the gentleman's name.

II. QUESTION 598, *by Maſter* J. Paty, *at the Mathematical Academy at* Briſtol.

A gentleman erecting a houſe, whoſe breadth was 28 feet and back wall 6 feet 9 inches higher than the front, had by him a ſufficient quantity of rafters for the front, each 24 feet long, which, to ſave timber, he was unwilling to cut, and therefore orders his builder to make the back rafters of ſuch a length as will make the declivity of them and front alike, for uniformity; but being at a loſs, is deſirous of having it propoſed in the Diary, that he may know how to proceed.

III. QUESTION 599, *by Mr.* Wm. Spicer.

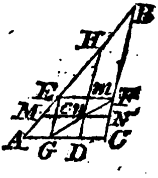
Suppoſe *A* lends *B* 2000 l. at 5 l. per cent. per ann. ſimple intereſt, which *B* is to pay again in the following manner, viz. 1 l. immediately down at the end of the 1ſt year, 2 l. at the end of the 2d year, 3 l. at the end of the 3d year, and ſo on, increaſing 1 l. every year; required at what time *B*'s debt will be the greateſt, and alſo in what time *A* will be indebted to *B* the ſum of 85 l. 15 s.

IV. QUESTION 600, *by Mr.* Paul Sharp.

A gentleman having a certain number of guineas and moidores, was aſked how many he had; to which he replied, that the ſquare root of the guineas multiplied by the cube root of the moidores when the product is the greateſt poſſible, will be = 39·1918; required how many he had of each ſort?

V. QUES-

V. QUESTION 601, *by Mr.* T. Moſs.



If one ſide *AB* of any plane triangle *ABC*, be divided into two parts, and from the point of diviſion (*E*) two right lines be drawn parallel to, and terminating at the other ſides, and the ſaid points of termination be joined by a right line, and there be likewiſe drawn another right line, as *DH* or *MN*, parallel to either of the two aforeſaid parallel lines, ſo as to interſect the other line, and terminate in the ſides of the triangle; then the two extreme parts *Hm*, *Dn* or *Nn*, *Mc* of the three parts into which the line, ſo drawn, is divided, will always be in the ratio of the two parts *BE*, *AE* of the line *AB* firſt divided; required the demonſtration?

VI. QUESTION 602, *by Mr.* J. Edwards, *of Magdalen College*, Cambridge.

A gentleman has a right-angled triangular garden, at the right angle of which grows a tree 40 feet high, whoſe ſhadow, I obſerved, on the 21ſt of June, at 2h. 30m. P. M. terminated in the hypothenuſe of the ſaid triangle in a perpendicular direction, and meaſuring the garden, I found that the difference of both its ſides from the hypothenuſe was 15 and 30 yards: Quere the latitude of the place, and area of the garden?

VII. QUESTION 603, *by Mr.* Wm. Gawith.

In a right-angled plane triangle *ABC*, ſuppoſe the two legs *AB* and *BC* $= x^5$ and y^5, the hypothenuſe *AC* $= \overline{xx+yy}|^{\frac{1}{2}} \times axy$, and the perpendicular *BD* upon the hypothenuſe $= x^3$; to determine the triangle, by quadratics.

VIII. QUESTION 604, *by Mr.* J. Dymond.

Given the line biſecting the vertical angle of a plane triangle and terminating in the baſe, the perpendicular falling thereon from one of the angles at the baſe, and the other angle at the baſe; to conſtruct the triangle.

IX. QUESTION 605, *by Mr.* J. Turner.

In any ellipſis the tranſverſe axis is the greateſt of all the diameters, and the conjugate axis the leaſt; and, of any other diameters, that which is neareſt to the tranſverſe axis is greater than thoſe which are fartheſt from it. The demonſtration hereof is required, geometrically, from the conſideration of the ſolid.

X. QUESTION 606, *by Monſ.* Fermat.

Let ANB be a ſemicircle deſcribed upon AB as a diameter, [*ſee the fig. to the ſolution*] and AD, BC two perpendiculars, each equal to AN, the chord of 90°. Alſo, let any point E be aſſumed in the ſemi-circumference, and two lines ED, EC be drawn from thence to the points D, C, interſecting the diameter in two points O, V: then $BO^2 + AV^2$ will be, univerſally, equal to AB^2; required the demonſtration?

⁂ *This queſtion was propoſed by Monſ.* Fermat *to Dr.* Wallis, *as may be ſeen at page* 188 *of the Commercium Epiſtolicum, publiſhed at Oxford in the year* 1658, *but was never yet publicly anſwered.*

XI. QUESTION 607, *by Mr.* Wm. Crakelt.

In a plane triangle there are given one of the angles at the baſe, the length of a line drawn from the other angle to the middle of its oppoſite ſide, and the perpendicular from the given angle upon its oppoſite ſide a maximum; to determine the triangle, by conſtruction.

XII. QUESTION 608, *by Mr.* J. Chipchaſe.

There is a hollow cylinder with a circular hole in its ſide $\frac{1}{10}$ of an inch in diameter, and at the diſtance of 3 feet from its bottom, out of which when the cylinder is full, the water will ſpout to the diſtance of 5 feet from its bottom on an horizontal plane, but after it has continued running for the ſpace of 15′, it will only ſpout to the diſtance of 3 feet: Required the content of the cylinder?

XIII. QUESTION 609, *by Mr.* Wm. Wales.

To determine the declination of that ſtar whoſe change in azimuth, in a given latitude, is the greateſt poſſible in a given time, reckoned from that of its riſing.

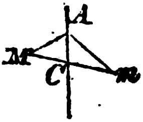
XIV. QUESTION 610, *by Mr.* C. Hutton.

To find the radius of a circle, whoſe area ſhall be equal to the ſurface of an elliptic ſpindle, or to that of any fruſtum or ſegment of it, having given the ellipſe from which it is generated, and the diſtance of the centers of the ellipſe and ſpindle.

XV. QUESTION 611, *by Mr.* Tho. Allen.

The ſum of the infinite ſeries $\frac{1}{2.4.2+3} + \frac{1.3}{2.4.6.4+5} + \frac{1.3.5}{2.4.6.8.6+7} + \frac{1.3.5.7}{2.4.6.8.10.8+9} +$ &c. is required, with the inveſtigation.

The PRIZE QUESTION, *by* Plus Minus.



Let a thread *MAm* be fixed by its ends to the ends *M*, *m* of a ruler, whoſe middle point is *C*, ſo as to ſlide over a tack fixed in the point *A*, whilſt the point *C* of the ruler keeps conſtantly in the line *AC*; then will the ends *M*, *m* of the ſaid ruler deſcribe a certain curve, which revolving about its axis *AC*, will generate a ſolid, the ratio of whoſe content to that of its circumſcribing cylinder is here required; the length of the thread being to that of the ruler in the ſub-duplicate ratio of 4 to 3.

Whoe'er is unable this truth to diſcover,
Either is *not at all*, or an *unhappy* lover.
Happy lovers do all (tho' not ſkill'd in the arts)
Know the *ſolid content* of *two conjugate hearts*.

1770.

Queſtions anſwered.

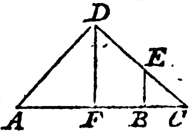
I. QUESTION 597 *anſwered by Mr.* Wm. Spicer.

FROM the 1ſt given equation w is $= a\ (52) - x - y - z$; and this value ſubſtituted in the other three, they become $ax - xx - xy - xz + yz\ (= 360) = b$, $az - xz - yz - zz + xy\ (= 280) = c$, and $ay - xy - yy - yz + xz\ (= 315) = d$: Whence, by addition, $a \times \overline{x+z} - \overline{x+z}^2 = b + c$, $a \times \overline{z+y} - \overline{z+y}^2 = c + d$, and $a \times \overline{x+y} - \overline{x+y}^2 = d + b$, and compleating the ſquare, &c. $x + z = \frac{1}{2}a - \frac{1}{2}\sqrt{aa - 4 \times \overline{b+c}} = 20$, $z + y = \frac{1}{2}a - \frac{1}{2}\sqrt{aa - 4 \times \overline{c+d}} = 17$, and $x + y = \frac{1}{2}a - \frac{1}{2}\sqrt{aa - 4 \times \overline{b+d}} = 25$, and conſequently $x = 14$, $y = 11$, $z = 6$, and $w = 21$; which ſhew the matchleſs hero's name required to be WOLF.

In a manner equally ingenious it is alſo anſwered by the Rev. Mr. *Crakelt*, Mr. *Gawith*, Mr. *R. Gibbons*, Mr. *Wm. Reynalds*, and Mr. *Tarratt*.

Meſſ. *T. Barker*, *J. Bartlett*, *E. Bayley*, *B. Brown*, *Wm. Cole*, *J. Coultbred*, *J. Cumbrid*, *R. Dening*, *W. Dent*, *J. Higgins*, *J. Mills*, *E. Reed*, *T. Robinſon*, *Alex. Rowe*, *S. Scott*, *P. Sharp*, *Wm. Stephens*, *Wm. Stoker*, Mrs. *E. Suggett*, and Capt. *Williams* have likewiſe anſwered it.

II. QUESTION 589 *anſwered by Maſter* J. Spencer, *at Mr.* Allen's *Boarding School, in* Spalding.



Put BE, the diff. of the heights of the walls $(= 6{\cdot}75) = a$; BA, the breadth of the building $(= 28) = b$; $AD\ (= DC)$ the length of the front rafters $(= 24$ feet$) = c$; and $BC = x$; then, drawing $DF \parallel EB$, FC will be $= \frac{1}{2} \times \overline{b+x}$, $EC = \sqrt{aa + xx}$, and, per ſim. $\triangle$s, $x\ (BC) : \sqrt{aa + xx}\ (EC) :: \frac{1}{2} \times \overline{b+x}\ (FC)$

: c

)C); ∵ $\sqrt{aa+xx}=\frac{2cx}{b+x}$, and conſequently $x=$, very nearly; and hence the length of the back rafters ſily found $=14{\cdot}02288$ &c. feet.*

ſſ. *J. Addiſon*, *T. Barker*. *J. Bartlett*, *E. Bayley*, *G. ron*, *t Cumbrid*, *H. Curtis*, *J. Dalby*, *R. Dening*, *W.* , *R. Gibbons*, Maſter *Jer. Oſborne*, *Pamphagus*, *E. el*, Maſter *J. Paty* (the propoſer), *Plus Minus*, *E.* *W. Reynolds*, *T. Robinſon*, *Aex. Rowe*, *Ruſticus*, *P.* , *W. Spicer*, *W. Stoker*, *J. Tarratt*, and Capt. *Wil-* have alſo anſwered it.

QUESTION 599 *anſwered by Mr.* Wm. Spicer (*the Propoſer*).

t $p=2000$, $r=0{\cdot}05$, $b=85{\cdot}75$l. and $x=$ the time in h *B*'s debt will be greateſt; then (by progreſſion) $\frac{1}{2}\times$ $1\times\overline{x+2}+\frac{1}{6}r\times\overline{x^3+3x^2+2x}=$ the ſum of *B*'s y payments and their intereſt, and $p+prx=$ the nt of the principal p in the ſame time, and conſequently $prx-\frac{1}{2}\times\overline{x+1}\times\overline{x+2}-\frac{1}{6}r\times\overline{x^3+3x^2+2x}=$ um he will then be indebted, a max. per queſt.——In and reduced, $x=45{\cdot}184086$, &c. which ſhews that ebt will be the greateſt juſt before the 46th payment is . Again, ſuppoſing x, now, to denote the time when indebted to *B* the ſum of 85·75l. (the reſt remaining ame as before), then will $\frac{1}{2}\times\overline{x+1}\times\overline{x+2}+\frac{1}{6}r\times$ $\overline{3x^2+2x}-p-prx=b$ (per queſt.); ∴ x^3+ $-11818x=250170$, and $x=93$ years, the time re- ed.†——*Pamphagus*'s ſolution is very little different.

is alſo anſwered by Meſſ. *T. Barker*, *G. Coughron*, *J. by*, *W. Dent*, *R. Gibbons*, *Giles Lacey*, *E. Reed*, *T. inſon*, *Alex. Rowe*, *S. Scott*, Capt. *Williams*, *R. William-* and *J. Young*.

IV. QUE

This is nearly the ſame as the prize queſtion for the year , to which ſeveral conſtructions were given the year following.

This ſolution ſeems to be falſe. For, the ſum of the pay- s being $1+2+3+\ldots\ldots x=\frac{1}{2}x.\overline{x+1}$, and the ſum

IV. QUESTION 600 *anſwered by Mr.* J. Addiſon.

It is eaſily proved, that two numbers to have the condition required in this queſtion (viz. that the cube root of the one multiplied into the ſquare root of the other may be a max. &c.) muſt be in the ratio of 2 to 3, and conſequently that if xx = the number of guineas, $\frac{2}{3}xx$ will = that of the moidores; whence $x\sqrt[3]{\frac{2}{3}xx} = 39{\cdot}1918$ (per queſt.), and $xx = 96$, and $\frac{2}{3}xx = 64$, the numbers required.

Meſſ. *J. Chipchaſe*, *J. Dalby*, *E. Parnel*, and *Paul Sharp* (the propoſer) put x and y for the number of guineas and moidores, and z = the ſum of them both; then will $y = z - x$, and $\sqrt[3]{z - x} \times \sqrt{x}$ be a max. (per queſt.). In flux. and reduced (z being conſtant), $x = \frac{3}{5}z$: Whence $y = \frac{2}{5}z$; and, $\sqrt{\frac{3}{5}z} \times \sqrt[3]{\frac{2}{5}z}$ being $= 39{\cdot}1918$ (per queſt.), $z = 160$, and conſequently $x = 96$, and $y = 64$, the ſame as before.

Meſſ. *T. Barker*, *G. Coughron*, *è Cumbriâ*, *W. Dent*, *R. Gibbons*, *Pamphagus*, *E. Reed*, *T. Robinſon*, and Capt. *Williams* have likewiſe anſwered it.

V. QUES-

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of the intereſts of theſe payments $= r\cdot\overline{x-1} + 2r\cdot\overline{x-2} + 3r\cdot\overline{x-3} + \ldots\ldots\overline{x-1}\cdot r\cdot\overline{x-\overline{x-1}} = rx \times : 1 + 2 + 3 \ldots\ldots\overline{x-1} - r \times : 1^2 + 2^2 + 3^2 + \ldots\ldots\overline{x-1}^2 = \frac{1}{2}rx\cdot\overline{x-1} - \frac{1}{6}rx\cdot\overline{x-1}\cdot\overline{2x-1} = \frac{1}{6}r\cdot\overline{x^3-x}$, ∵ the annual payments and their intereſts together are $\frac{1}{2}x\cdot\overline{x+1} + \frac{1}{6}r\cdot\overline{x^3-x}$, which taken from $p + prx$, and the flux. of the remainder made $= 0$, we obtain $x = \sqrt{2p + \frac{1}{3} + \frac{1}{rr} - \frac{1}{r}} - \frac{1}{r} = \sqrt{4380\frac{1}{3}} - 20 = 46{\cdot}18408$, the time when the debt is greateſt.

Again, the root of the equation $\frac{1}{2}x\cdot\overline{x+1} + \frac{1}{6}r\cdot\overline{x^3-x} - p - prx = b$, or $x^3 + 60x^2 - 11941x = 250290$, is $x = 94\frac{1}{2}$ years, the time when $85{\cdot}75$ is due to B.

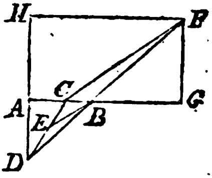
The time when the whole debt is juſt cleared, will be the root of this equation $\frac{1}{2}x\cdot\overline{x+1} + \frac{1}{6}r\cdot x^3 - x = p + prx$, or $x^3 + 60x^2 - 11941x = 240000$, where $x = 94{\cdot}006$, the time when the debt is cleared.

V. QUESTION 601 *anſwered by Mr.* E. Williams, *Captain in the Royal Artillery.*

By the conſtruction of the propoſer's figure (ſee the laſt year's Diary) the following analogies are evident, viz. $BF : Hm :: FE : mE :: EG\ (FC) : cG\ (Dn)$; therefore $Hm : Dn :: (BF : FC) :: BE : AE$. In like manner, $AG : Mc :: GE : cE :: EF\ (GC) : mF\ (Nn)$, and conſequently $Mc : Nn :: (AG : GC) :: AE : EB$.

The demonſtrations by Meſſrs. *T. Barker*, *W. Cole*, *G. Cougbron*, the Rev. Mr. *Crakelt*, *J. Dalby*, *T. Moſs* (the propoſer), *T. Robinſon*, and *W. Spicer* are ſo little different from the preceding, that it is needleſs to repeat them.

VI. QUESTION 602 *anſwered by the Rev. Mr.* Crakelt.



CONSTRUCTION. Let BA (30 yards) repreſent the difference betwixt the hypothenuſe and leſs leg, and CA (15) that betwixt the hypothenuſe and greater; then, at the point A, erect the $\perp AD$, meeting BD, making with AB an angle of 45°, in D, and join the points D, C: This done, from B apply BE (to DC) $= BA$, and from the point F, where CF drawn $\parallel BE$ meets DB produced, let fall the $\perp FG$ upon AB produced; ſo will CFG be the right-angled triangular garden required.

DEMONSTRATION. Produce DA till it meets FH, drawn $\parallel AG$, in the point H: Then, by ſimilarity of Δs, $AB : HF :: DB : DF :: EB : CF$; but (by conſtruc.) $EB = AB$, and conſequently $CF = HF = AG = AC + CG = AB + BG = AB + FG$.

The calculation from this conſtruction is extremely eaſy, whereby the length of the tree's ſhadow, at the given time, comes out $= 36$ yards, and conſequently the ſun's altitude $= 20° 19' 23''$; whence, per ſpherics, the latitude required is readily found to be 37° 33′ 38″ ſouth, &c.

Meſſ. *J. Addiſon*, *T. Barker*, *J. Bartlett*, *J. Chipchaſe*, *W. Cole*, *è Cumbriâ*, *J. Dalby*, *R. Gibbons*, *J. Haycock*, *E. Parnel*, *W. Pearſon*, *E. Reed*, *T. Robinſon*, *Alex. Rowe*, *W. Spicer*, *W. Stoker*, *J. Tarratt*, Capt. *Williams*, *R. Williamſon*,

ſon, and *J. Young* have given ingenious algebraic ſolutions to it.

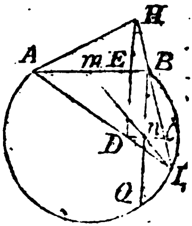
Mr. *G. Coughron* obſerves that, as the exceſs of the hypothenuſe above one of the legs is double its exceſs above the other leg, the ſides of the required triangle are in arithmetical progreſſion; and from thence derives every thing (without algebra) nearly the ſame as above.

VII. QUESTION 603 *anſwered by Mr.* G. Coughron.

By the data and properties of a right-angled triangle, $x^{10} + y^{10} = \overline{xx+yy}|^{3} \times aaxxyy$, and $x^5 y^5 = \overline{xx+yy}|^{\frac{3}{2}} \times axy \times x^3$. Subſtitute zxx for yy and its powers in the firſt equation, and it becomes $x^{10} \times \overline{z^5+1} = x^{10} \times \overline{z+1}|^{3} \times aaz$; $\because z^5 + 1 - \overline{z+1}|^{3} \times aaz = 0$; which, divided by $zz \times \overline{z+1}$ and reduced, gives $zz + \frac{1}{zz} - \overline{1+aa} \times \overline{z + \frac{1}{z}} + 1 - 2aa = 0$; or, putting $v = z + \frac{1}{z}$, $vv - \overline{1+aa} \times v - \overline{1+2aa} = 0$; $\because v = \sqrt{\frac{\overline{1+aa}|^{2}}{4} + 1 + 2aa} + \frac{1+aa}{2}$, which put $= 2c$, and then, $z + \frac{1}{z}$ being $= 2c$, z will be found $= \pm\sqrt{\overline{cc-1}} + c$. Laſtly, from the ſecond equation is had $z^{\frac{5}{2}} x^{10} = x^8 \times \overline{z+1}|^{\frac{3}{2}} \times az^{\frac{1}{2}}$; $\because x = \overline{z+1}|^{\frac{3}{4}} \times \frac{\sqrt{a}}{z}$; which, as z is now known, will likewiſe be known, and from thence the triangle becomes known.

The ſolutions given by the Rev. Mr. *Crakelt* and Mr. *Gawith* (the propoſer) are alſo very ingenious.

VIII. QUESTION 604 *anſwered by Mr.* T. Moſs.



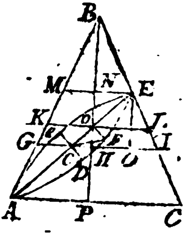
CONSTRUCTION. Upon the given biſecting line AB deſcribe a ſegment of a circle to contain the given angle of the required triangle, and in $BQ \perp AB$, take Bn = the given perpendicular, Biſect AB in m, and through the points m and n draw a right line meeting the arch of the ſaid ſegment in I: Join I, A, and I, B, and draw $DnC \parallel AB$; in IB, produced, take $BH = BC$: Draw AH, and AHI will be the triangle required.

DEMONSTRATION. Draw HD, cutting AB in E: Then, becauſe (by conſtruc.) AB is biſected in m, and DC is $\parallel AB$, Dn is $= nC$; alſo (by conſtruction) BH is $= BC$; whence HD is $\parallel Bn$, and conſequently HE $(= \frac{1}{2} HD) = Bn$, the given perpendicular, by conſtruc. Moreover, becauſe the given line AB is $\perp HD$ and biſects it at E, the $\angle EAD = EAH$; and AIH is = the given angle, by conſtruction. *Q.E.D.*

Elegant conſtructions to this queſtion have alſo been received from the Rev. Mr. *Crakelt*, and Meſſ. *G. Coughron*, *H. Curtis*, *J. Dalby*, *P. George*, and *W. Wales*.——It is alſo anſwered by Meſſ. *T. Barker*, *J. Chipchaſe*, *Alex. Rowe*, and *W. Spicer*.

IX. QUESTION 605 *anſwered by the Rev. Mr.* Crakelt.

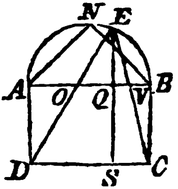
Let ABC be the repreſentation of any upright cone; $ADEA$ that of any elliptic ſection made therein; $Ac = EC$ = the ſemi-tranſverſe of ſuch ſection; $De = ce$ its ſemi-conjugate; Fc any other ſemi-diameter; GHI, KoL, MNE the diameters of three circular ſections made by planes paſſing through the points D, F, E, and EO a parallel to the axis of the cone, BP: Then, by ſimilarity of Δs, $Lo : Ic :: Ec - co$ $(Eo) : Ec$, and $Ko : Gc :: Ec + co$ $(Ao) : Ec$ (Ac), and conſequently $Lo \times Ko =$ (by prop. of the circle) $Fo^2 : Ic \times Gc$ or $Dc^2 ::$



$Ec^2 - co^2 : Ec^2$, the principal prop. of the ellipſe: But Dc^2 or $Ic \times Gc =$ (by ſimilarity of triangles and the nature of parallels) $Ic \times MN$ or $EN = Ic \times HO = \overline{HO + 2OI} \times HO = HO^2 + HO \times 2OI$, and $Ec^2 = OI^2$ (cH^2) $+ HO^2 + HO \times 2OI + EO^2$, and conſequently $Ec^2 - Dc^2 = OI^2 + EO^2 = EI^2$, or $Dc^2 = Ec^2 - EI^2$: Wherefore $Ec^2 \times Fo^2 = Dc^2 \times Ec^2 - Dc^2 \times co^2 = Dc^2 \times Ec^2 - Ec^2 \times co^2 + EI^2 \times co^2$, or, by adding $Ec^2 \times co^2$ to both, $Ec^2 \times Fc^2 = Ec^2 \times Dc^2 + EI^2 \times co^2$: Whence, as a ſimilar equality will obtain for all the other points of of the curve, DFE, and the greater co^2 becomes the greater will $Ec^2 \times Fc^2$ become likewiſe, the truth of the prob. is manifeſt.

Meſſ. *T. Barker* and *J. Dalby* have alſo demonſtrated it.

X. QUESTION 606 *anſwered by* Clericus.

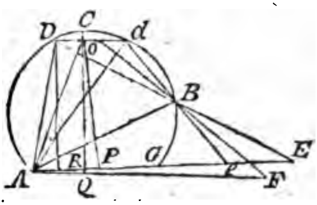


Join the points B, N; D, C; and draw $ES \perp DC$ interſecting AB in Q: Then, per ſim. Δs, $DA : AO :: ES : DS = AQ$, and $BC\,(=DA) : BV :: ES : SC = BQ$, and conſequently $DA \times DA : AO \times BV :: ES \times ES : AQ \times BQ = EQ^2$ (by property of the circle): Whence $2AO \times BV \left(= \frac{2DA^2 \times EQ^2}{ES^2}\right) = \frac{AB^2 \times EQ^2}{ES^2}$, becauſe $2DA^2\,(=2AN^2) = AB^2$ (47 Euc. 1.). But $OV^2 : DC\,(AB^2) :: EQ^2 : ES^2$ (per ſim. Δs OEV, DEC), and conſequently $OV^2 \left(= \frac{AB^2 \times EQ^2}{ES^2}\right) = 2AO \times BV\,(= 2AO \times \overline{OB - OV}) = 2AO \times OB - 2AO \times OV$; $\because OV^2 + 2AO \times OV = 2AO \times OB$, and $AO^2 + OV^2 + 2AO \times OV + OB^2 = AO^2 + 2AO \times OB + OB^2$; i. e. $\overline{AO + OV}|^2 + OB^2 = \overline{AO + OB}|^2$, or $AV^2 + BO^2 = AB^2$

Demonſtrations of this theorem have alſo been received from Meſſ. *T. Allen*, *T. Barker*, *J. Chipchaſe*, *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Dalby*, *P. George*, *R. Gibbons*, *M. Halls*, the Rev. Mr. *Lawſon*, *Plus Minus*, *T. Robinſon*, *W. Spicer*, the Rev. Mr. *Wildbore*, and *E. Williams*, Captain in the Royal Artillery.

XI. QUESTION 607 *anſwered by the Rev. Mr.* Lawſon.

CONSTRUCTION. On *AB* the given line, deſcribe a ſegment to contain the given angle. Biſect the circumference in *D*, and join *A*, *D*; *D*, *B*: Continue *DB* till *BE* equals *BD*; then, draw *AE* cutting the circumference continued in *G*. Biſect again the circumference *ADG* in *C*, and join *A*, *C*; *C*, *B*; and continue *CB* till *BF* equals *CB*; then join *A*, *F*, and I ſay *ACF* is the triangle required, whoſe perpendicular *CQ* is a maximum.

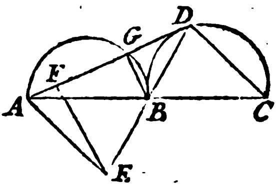


DEMONSTRATION. From *C* let fall $CP \perp AG$, and it will be the greateſt perpendicular in the ſegment *ACG*. But *CQ* is ſtill greater, becauſe the locus of the remaining angle of the required triangle (viz. *EFe*) is a circumference ſimilar to *ADB*, to which *AF* is a tangent.

NOTE, If the perpendicular be given in magnitude, the ſame conſtruction may be uſed; and if *Po* be ſet off on *PC* equal thereto, and through *o*, *Dod* drawn ∥ *AG*, the points *D* and *d* will each give a ſolution to the problem.

CONSTRUCTION *to the ſame by the Rev. Mr.* Crakelt (*the Propoſer*).

Let *AB* be ſuppoſed the given line drawn from the unknown angle at the baſe to the middle of its oppoſite ſide, and having produced it to *C* ſo that *BC* may be = *AB*, deſcribe upon *BC* a ſegment of a circle capable of containing an angle = the given one at the baſe, and draw the tangent thereto *AD*: Then, drawing *DC*, through *A* draw a parallel thereto to meet *DB* produced in *E*, and the $\triangle AED$, formed thereby, will be that required.



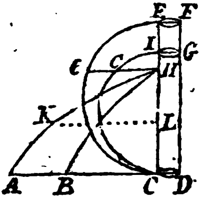
DEMONSTRATION. The line AE being $\parallel DC$, the angle AEB will be $=$ the $\angle GDB =$ the given angle at the baſe (by conſtruction), and the $\triangle$s ABE, CBD ſimilar; whence $AB : BC :: BE : BD$; but $AB = BC$ (by conſtruction); wherefore $BE = BD$. And, that the perpend. EF will in this caſe be a maximum, is very plain, ſince in all other (poſſible) caſes the line AD will cut the ſegment BDC in two points, and fall nearer the point B, and conſequently not give the $\perp BG$ or its double, EF, a maximum.

SCHOLIUM. If it was required that EB ſhould be to BD in a given ratio, or that the perpendicular inſtead of a maximum ſhould be of a given magnitude, the conſtruction would be the ſame very nearly.

This queſtion is alſo ſolved in a curious manner by the Rev. Mr. *Wildbore*; and the conſtructions by Meſſrs. *G. Coughron* and *T. Moſs* are not eſſentially different from the preceding, except in their ſtrictly demonſtrating the perpendicular to be a maximum.

XII. QUESTION 608 *anſwered by the Rev. Mr.* Wildbore.

If $HL = EH$, KL will $= 2EH$, and (per conics) $KL^2 : HL$, or $4EH^2 : EH :: AC^2 : HC$; $\because HE = \frac{AC^2}{4HC} = 25$ inches; and in like manner is found $IH = 9$ inches. Alſo (per Newton: Princip. edit. opt. p. 329, 330, 331) the times in which EH and IH may be emptied will be $= \frac{100EF^2\sqrt{2EH}}{\sqrt{193}}$ and $\frac{100EF^2\sqrt{2IH}}{\sqrt{193}}$ respectively; the difference of which is evidently $= 900''$ (the time in which EI is run out at H) per queſtion; whence $EF = 6{\cdot}64872$, and the content of the cylinder $2117{\cdot}8266$ cubic inches.

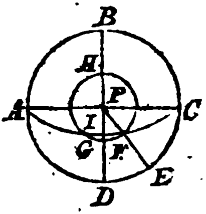


Anſwer to the ſame by Mr. T. Allen.

Let $CEFD$ repreſent the cylinder, H the hole, and HA, HB the ſpouting water, when the ſurface occupies EF and IG reſpectively. Upon the diameters EC, IC deſcribe the ſemicircles EeC, IcC, and draw $ecH \perp EC$: Then, it is well known that the horizontal diſtances AC, BC, of the ſpouting fluid, are reſpectively $= 2 \times eH$ and $2 \times cH$. Therefore, ſince eH and cH are given $= 2{\cdot}5$ and $1{\cdot}5$ feet reſpectively, EH and IH are eaſily found $= 2{\cdot}0833$ &c. and $0{\cdot}75$ feet: Therefore CE, the height of the cylinder, is $= 5{\cdot}0833$ &c. feet.——Put now, $EH = h$, $F = {\cdot}0000545415$ the area of the orifice in feet, $s = 16\frac{1}{12}$, $t =$ time of emptying $EFGI$ with the firſt or greateſt velocity, $IH = c$, $EI = 1{\cdot}33$ &c. $= d$, the given time $900'' = a$, $p = {\cdot}785397$, and EF (the diam. of the cylinder) $= x$. Then, per Emerſ. Mech. 1 edit. pa. 173, $tF\sqrt{2hs} = pdxx$; $\because t = \frac{pdxx}{F\sqrt{2hs}}$: And, per Emerſ. Flux. 2 edit. p. 139, $\frac{2pdx\dot{x}}{F\sqrt{2hs}} - \frac{2pdxx\sqrt{c}}{t\sqrt{2hhs}} = a$; whence, &c.

Ingenious anſwers to this queſtion have alſo been received from Meſſ. *J. Addiſon*, *T. Barker*, *J. Chipchaſe* (the propoſer), *W. Cole*, *G. Coughron*, and the Rev. Mr. *Crakelt*.

XIII. QUESTION 609 *anſwered by Mr.* W. Wales (*the Propoſer*).



PROJECTION. Deſcribe the primitive $ABCD$ for the equator, and draw BD for the meridian; from D, ſet off $DE = \frac{1}{2}$ the arc of the equator paſſed over in the given interval, and join P, E: Deſcribe the horizon $AGFC$ interſecting PE in F; then, from P, the pole, as a center, through F, deſcribe the parallel of declination FGH, which ſhall be that of the ſtar required.

DEMONSTRATION. It is a thing too well known to need demonſtrating here, that every phænomenon whatever changes its azimuth the faſteſt poſſible when on the meridian, and with equal velocities at equal diſtances therefrom; conſequently, the change in azimuth will be the greateſt poſſible during the whole of any given interval, when the phænomenon is on the meridian preciſely at the middle of that interval, or when the ſemi-diurnal arc DE, turned into time, is $= \frac{1}{2}$ the given interval.

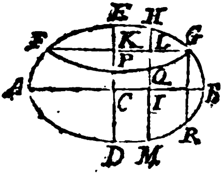
SCHOLIUM. It is manifeſt that when the given interval exceeds 12 hours, the declination will be of the ſame name with the latitude, but of a contrary name when the interval is leſs.

REMARK. It may at firſt ſight ſeem to ſome, as if this method of projection would fail in particular caſes; ſuch as, if it was required to find the ſun's declination when his change in azimuth was the greateſt poſſible in any given time, reckoned from his riſing, and leſs than the ſhorteſt day in the latitude given: But on more mature conſideration this will be found a miſtake; for in all thoſe caſes, the ſun's declination muſt be the greateſt poſſible, and of a different name with the latitude, becauſe then the whole of the given interval is the neareſt poſſible to the meridian, which is the very principle on which the preceding projection is founded.

I ſcarce need add that coſ. *FPI* : rad. : : tang. *IP* (the given lat.) : tang. *PF* the co-declination required.

The Rev. Mr. *Wildbore* alſo gives a curious projection of this problem on the plane of the equinoctial; and the Rev. Mr. *Crakelt* and Mr. *G. Coughron* have given ingenious algebraic ſolutions to it.

XIV. QUESTION 610 *anſwered by Mr.* T. Allen.



Let *AEBD* repreſent the given ellipſis, and *FEGP* the ſpindle, generated by the revolution of the elliptic arc *FEG* about its axis *FG*.——Put $CB = a$, $CE = b$, $KC = LI = d$, $CI = KL = x$, $IH = y$, arc $EH = z$, and $p = 3{\cdot}14159$; then, per conics, $y = \frac{b}{a}\sqrt{aa - xx}$, and therefore $HL = LQ = \frac{b}{a}\sqrt{aa - xx} - d$;

Whence $\dot{z}\,(= \sqrt{\dot{x}^2 + \dot{y}^2}) = \frac{\dot{x}}{a}\sqrt{\frac{a^4 - ccxx}{aa - xx}}$ (by putting $bb - aa = -cc$) and $\frac{2pb\dot{x}}{aa} \times \sqrt{a^4 - ccxx} - 2pd\dot{z} =$ the flux. of the ſpindle; whoſe fluent is $\frac{pb}{aa}\sqrt{a^4x^2 - c^2x^4}$ $+ \frac{pbaa}{c} \times$ circ. arc, whoſe rad. is 1 and ſine $\frac{cx}{aa}$, $- 2pdz$, a general expreſſion for the ſurface, either of the whole, or of any fruſtum of the ſpindle: But the two firſt terms thereof expreſs the ſurface of the portion *EHMD* of the ſpheroid *AEBD*; wherefore it is evident that the radius of a circle = the ſurface $\left\{\begin{matrix} EHQP \\ EGQ \\ HGQ \end{matrix}\right\}$ will be a mean proport. between the ſurf. $\left\{\begin{matrix} EHMD \\ EGRD \\ HGRM \end{matrix}\right\} - 2p \times KC \times \left\{\begin{matrix} EH \\ EG \\ HG \end{matrix}\right\}$ and $\frac{1}{p}$.

The Rev. Meſſ. *Crakelt* and *Wildbore* have likewiſe given ingenious and general ſolutions to this queſtion.——Mr. *G. Coughron* gives a general conſtruction to it, and refers to pages 291 and 292 of Mr. Hutton's Menſuration for the demonſtration.

XV. QUESTION 611 *anſwered by the Rev. Mr.* Wildbore.

The terms of the propoſed ſeries being multiplied ſeparately by x^5, x^9, x^{13}, &c. reſpectively, and the fluxions taken, it becomes $\frac{x^4\dot{x}}{2.4} + \frac{1.3\,x^8\dot{x}}{2.4.6} + \frac{1.3.5\,x^{12}\dot{x}}{2.4.6.8}$ + &c. = $\frac{\dot{x}}{x^4} - \frac{\dot{x}}{2} - \frac{\dot{x}\sqrt{1-x^4}}{x^4} = \frac{\dot{x}}{x^4} - \frac{\dot{x}}{2} - \frac{\dot{x}\sqrt{1-x^4}}{x^4} - \frac{\frac{2}{3}\dot{x}}{\sqrt{1-x^4}} + \frac{\frac{2}{3}\dot{x}}{\sqrt{1-x^4}}$; the fluent of which, when $x = 1$, is $\frac{e+\sqrt{ee-2f}}{3} - \frac{5}{6} =$ ·640685, the ſum required; where $e = \frac{1}{4}$ the periphery of the ellipſis whoſe ſemi-axes are $\sqrt{2}$ and 1, and $f = \frac{1}{4}$ of that of the circle, whoſe rad. is 1.

Mr. *Allen* (the propoſer) and the Rev. Mr. *Crakelt* alſo bring out the ſame concluſion exactly, by means of theo. 1. pa. 146, of Mr. Landen's Lucubrations.

The PRIZE QUESTION *anſwered by Mr.* Tho. Allen, *of* Spalding.

Let DF (= $2a$) repreſent any indefinite poſition of the ruler, and DAF (= s) the correſponding poſition of the thread. Put AH, the diſtance of the tack A from the middle of the ruler, = z, $HG = HE = x$, and p = 3·14159 &c. then will $FG^2 = DE^2 = aa - xx$

Mr. *Da. Kinnebrook*'s a
And much after the ſame
by Meſſ. *J. Barnard, W*
W. Spicer, and ſome othe

The ſame anſwered otherw

Let *A* and *O* be the pla
point in the required curve
the exceſs of the angle at
at the eak. Draw *MT* ⊥
whence we have the met
curve, viz. draw *OD* ⊥ *A*
the center *C*, and radius *C*
a circle which will cut *AG*
quired curve. Call *AO*, *a*

then (per ſim. triangles *A*

and $CE = \frac{ay}{x} - y$; but *E*
i.e. $2ayy - xyy = x^3 -$
tion for the hyp². defectiv
Newton's Enumeratio Lin.

$$= \frac{\sqrt{x \times \overline{a-x}^2}}{\sqrt{2a-x}} = \frac{}{x^0\sqrt{-}}$$

which belongs to tab. 2, f
Quad. Curv. or $y = \frac{ax-}{\sqrt{2ax}}$
and 3 of form 8; and by e
you deſcribe a quad. *AD* o
radius, and produce *PM* m
Q t parallel to *OA* and *OD*
to the area *tSQ*; and the
— a ſemicircle whoſe rad.
the whole of the area is p
$2ayy - xyy = x^3 - 2ax$
or *a*; ſo that the trees ſtand
part of the curve beyond *O*
ing to the field, is alſo p
angle at the fence muſt n
When *y* is infinite, $x = 2$

re anſwered by the Rev. Mr. Crakelt, *of* North-
fleet, *in* Kent.

ſe the lines drawn as in the preceding diagram, and
r to be moved from its firſt poſition Vv, into the
DF, and put $FA + AD$ (the length of the thread)
$A - AD = 2x$, DF (the length of the ruler) =
$= FG = y$, and $p = 3{\cdot}14159$ &c. Then, by
11, book 2d, of Simpſon's Geometry, 2d edit. and
re of the problem, will be found $Gv = a + b$
$\frac{aa - bb}{2a - bb + yy} - \sqrt{bb - yy}$, $EV = b - a +$
$\frac{a - bb}{- bb + yy} - \sqrt{bb - yy}$, and, ſuppoſing the ſolid
ided into two parts by a plane coinciding with its
ordinate, the ſum of the fluxion of thoſe parts
$\frac{\dot{y}}{- yy}$; the correct flu. whereof $(- \frac{2p}{3} yy \sqrt{bb - yy}$
$\sqrt{bb - yy} + \frac{4pb^3}{3})$ gives (when $y = b$) the ſoli-
uired $= \frac{4pb^3}{3} = \frac{8pb^3}{6}$, the ſolidity of a ſphere,
ameter is = the length of the ruler.——Again,
$\overline{+ xx - ax} \times \overline{aa - bb + xx}|^{-\frac{1}{2}}$ repreſenting the
AE, its flux. made $= 0$, and properly reduced,
$+ \overline{aa - bb} \times x = \overline{aa - bb} \times a$, or (ſince $bb = \frac{3aa}{4}$,
ta) $x^3 + \frac{a^2}{4} x = \frac{a^3}{4}$; and the root $\frac{a}{2}$, by ſubſti-
hews that, in the preſent caſe, the points E, A
or that the altitude of the circumſcribing cylinder
and conſequently that its ſolidity is to that of the
l ſolid as $1 + \frac{2}{\sqrt{3}}$ to $\frac{4}{3}$.——The ſeveral curious
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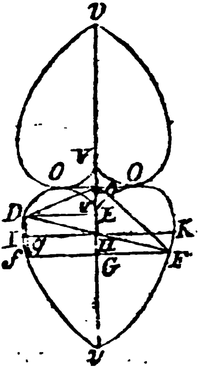
d $4pyy\sqrt{bb - yy}$ = the fluxion of the ſolid

E e 3 gene-

(47 Euc. 1.), $AE = z - x$, and $AG = z + x$, and conſequently $\sqrt{aa - xx + \overline{z+x}|^2} + \sqrt{aa - xx + \overline{z-x}|^2} = s$; $\because z = \sqrt{\frac{s^4 - 4ssaa}{4ss - 16xx}}$ (generally) $= \frac{1}{2\sqrt{1-xx}}$ in the preſent caſe, where $a = \frac{\sqrt{3}}{2}$ and $s = 2$ (per queſt.).——Further, when the extremity of the ruler paſſes the point O, the fluxion of EA, or $\dot{x} - \dot{z}$ will evidently be $= 0$; therefore $-\frac{1}{2}x\dot{x} \times \overline{1-xx}|^{-\frac{1}{2}} + \dot{x} = 0$, or $4 \times \overline{1-xx}|^{1} - xx = 0$; in which equation one root, or value, of x (which is that here required) is evidently $= \sqrt{\frac{1}{2}}$.——Now, $p \times : aa\dot{x} - x^2\dot{x} + \frac{1}{2}x^3\dot{x} - \frac{1}{2}aax\dot{x} \times \overline{1-xx}|^{-\frac{1}{2}}$, $p \times : x^2\dot{x} - aa\dot{x} + \frac{1}{2}aax\dot{x} - \frac{1}{2}x^3\dot{x} \times \overline{1-xx}|^{-\frac{1}{2}}$ and $p \times : aa\dot{x} - x^2\dot{x} + \frac{1}{2}aax\dot{x} - \frac{1}{2}x^3\dot{x} \times \overline{1-xx}|^{-\frac{1}{2}}$ being the fluxions of the ſolids whereof OVO, $IOAOK$, and IvK are the reſpective ſections (IK being the greateſt ordinate), their fluents, by proper correction and ſubſtitution, will be $p \times : \frac{2}{3\sqrt{\frac{1}{2}}} - \frac{2+\sqrt{3}}{4}$, $p \times : \frac{2}{3\sqrt{\frac{1}{2}}} - \frac{5}{8}$, and $p \times : \frac{5}{8} + \frac{\sqrt{3}-2}{4}$; the ſum of the two laſt of which minus the firſt gives $p \times \frac{\sqrt{3}}{2}$ = the content of the ſolid required; the ratio of which to that of its circumſcribing cylinder will be as $1 : \frac{\sqrt{3}}{2} + \frac{3}{4}$, as is evident from what has been determined above.



Ingenious *Plus Minus*, you ſee how I prove
Your *heart's true content* from what's given above:
And ſince I'm ſo happy this truth to diſcover,
I hope you'll eſteem me, a happy true lover.

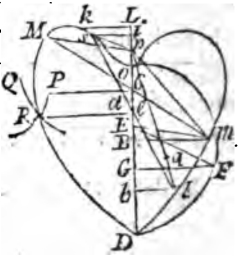
The ſame anſwered by the Rev. Mr. Crakelt, *of* Northfleet, *in* Kent.

Suppoſe the lines drawn as in the preceding diagram, and the ruler to be moved from its firſt poſition Vv, into the poſition DF, and put $FA + AD$ (the length of the thread) $= 2a$, $FA - AD = 2x$, DF (the length of the ruler) $= 2b$, $DE = FG = y$, and $p = 3{\cdot}14159$ &c. Then, by theo. 10, 11, book 2d, of Simpſon's Geometry, 2d edit. and the nature of the problem, will be found $Gv = a + b - a\sqrt{\frac{aa-bb}{aa-bb+yy}} - \sqrt{bb-yy}$, $EV = b - a + a\sqrt{\frac{aa-bb}{aa-bb+yy}} - \sqrt{bb-yy}$, and, ſuppoſing the ſolid to be divided into two parts by a plane coinciding with its greateſt ordinate, the ſum of the fluxion of thoſe parts $= \frac{2py^3\dot{y}}{\sqrt{bb-yy}}$; the correct flu. whereof $\left(-\frac{2p}{3}yy\sqrt{bb-yy} - \frac{4pb^2}{3}\sqrt{bb-yy} + \frac{4pb^3}{3}\right)$ gives (when $y = b$) the ſolidity required $= \frac{4pb^3}{3} = \frac{8pb^3}{6}$, the ſolidity of a ſphere, whoſe diameter is $=$ the length of the ruler.——Again, $\overline{aa-bb+xx-ax} \times \overline{aa-bb+xx}^{\,-\frac{1}{2}}$ repreſenting the value of AE, its flux. made $= 0$, and properly reduced, gives $x^3 + \overline{aa-bb} \times x = \overline{aa-bb} \times a$, or (ſince $bb = \frac{3aa}{4}$, by the data) $x^3 + \frac{a^2}{4}x = \frac{a^3}{4}$; and the root $\frac{a}{2}$, by ſubſtitution, ſhews that, in the preſent caſe, the points E, A coincide, or that the altitude of the circumſcribing cylinder is $= vA$, and conſequently that its ſolidity is to that of the generated ſolid as $1 + \frac{2}{\sqrt{3}}$ to $\frac{4}{3}$.——The ſeveral curious corollaries deduced from this ſolution our narrow limits oblige us to omit; and Mr. *G. Coughron*'s anſwer is ſo nearly the ſame that it would be needleſs to repeat it.

Mr. *James Yeret* produces FG to meet the curve again in f; then, retaining the laſt ſubſtitution, and joining D, f and drawing $HG \perp$ thereto, $\sqrt{bb-yy} = EH = HG$ (per 47 Euc. 1.) and $4pyy\sqrt{bb-yy} =$ the fluxion of the ſolid generated by the revolution of the ſpace $VDgfv$ about the axis Vv; the correct fluent whereof ($\frac{4p}{3} \times b^3 - \overline{bb-yy}|^{\frac{3}{2}}$) when $y = b$, gives $\frac{4p}{3} \times b^3 = \frac{p}{6} \times 8b^3$, the content of the whole ſolid required, let the length of the thread be what it will, &c.

The ingenious propoſer *Plus Minus*, after his curious ſolution, makes the following remark, viz. 'I have given 'the length of the thread in proportion to that of the ruler 'as 2 : √3, that the conjugate hearts may oſculate, as in O 'and O; had the thread been ſhorter, they would have 'interſected; had it been longer, they would not have 'touched; and had it been much longer, they would not 'have had this form of an heart, but would have been two 'ovals. Yet in all caſes the ſolid generated by the rotation 'of one of the curves is equal to a ſphere, whoſe diameter 'is equal to the length of the ruler.'

General Anſwer to the ſame by the Rev. Mr. Wildbore.



The curve being generated by the motion of the point m from D to O, let the ſemicircle DFO, whoſe rad. is ED ($= \frac{1}{2}OD = \frac{1}{4}Mm = Cm$) be ſuppoſed to be generated by that of the point F. Through the center E draw $EF \parallel Cm$, and then, the ∠s mCD, FED being equal, m and F muſt be co-temporary poſitions of the two points. Moreover, $p.B\dot{D}.Bm^2 = p.B\dot{G}.Bm^2 + p.G\dot{D}.Bm^2$ is evidently = the flux. of the ſolid generated by the revolution of BmD about $BD = p.B\dot{G}.GF^2 + p.G\dot{D}.GF^2$ (becauſe $Bm = GF$): But $p.G\dot{D}.GF^2$ = the flux. of the ſpheric ſegment generated by the revolution of GFD about GD; whence the corrected fluent of $p.B\dot{G}.GF^2$ + the ſaid ſegment = the ſaid ſolid; but, let the relation of BG to GF be what it will, when each of them is = nothing at the ſame time, the corrected fluent of $p.B\dot{G}.GF^2$ muſt neceſſarily be = 0: In which caſe therefore, the ſegment will be = the ſolid. Now m and F evidently arrive at O in the ſame time, making GF and BG each equal to nothing; and, by conſequence, the ſphere generated by $OFD =$ the ſolid generated by OmD.——Which property is not at all affected by $AM + Am$, but it is univerſal to the whole family of curves, when Cm is a conſtant quantity. Further, Mm is evidently equal to the greateſt diameter of the ſolid, or to that of its circumſcribing cylinder, and $LD =$ its height; ∵ the ratio of the ſolidities will be $\frac{2}{3}OD : \frac{1}{2}LD$.

Now, to determine the ſolid anſwering to any particular data; from the center i with the rad. $iQ = \frac{1}{2}.\overline{AM + Am}$ deſcribe a circle, and from A as a center, with the power $Ad^2 = iQ\sqrt{iQ^2 - Cm^2}$, deſcribe the equilateral hyperbola PR, and where it cuts the circle at R let fall the perpendicular Re; at i erect the $\perp is$, and through e draw sl, which will be $= Mm$; for (per trigonom.) $iQ : AE :: ie : \frac{1}{2}.\overline{Al - As} = ql$ (taking $Aq = iQ$) and $\overline{ql + Aq}^2 = Al^2 = \overline{Ae + eb}^2 + es^2 - ie^2 = Ae^2 + 2Ae.ei + es^2$; ∵ $Ae^2.Aq^2 - ie^2.Ae^2 = Aq^2 \times \overline{Aq^2 - es^2} = Ad^4$: But $Aq^2\ (= iQ^2) - ie^2 = Re^2$ (47 Euc. 1.); therefore $Re.Ae = Ad^2$, a known property of the hyperbola, whoſe aſymptote is AD and power Ad^2. It is moreover evident that, when Ai is the greateſt poſſible, or when i coincides with L, the hyperbola will not cut but touch the circle, AL being univerſally the height of the circumſcribing cylinder above AD; and, in the particular caſe propoſed, becauſe $\sqrt{iQ^2 - Cm^2} = \frac{1}{2}iQ$, $Qd = Re = Ad$, $iQ^2 - Ad^2 = Ad^2$, $\frac{1}{2}iQ^2 = Ad^2 = ie^2$, $Ai = 0$, $AL = 0$, AD is the height of the circumſcribing cylinder, and the ratio above will become $\frac{2}{3}OD : \frac{1}{2}AD$ or $1 : \frac{3 + 2\sqrt{3}}{4} ::$ the ſolid content of the heart to that of its circumſcribing cylinder.

COR. 1. The fluxion of the area $BmD = B\dot{D}.Bm = B\dot{G}.GF + G\dot{D}.GF = B\dot{G}.GF +$ flux. of the circ. ſegment GFD; whence, when BG and GF each $= 0$, the area $BmD =$ circular ſegment GFD; wherefore the area of the whole circle $=$ that of the curve.

COR. 2. From what is ſhewn above, the ſolidity of any ſegment of the heart, cut perpendicular to the axis OD, is eaſily derived, being $=$ the ſpheric ſegment above $+$ the correct fluent of $p.B\dot{G}.GF^2 = \frac{1}{6}p.GD \times \overline{3FG^2 + GD^2} + \frac{1}{3}p \times \overline{\frac{AE^4}{AC} + AE^2.AC - AE^3} =$ the ſolid generated by revolution of BmD about BD.

COR. 3. In the ſame manner it will be found that the fluent of $\dot{B}G . GF$ is $= \frac{1}{2} AE \sqrt{AE^2 - AC^2} - \frac{1}{2} AE^2 \times$ hyp. log. of $\frac{AE - \sqrt{AE^2 - AC^2}}{AE + \sqrt{AE^2 - AC^2}}$; which + the circ. ſegment GFD is = the area of the curve BmD.

COR. 4. If Cm is not $= ED$, but in a given ratio $n . Cm = ED$, then will $\frac{1}{nn} \times$ globe = the ſolid; as is evident from what is demonſtrated above; and the ſame univerſal property may be ſtill farther extended.

SCHOLIUM. The above univerſal properties may be applied to the finding a number of fluents of very difficult forms; but the ſubject is much too copious for this place.

Can I miſtake the problem, miſs the prize!
On whom my Lucy's choiceſt bleſſings riſe;
For whoſe content ſhe labours and ſhe lives:
Friendſhip, truth conjugate ſhe daily gives
Entwin'd with love unſpotted;——then may I
E'er guide her on to bliſs;——to endleſs joy.

NOTE, The initials anſwer the *Prize Enigma*.

Ingenious anſwers to this queſtion have alſo been received from Meſſ. *T. Barker*, *J. Chipchaſe*, *P. George*, *Wm. Hardy*, and *Wm. Sewell*; but the firſt prize of 12 Diaries for the ſolution of it are fallen to the lot of the Rev. Mr. *Wildbore*, and that of 8 to *Plus Minus* the propoſer.

The Eclipſes calculated for 1770.

There will happen only two eclipſes this year; both of the ſun, and inviſible in Europe.

The firſt, on Friday the 25th day of May, at two in the morning, viſible in America, towards the tropic of Cancer.

The ſecond, on Saturday the 17th day of November, at 10 in the morning, viſible in the ſouthern parts of Africa and America.

J. KEACH.

On the 28th of Auguſt, Jupiter will be occulted by the moon, according to the following computation for Greenwich Obſervatory, viz. immerſion 11 h. 39′ 20″, and emer- 11 h. 57′ 47″ apparent time.

REUBEN ROBINS.

New Queſtions.

I. QUESTION 612, *by Mr.* Tho. Sadler.

In Whitchurch now a maid doth dwell,
Her neighbours ſtile her bonny Nell:
She likes to live a maiden's life,
And won't conſent to be a wife,
Tho' Harry, Richard, Ben, and Joe
To Nelly oft a courting go.
Above a ſcore ſue for the maid,
Fop, clown, and leering ganymede,
But to them all 'tis Nelly's ſong,
To marry ſhe is yet too young.
Her age and fortune you will find
From the * equations here ſubjoin'd.
Come artiſts ſay, from what is ſaid,
Is Nelly old enough to wed?
For ſhould the maiden ſtay too long,
She may forget her fav'rite ſong.

* Given $\left\{\begin{array}{l} x^2+y^2=13900+x^3, \text{ and} \\ x^3+y^3=6545080000000+y^4 \end{array}\right\}$: To find x, her age in years, and y her fortune in pounds

II. QUESTION 613, *by Mr.* Paul Sharp.

Given the difference of the tranſverſe and conjugate diameters of an ellipſis = 8, and the diff. of the length of the greateſt inſcribed parallelogram and radius of curvature at the end of the conjugate diameter = 3·28426, to find the ſaid diameters and rad. of curvature at the ſaid point.

III. QUESTION 614, *by Mr.* Wm. Gawith.

Given $\overline{x^{15}+y^{15}+z^{15}}|^{3}+\overline{x^3+y^3}|^{2}=a$, $\overline{x^{\cdot5}+y^{\cdot5}+z^{\cdot5}}| \times \overline{x^3+y^3+z^3}|^{2}=b$, and $\overline{x^3+y^3}|^{2}+\overline{x^3+y^3+z^3}|^{3}=c$; to find x, y, and z by means of quadratics.

IV. QUESTION 615, *by Mr.* J. Chipchaſe.

In the midſt of the Market-place at Stockton is erected a fine column of the Doric order, on the top of which ſtands an urn 5 feet in height, that appears the greateſt poſſible to a perſon (the height of whoſe eye is $5\frac{1}{2}$ feet) ſtanding at the diſtance of 32 feet from the baſe of the column, on level ground: From whence the column's height is required.

V. QUESTION 616, *by Mr.* Wm. Spicer.

Required the area of the exponential curve, whoſe equation is $\overline{x}|^{x^2} = b^{\frac{y^{\frac{1}{y}}}{x}}$.

VI. QUESTION 617, *by Mr.* Samuel Vince.

If the ſubtang. of a curve be expreſſed by $\frac{\overline{dx - x^4}|^2 \times y\dot{y}}{\overline{a - bx^2}|^2 \times \dot{x}}$; required the value of the ſemi-ordinate y thereof, when the abſciſſa x is $=$ a given quantity s.

VII. QUESTION 618, *by Mr.* J. Addiſon.

Suppoſe a triangular priſm, whoſe length is 12 feet, and baſe a right-angled iſoſceles triangle, the longeſt ſide of which is alſo 12 feet, to be placed perpendicularly in a ſtream of water 12 feet deep, with its right angle directly facing the ſtream; required the velocity of the ſtream per ſecond, when its force againſt the priſm is = 387072 lb. avo.

VIII. QUESTION 619, *by Mr.* Steph. Ogle.

Given an angle A in magnitude, and a point P in poſition; to draw a right line PLS in ſuch a direction, that $PLS + M$ may be $= AS + AL$, M being a given right line. [*See the fig. to the ſolution.*]

IX. QUESTION 620, *by Mr.* Edw. Williams, *Captain in the Royal Artillery.*

The ſame conſtruction remaining as in Monſ. Fermat's theorem (ſee the laſt year's Diary); it is required to demonſtrate, geometrically, when OV will be a maximum.

X. QUESTION 621, *by the Rev. Mr.* Wildbore.

Given the right line DC and the angle D, it is propoſed geometrically to determine the point B ſo, that BA, DA

being in a given ratio, the ſum of *BA* and *BC* may be a given quantity. [*See the fig. to the ſolution.*]

IX. QUESTION 622, *by the Rev. Mr.* Crakelt.

Given the right line biſecting the vertical angle of any plane triangle and terminating in its baſe, and the rectangle under each ſide and its adjacent ſegment of the baſe made by the ſaid biſecting line; to conſtruct the triangle.

XII. QUESTION 623, *by Mr.* T. Moſs.

If from the extremities of the diameter of a circle ever ſo many chords are drawn, two and two, interſecting each other in an ordinate perpendicular to that diameter, the chords joining the extremities of every correſponding two of them, being produced, will all interſect the ſaid diameter produced in one point; query the demonſtration?

XIII. QUESTION 624, *by Mr.* J. Powle.

Given $1^3 . 1^2 + 2^3 . 3^2 + 3^3 . 5^2 + 4^3 . 7^2$ &c. .... to x terms $= 845815702$, to find x;

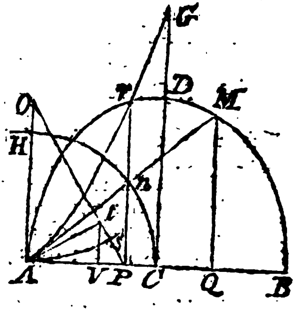
Where x is the age of my partner for life:
A fond mother, true friend, and good wife.

XIV. QUESTION 625, *by* Plus Minus.

Juſt after reading in the book of Geneſis of the tower of Babel, whoſe top was to reach up to heaven, I fell aſleep, and dreamed, that I was got upon the top of this tower. When St. Paul's clock, in London, began to ſtrike 12, I ſaw my own ſhadow fall on that church, and ſome time after ſaw it quit the earth's ſurface, juſt at the ſpot where a man ſeemed to be very hard at work. I called out (as loud as I could bawl) ſurely, friend, tis time to leave off; I think ſo too, ſays he, for our clock wants but a quarter of 7. This adventure happened on the firſt of May, as I knew by the garlands on the May-polls all about the country, and ſo the ſun's declination was 15° 19′. Hence I would know the height of the tower, the place where it ſtood, and the habitation of my friend aforeſaid.

XV. QUESTION 626, *by Mr.* Tho. Allen.

Let *ADB* be a given ſemi-ellipſe, *CnH* a circular arc deſcribed on the center *A* with the rad. *AC* = the ſemi-conjugate, and ſuppoſe the right line *AnM* cutting the circle in *n* and the ellipſe in *M*, to revolve about the point *A*, until it coincides with *AB*. Let, alſo, *QM*, *Pn* be drawn perpendicular to *AB* in the points *Q* and *P*, and on *Pn* (produced when neceſſary) let *Pr* be taken always = *AQ*; then, whilſt the point *M* deſcribes the ſemi-elliptic arc *ADB*, the point *r* will deſcribe the curve line *ArG*. It is required to determine the content of the ſolid generated by the revolution of the ſaid curve about its axis *AC*.



The PRIZE QUESTION, *by the Rev. Mr.* Wildbore.

A lady ſent her ſervant to a well famous for its excellent water, who filled the bucket, and drew it up very uniformly and ſteadily; but when the bottom arrived at land, found that the laſt drop of water was running out of it; for ſome unlucky boys had bored an hole therein 1·066105 inch diameter. Hence the time it was in drawing up is required, the diameter and depth of the bucket being each of them one, and the depth of the well to the ſurface of the water 29 feet.

1771.

Queſtions anſwered.

I. QUESTION 612 *anſwered by Mr.* Wm. Wilkin.

PUT $a = 13900$ and $b = 654508000000$; then, from the 1ſt equation, $y = \overline{a + x^3 - x^2}|^{\frac{1}{2}}$, which value ſubſtituted in the 2d gives $x^8 + \overline{a + x^3 - x^2}|^{\frac{3}{2}} = b + \overline{a + x^3 - x^2}|^2$: From whence x is found $= 30$, her age in years, and thence $y = 200$, her fortune in pounds.

In this manner, nearly, it is alſo anſwered by Meſſ. *J. Bartlet*, *G. Coughron*, *Curioſus*, *R. Dening*, *J. Hellings*, *J. Hooper*, jun. *Pamphagus*, *E. Parnel*, *E. Reed*, *Wm. Reynolds*, *Alex. Rowe*, *T. Sadler* (the propoſer), *J. Shadgett*, *T. Smith*, *Wm. Spicer*, *Mich. Taylor*, and *J. Young*, who are all greatly ſurpriſed that Nelly ſhould think herſelf too young to marry!

II. QUESTION 613 *anſwered by Mr.* Wm. Spicer.

Let $x =$ the ſemi-tranſverſe; then will $x\sqrt{2} =$ the length of the greateſt inſcribed parallelogram, and (by art. 71 of Simpſon's Flux.) $\frac{xx}{x - 4} =$ rad. of curvature at the extremity of the conjugate diam. whence (putting $a = 3{\cdot}28426$) $\frac{xx}{x - 4} = x\sqrt{2} - a$, per queſt. and $x = 20$: Conſequently the tranſverſe diameter $= 40$, the conjugate $= 32$, and the rad. of curvature $= 25$.

Much after this manner it is likewiſe anſwered by Meſſ. *J. Addiſon*, *J. Chipchaſe*, *G. Coughron*, *J. Hellings*, *Pamphagus*, *E. Parnel*, *E. Reed*, *Tho. Robinſon*, *Alex. Rowe*, *P. Sharp* (the propoſer), *W. Wilkin*, and *Ja. Young*.

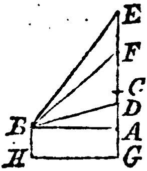
III. QUESTION 614 *anſwered by Mr.* Geo. Coughron.

Put $\overline{x^{1.5}+y^{1.5}+z^{1.5}}|^{3}=u$, $\overline{x^{3}+y^{3}}|^{2}=v$, and $\overline{x^{3}+y^{3}+z^{3}}|^{3}=w$; then the given equations will become $u+v=a$, $uw=b$, and $v+w=c$: Take the laſt equation from the firſt, and $u-w=a-c$, from which and the 2d equation u is found $=\frac{1}{2}\times\overline{\sqrt{\overline{a-c}|^{2}+4b}+a-c}$, $w=\frac{1}{2}\times\overline{\sqrt{\overline{a-c}|^{2}+4b}-a+c}$, and from thence v $(=a-u)=\frac{1}{2}\times\overline{a+c-\sqrt{\overline{a-c}|^{2}+4b}}$; whence $x^{1.5}+y^{1.5}+z^{1.5}$ $(=u^{\frac{1}{3}})$, $x^{3}+y^{3}$ $(=v^{\frac{1}{2}})$ and $x^{3}+y^{3}+z^{3}$ $(=w^{\frac{1}{3}})$ all become known, and from the 2d and 3d of theſe equations $z=\overline{w^{\frac{1}{3}}-v^{\frac{1}{2}}}|^{\frac{1}{3}}$; whence $x^{1.5}+y^{1.5}$ $(=u^{\frac{1}{3}}-z^{1.5})$ alſo becomes known, which put $=r$ and $x^{3}+y^{3}$ $(=v^{\frac{1}{2}})=2s$, and then (by queſt. 102 p. 63 of Simp. Select Exerc.) $x^{3}-y^{3}$ will eaſily be found $=2\times\overline{\overline{\tfrac{1}{10}rs^{-1}+\tfrac{4}{5}s^{4}}|^{\frac{1}{2}}-s^{2}}|^{\frac{1}{2}}$ (ſuppoſing x to be greater than y) which put $=2d$, and then x and $y=\overline{s+d}|^{\frac{1}{3}}$ and $\overline{s-d}|^{\frac{1}{3}}$ reſpectively.

In this manner, exceeding nearly, it is alſo anſwered by the Rev. Mr. *Crakelt*, Mr. *Gawith* (the propoſer), Mr. *Tho. Robinſon*, of *Biddick*, and Mr. *Wm. Spicer.*

Mr. *Mich. Taylor* has alſo ſent a ſolution to it.

IV. QUESTION 615 *anſwered by the Rev. Mr.* Crakelt.



CONSTRUC. Take $AB=32$ feet (the diſt. of the obſerver from the column), and in an indefinite perpendicular thereto, at the point A, the diſt. $AC=5$ feet (the length of the urn): Biſect AC in D, and take $DE=DB$; alſo take $EF=AC$ and $AG=5\frac{1}{4}$ feet, the height of the obſerver; then GF will be the required height of the column.

DEMONST. Since $BA^{2}=BD^{2}-AD^{2}$ $=\overline{BD+AD}\times\overline{BD-AD}=\overline{ED+DA}$ $\times\overline{ED-DC}$ (by conſtruc.) $=EA\times EC=EA\times AF$, a circle deſcribed through the three points *F*, *E*, *B* will have the line *AB* for a tangent, and by Simp. Algebra, p. 358, 2d edit. the line *FE* will appear under the greateſt angle to an eye at *B*, and conſequently *FA* + *AG* or *FG* will be the perpendicular height of the column.*

Upon this principle it is alſo anſwered by Meſſ. *J. Chipchaſe* (the propoſer), *Gemini*, *J. Hellings*, *E. Reed*, *Wm. Reynolds*, *Tho. Robinſon*, *Alex. Rowe*, *J. Spencer* (Diſcip. *T. Allen*), *W. Spicer*, and the Rev. Mr. *Wildbore*.

Mr. *Geo. Coughron* gives an ingenious conſtruction to it, founded on art. 55 of Rowe's Flux. 3d edit. and from thence calculates the height of the column required = 35·09751 &c. feet.

V. QUESTION 616 *anſwered by Mr.* J. Spencer, *Diſcip.* T. Allen.

Put z and a for the hyp. logs. of x and b reſpectively, and then $zx^2 = \frac{ay^{\frac{1}{2}}}{x}$, and $y = a^{-2}z^2x^6$; whence $y\dot{x} = a^{-2}z^2x^6\dot{x}$, and the fluent (by art. 341 of Simp. Flux.) is $\frac{1}{7}a^{-2}x^7 \times z^2 - \frac{2}{7}z + \frac{2}{49}$ = the area of the curve required.

And in this manner the anſwer is alſo given by Meſſ. *J. Addiſon*, *G. Coughron*, the Rev. Mr. *Crakelt*, Meſſ. *E. Parnel*, *Tho. Robinſon*, *Alex. Rowe*, *Paul Sharp*, *Wm. Spicer* (the propoſer), *Wm. Wilkin*, and the Rev. Mr. *Wildbore*.

VI. QUESTION 617 *anſwered by Mr.* Geo. Coughron.

Per queſt. $\frac{\overline{dx - x^4}|^2 \times y\dot{y}}{\overline{a - bx^2}|^2 \times x\dot{x}} = \frac{y\dot{x}}{\dot{y}}$; ∵ $\dot{y} = \frac{a\dot{x} - bx^2\dot{x}}{dx - x^4} = \frac{a\dot{x}}{dx} + \frac{a}{d} \times \frac{x^2\dot{x}}{d - x^3} - \frac{bx\dot{x}}{d - x^3}$, and, taking the correct fluent, $y =$

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* See Mr. Crakelt's remark on this queſtion, at the bottom of p. 37, Diary 1774.

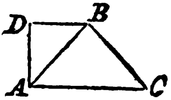
$y = \frac{a}{d} \times$ hyp. log. of $x + \frac{a}{3d} \times$ hyp. log. of $\frac{d}{d - x^3} + \frac{b}{3r} \times \overline{N\sqrt{3} + M - M'}$ (where $N =$ the circular arc whoſe ſine is $\frac{x\sqrt{\frac{3}{4}}}{\sqrt{rr + rx + xx}}$ to rad. 1, $r = d^{\frac{1}{3}}$ and $M, M' =$ the hyp. logs. of $\frac{r - x}{r}$ and $\frac{\sqrt{rr + rx + xx}}{r}$ reſpectively, ſee arts. 329 and 330 of Simpſon's Flux. 2d edit.); whence the value of y may be determined let that of x be what it will.

The Rev. Mr. *Wildbore* ſolves it thus, viz. $\dot{y} = \dot{x} . \frac{a - bx^2}{dx - x^4}$ (per queſtion) $= \frac{a\dot{x}}{dx - x^4} - \frac{bx\dot{x}}{d - x^3} = \frac{a\dot{x}}{dx - x^4} - b\dot{x} \times : \frac{-\frac{1}{3}x + \frac{1}{3}d^{\frac{1}{3}}}{d^{\frac{1}{3}}x^2 + d^{\frac{2}{3}}x + d} + \frac{\frac{1}{3}}{d^{\frac{1}{3}}x - d^{\frac{2}{3}}} = \frac{a}{d} \frac{\dot{x} - 4d^{-1}x^3\dot{x}}{x - x^4d^{-1}} + \frac{a}{d} \times \frac{4d^{-1}x^2\dot{x}}{1 - x^3d^{-1}} + \frac{b\dot{x}}{3d^{\frac{2}{3}} . \overline{d^{\frac{1}{3}} - x}} + \frac{b}{3d^{\frac{1}{3}}} \times \frac{t\dot{t}}{cc + tt} - \frac{1}{2}b \times \frac{\dot{t}}{cc + tt}$ (putting $cc = \frac{3}{4}d^{\frac{2}{3}}$ and $t = x + \frac{1}{2}d^{\frac{1}{3}}$), the correct fluent of which is $y = \frac{a}{d} \times$ h. l. $\overline{x - \frac{x^4}{d}} - \frac{4}{3} \times$ h. l. $\overline{1 - \frac{x^3}{d}} - \frac{b}{3d^{\frac{1}{3}}} \times$ h. l. $\overline{d^{\frac{1}{3}} - x} + \frac{b}{6d^{\frac{1}{3}}} \times$ h. l. $\overline{cc + tt} - \frac{b}{2c} \times$ circular arc whoſe tang. is $\frac{t}{c}$ to rad. $1 + \frac{b}{2c} \times$ circular arc tang. $\frac{1}{\sqrt{3}}$ and rad. 1; which, placing s for x, gives what is required.

The Rev. Mr. *Crakelt* and Meſſ. *Alex. Rowe* and *S. Vince* (the propoſer) have likewiſe anſwered it.

VII. QUES-

VII. QUESTION 618 *anſwered by Mr.* J. Addiſon (*the Propoſer*).



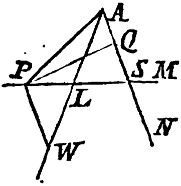
ABC repreſent a ſection of the priſm parallel to its ends, and *BD* and *AD* be parallel and perpendicular to *AC* reſpectively; then, the number of forcing particles of the fluid acting upon the ſide *AB*, being as *BD*, and their force as the ſquare of the ſine of the angle of incidence *BAD* (45°), by mechanics, the whole force upon the two ſides thereof, *AB*, *BC*, will manifeſtly be = half that upon its baſe *AC* (ſuppoſing *AC*, directly, to face the ſtream): But, putting v = the velocity of the water (per ſecond) reckoned in feet, and $s = 16\frac{1}{12}$ feet, the force againſt *AC* is $= \frac{144vv}{2s} \times 62{\cdot}5$ lb. Av. (per prop. 107 of Emerſon's Mechanics, 1ſt edit. a cubic foot of water weighing $62\frac{1}{2}$ lb. Av.); therefore $\frac{144vv}{4s} \times 62{\cdot}5 =$ the force againſt *ABC* = 387072 lb. Av. per queſt. Whence $v = 52{\cdot}6008$ &c. feet, the velocity of the ſtream per ſecond, which was required.

Upon the ſame principle, and in the very ſame manner nearly, the anſwer is alſo given by the Rev. Mr. *Crakelt* and Meſſ. *J. Bartlett*, *J. Chipchaſe*, *G. Coughron*, *J. Spencer* (Diſcip. *T. Allen*), and *S. Vince*.

The Rev. Mr. *Wildbore* has alſo favoured us with a curious ſolution to this queſt. founded on the principle, that the effect which the preſſure of the water has to move the priſm in the direction of the ſtream ſhall be equal to that of a weight of 387072 lb. Av. from which, and the conſideration of the action of the back water, he proves that the effect of the preſſure againſt low-arched bridges, in time of floods, ſuch as thoſe made by Mr. *Brindley* for conveying his aqueducts over rivers, is not ſo great as is commonly imagined, not varying according to the depth of the water above, but nearly as the ſquare of the difference of the depth above and below the bridge: Alſo, that the arch ought in this caſe to be a parabola, that ſo its width may be every where proportional to the quantity of water which is to be diſcharged through it; for the water runs with the greateſt velocity at the bottom, and when the upper part of the arch is choaked up with hay, ice, &c. if it be thus formed it will diſcharge the moſt water when and where there is the greateſt neceſſity for it, &c.——but we cannot ſpare room for the ſolution at preſent.

VIII. QUEST. 619 *anſwered by the Rev. Mr.* Wildbore.

If the conchoid ruler revolve about the pole P upon the given line ASN with the given line or diſt. $SM = M$, till $PM = AL + AS$, it will give the poſition, or poſitions, of PM required, with much more eaſe than any ſolid conſtruction derived from the following



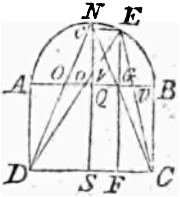
ANALYSIS. Join A, P, and draw PQ perpendicular and PW parallel to AS. Put $AP = a$, $AW = b$, $AQ = g$, $PW = d$, $As = x$, and $AL = y$; then, per ſim. triangles, $b - y$ $(AW - AL) : d$ $(PW) :: y$ $(AL) : x$ (AS), whence $x = \frac{dy}{b-y}$. Alſo, by Euc. 2. 13, and the conditions of the queſtion, $x + y - M = \sqrt{aa + xx - 2gx}$; reduced, $y^3 - \overline{2d + 2M + b}.y^2 + \overline{2Md + 2Mb + M^2 - a^2 - 2gd}.y + \overline{a^2 - M^2}.b = 0$: Which equation not being of the kind that admits of reduction by any of the methods of diviſors, except in particular caſes, it ſhould ſeem as if the problem could not generally be reduced to a plane one.——But the roots of the equation may be found with great facility after the manner of Newton and Leibnitz by means of the above mechaniſm.

Mr. *G. Coughron*'s ſolution is not eſſentially different from this.——He alſo ſhews, by way of corollary, that where the point P coincides with L, the cubic reduces to a ſimple equation.

It is likewiſe anſwered by the Rev. Mr. *Crakelt* and Meſſ. *J. Hellings* and *Alex. Rowe*.

IX. QUESTION 620 *anſwered by the Rev. Mr.* Crakelt.

CONSTRUC. From the vertices, D, C, of the given perpendiculars, AD, BC, to the middle N of the ſemicircle ANB draw the right lines DN, CN, and the part, OV, which they intercept on the diameter AB will be the maximum required.



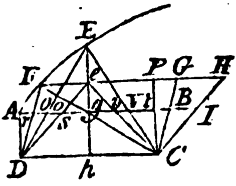
DEMONSTRA. For to any other point, E, of the ſaid ſemicircle draw the right lines DoE, CvE, and from N and E let fall the perpendiculars NQS and EGF upon DC, and draw $Ec \parallel BA$; then, by ſimilarity of triangles, $NS : NQ :: DC : OV$, and $EG : EF :: ov : DC$, or, by taking the rectangles of the correſponding terms, $NS \times EG = \overline{Nc + cS} \times cQ : NQ \times EF = \overline{Nc + cQ} \times cS :: ov : OV$: But $\overline{Nc + cQ} \times cS$ is greater than $\overline{Nc + cS} \times cQ$; wherefore OV will be greater than ov.

SCHOLIUM. Whatever the ſegment ANB, and the lengths of the perpendiculars AD, BC, are, the conſtruction and demonſtration will hold good.

The demonſtration by Capt. *Edw. Williams* (the propoſer) is alſo very curious, and purely geometrical.

The Rev. Mr. Wildbore*'s Demonſtration is thus, viz.*

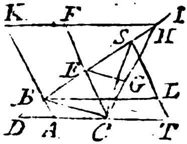
At any given diſt. whatever, draw $AB \parallel$ to DC and $Eh \perp$ to the ſame; then 'tis ſelf-evident that the higher the point e or E is taken above h, the greater will ov or OV be; through e draw a right line parallel to DC, in which take any point F; join FD, FC, cutting AB in r and s: Compleat the parallelograms DG, DH; then, Fe being $= GH$, and PC ($\perp$ to FeH) common (to the $\triangle$s CGH, CFe, DFe) BI will be $= sv = ro$, and adding os (common), rs will evidently be $= ov$; therefore 'tis manifeſt that OV will be the greateſt at the vertex, and the ſame at equal heights above DC, let gh and the curve AFE be what they will.



Meſſ. *J. Chipchaſe*, *G. Coughron*, *J. Hellings*, *E. Parnel*, *T. Robinſon*, *Alex. Rowe*, *J. Spencer* (Diſcip. *T. Allen*), *W. Spicer*, *S. Vince*, and *W. Wilkin* have likewiſe demonſtrated it.

X. QUESTION 621 *anſwered by Mr.* Steph. Ogle.

CONSTRUC. Make the $\angle CDI =$ the given $\angle D$, and draw CE to terminate in DI ſo, as to be to DC in the given ratio of BA to DA; produce it till $CF =$ the given quantity, and through F draw $KH \parallel$ to DC meeting DI in H; join C, H, and draw EG to terminate therein ſo as to be $= EF$; then, drawing CB, $BA \parallel$ to EG, CE reſpectively, the thing will be done.

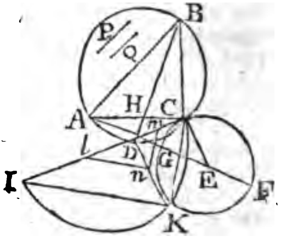


DEMONTST. Produce AB to meet KH in K; then, per ſim. triangles, $EF : BK (:: HE : HB) :: EG : BC$, where the antecedents EF, EG being equal (by conſtruction) the conſequents BK, BC muſt be equal alſo: Conſequently $AB + BC (= AB + BK) = CF$, the given quantity by conſtruction.——The Rev. Mr. *Crakelt*'s conſtruction is not materially different.

Elegant conſtructions to this problem have alſo been received from Meſſ. *G. Coughron*, *Plus Minus*, *T. Moſs*, and the Rev. Mr. Wildbore (the propoſer); and Meſſ. *T. Robinſon*, *Alex. Rowe*, *W. Wilkin*, and *Ja. Young* anſwer it algebraically.

XI. QUESTION 622 *anſwered by the Rev. Mr.* Crakelt (*the Propoſer*).

CONSTRUC. Let P and Q repreſent the ſides of two ſquares equal in magnitude to the two given rectangles, and find a fourth proportional, DC, to the given biſecting line, P and Q; perpendicular to which erect $CE = \frac{1}{2}$ the ſaid biſecting line, and therewith as rad. deſcribing a circle, draw $DGEF$: In CD produced take $DI = DF$, and on CI as diam. deſcribe the ſemicircle CKI; erect the per-



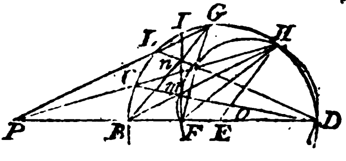
pendicular DK, and having taken thereon $Dn = Q$, draw nl, nm ∥ to KI, KC reſpectively; then with DC, DG, and Dm conſtitute the △ DHC, and in DH produced take $HB = GF$, and join the points B, C: Circumſcribe the △ BDC with a circle, and produce CH to meet it in A; then, joining the points A, B, ABC will be the required triangle.

DEMONSTRA. By conſtruc. $BD \times DH = DF \times DG = DC^2$; whence the △s BDC and HDC are ſimilar, and conſequently $BD : DC :: BC : CH$: But, by reaſon of parallels, we alſo have BD (lD) $: DC :: lD : CH$ (Dm); whence $BC = Dl$, and $BC \times CH = Dl \times Dm =$ (ſince the $\angle lnm$ is right) $Dn^2 = Q^2$, by conſtruction. The $\angle ABD = \angle ACD = \angle DBC$; whence BH $(= GF = 2CE =$ the given biſecting line) biſects the $\angle ABC$: Therefore $CB : HB :: DC$ (AD) $: AH$, and $CH : HB :: DC : AB$, and conſequently $CB \times CH : HB^2 :: DC^2 : AH \times AB$: But, by conſtruction, $HB^2 : P^2 :: Q^2 : CD^2$; wherefore, *ex æquo perturbatè*, $CB \times CH : P^2 :: Q^2 : AH \times AB$; but, from what has been already proved, $CB \times CH = Q^2$, and of conſequence $AH \times AB = P^2$.

Ingenious conſtructions to this probem have likewiſe been received from Mr. *G. Coughron*, *Gemini*, and the Rev. Mr. *Wildbore*; and Meſſ. *Wm. Reynolds*, *T. Robinſon*, *Alex. Rowe*, *Wm. Spicer*, *Mich. Taylor*, *Wm. Wilkin*, and *Ja. Young* have given neat algebraic ſolutions to it.

XII. QUESTION 623 *anſwered by Mr.* T. Moſs (*the Propoſer*).

DEMONSTRA. Let any two (unequal) chords BH, DG be drawn, interſecting each other in the point m; thro' which point draw the perpendicular (or ſemi-chord) FI; draw the radius EH, and alſo FH and DH: Then it is evident that the $\angle FHB$ is equal to $\angle BHP$, each being $= \angle FDm$ $(= \angle BDC)$. Moreover, the $\angle EBH + \angle FHB = \angle EFH$; but $\angle EBH$ is $= \angle EHB$: Therefore $\angle EHB + \angle BHP$ (FHB) $= \angle EHP = \angle EFH$; whence the $\angle E$ being



being common, the $\triangle$s EHP and EHF are ſimilar, and ſo $HE : EF :: PE : HE$, or $EB\,(HE) : EF :: PE : EB\,(HE)$, or (dividedly) $EB : EB - EF\,(FB) :: PE : PE - BE\,(BP)$, or, alternately, $EB : PE :: FB : BP$: Again, by diviſion of ratios, $EB : PE - EB :: FB : BP - FB$, i. e. $EB : BP :: FB : BP - FB$; in which circumſtance it is demonſtrated (ſee theo. 15. p. 74. Simp. Geom. 1ſt edit.) that if any two right lines be drawn from the points F, P meeting any where in the periphery of the circle deſcribed with the rad. EB, they will be in the conſtant ratio of FB to BP, and conſequently a right line (BG or BH) drawn from B to the point where the two ſaid lines meet in the periphery will always biſect the angle formed by thoſe lines. ——Therefore, through any other point n, in the perpend. FI, let now two chords be drawn from B and D to meet the periphery in G and L; then, having drawn GF and GL, it is evident from the former part of this demonſtration, that the $\angle FGB\ (= FDn) = \angle BGL$, and conſequently that (by the above-mentioned theorem) GL produced muſt neceſſarily meet DB produced, in the very ſame point P where HC produced met it.

The Rev. Mr. Wildbore's *Demonſtration is as follows, viz.*

In the given ordinate FI take any point m, and draw DmC, BmH; and HCP meeting DB produced in P. Biſect Dm in O, and with OD rad. deſcribe the circ. FmD, which, per Euc. 3. 31, will circumſcribe the quadrilateral $FmHD$; then, per 1. 32. the $\angle HBE = \angle BPH + \angle BHP$, the $\angle HED = \angle EHP + \angle EPH$, and, per 3. 21, the $\angle CHB = \angle CDB\,(mDF) = mHF$; therefore the $\angle PHE = \angle EBH\ (BHE) + \angle PHB\,(CDB)$, the $\angle HPD = \angle HBE - \angle CDB$, and the $\angle EFH = \angle EBH + \angle mHF\,(mDB)$: Conſequently the $\angle EFH = \angle PHE$, and the $\angle E$ being common, the $\triangle$s FEH, HEP (per 1. 32.) are ſimilar, and (per 6. 4.) $FE : EH\ (= BE) :: EH : PE$; where 'tis manifeſt that FE and EH will continue the ſame wherever in the given ordinate FI the point m is taken, and conſequently that their third proportional PE muſt continue the ſame likewiſe.

COR. 1. If, in like manner, a circle be deſcribed about the quadrilateral $BCmF$, it will appear that the $\angle FCm$ is $= \angle mBF = \angle HCm$; $\because CD$ biſects the $\angle FCH$: and it is ſhewn above that BH biſects the $\angle CHF$; $\because IF$ biſects the $\angle CFH$, and m is the cent. of the circle inſcribed in the $\triangle CFH$. It is evident likewiſe that PI is a tangent at I.

COR. 2. Becauſe $FE : BE = EH :: FH : PH :: BF : BP$, $\because$ by converſion and diviſion, $BP : BP - BF :: BE : BE - FE = BF$. Wherefore if, from any two given points P and F, two right lines in a given ratio are to be drawn, divide PF in that ratio in B, and take $BE : BF :: BP : BP - BF$; then two lines meeting any where in the circumference of the circle deſcribed with the rad. BE, will be in the required ratio, which is the property laid down at *P.* 336 of Simp. Alg. 3d edit. and applied to the ſolution of many difficult problems by that ingenious author.——And with the ſame eaſe may other curious and uſeful proportions be derived.

We are alſo favoured with demonſtrations of this curious property by Mr. *J. Chipchaſe*, the Rev. Mr. *Crakelt*, Mr. *G. Coughron*, *Plus Minus*, and Mr. *Ja. Young*.

XIII. QUESTION 624 *anſwered by Mr.* Cha. Hutton.

Put s for the given ſum of x terms of the ſeries; then the $x+1$ term or $\underset{\cdot}{s}$ is $= \overline{x+1}|^3 . \overline{2x+1}|^2 =$ (putting $z = x+1$) $\underset{\cdot}{z}^3 . \overline{2z-1}|^2 = 4\underset{\cdot}{z}^5 - 4\underset{\cdot}{z}^4 + \underset{\cdot}{z}^3 = 4z\underset{1}{z}\underset{2}{z}\underset{3}{z}\underset{4}{z} - 40\underset{\cdot}{z} + 4 . z\underset{1}{z}\underset{2}{z}\underset{3}{z} + \overline{100\underset{\cdot}{z}^2 + 24\underset{\cdot}{z} + 1} . z\underset{1}{z}\underset{2}{z} - \overline{60\underset{\cdot}{z}^3 + 28\underset{\cdot}{z}^2 + 3\underset{\cdot}{z} . z\underset{1}{z}} + \overline{4\underset{\cdot}{z}^4 + 4\underset{\cdot}{z}^3 + \underset{\cdot}{z}^2} . z$; and, by taking the integrals, &c. we then obtain $s = \frac{1}{60} \times : 40x^6 + 72x^5 - 5x^4 - 50x^3 - 5x^2 + 8x$, the ſum of the ſeries propoſed ($\underset{\cdot}{z}$ being $= 1$ and $x = z - 1$); from which the remaining part of the queſt. might be eaſily determined, &c.

And by other different but ingenious methods the ſum of the ſeries is likewiſe determined by Mr. *G. Coughron*, the Rev. Mr. *Crakelt*, Meſſ. *J. Hellings*, *E. Reed*, *Alex. Rowe*, *W. Spicer*, *S. Vince*, and *W. Wilkin*: But the ingenious propoſer having, by ſome miſchance, given us a wrong number (*845815702*), none of them could find the right age, that ſent by the propoſer along with the queſt. in the year 1767 being 48.

Anſwer to the ſame by the Rev. Mr. Wildbore.

The propoſed ſeries $1^3 . 1^2 + 2^3 . 3^2 + 3^3 . 5^2 + 4^3 . 7^2$ .... to x terms is evidently $= 1^3 . \overline{2.1-1}|^2 + 2^3 . \overline{2.2-1}|^2$

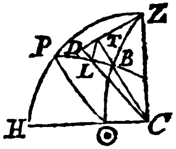
+

$+ 3^3 \cdot \overline{2 \cdot 3 - 1}|^2 + 4^3 \cdot \overline{2 \cdot 4 - 1}|^2 \ldots\ldots$ to x terms $= 4 \times \overline{1^5 + 2^5 + 3^5 \ldots\ldots x^5} - 4 \times \overline{1^4 + 2^4 + 3^4 \ldots\ldots x^4} + 1^3 + 2^3 + 3^3 \ldots\ldots x^3 =$ (by caſe 3d on p. 206 of Simp. Alg. 2d edit. &c.) $\frac{2}{3}x^6 + \frac{6}{5}x^5 - \frac{1}{12}x^4 - \frac{5}{6}x^3 - \frac{1}{12}x^2 + \frac{2}{15}x$, which ſuppoſe $= 8458957024$,* and then $x = 48$, the truly valuable lady's age required by (the propoſer) Mr. *Powle*.

* NOTE, This truly ingenious gentleman having diſcovered this number ſo as to give the value of x a whole number, it happens luckily to be the very number that gives $x =$ the age meant by the propoſer, &c.

XIV. QUESTION 625 *anſwered by Mr*. W. Wales.

From the data of this queſt. it appears that the tower ſtood to the ſouth of London, and under the ſame meridian, and that the ſun ſet at 6 h. 45′ by the time at the labourer's meridian, which let the arc ZPH repreſent, where Z is the zenith, or geographical ſituation, P the pole, and ⊙ the ſun in the horizon HC. Draw P⊙, the ſun's polar diſtance, the vertical Z⊙ and PLB for the meridian of London,

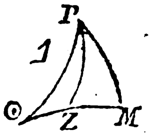


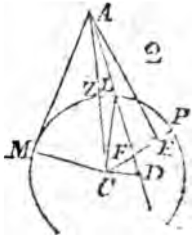
and the point B where theſe two laſt interſect will be the ſituation of the tower, and L that of London. Now, in the right-angled ſpherical △ PH⊙, having given P⊙ and the ∠ P, HP, the labourer's latitude, is readily found $= 35° \, 27' \, 46''$ north; and if ZT be a tangent to the arc ZB in Z, and meeting CB produced in T, TB will be the height of the tower. Join T, L, and from T, on CL produced, let fall the perpendicular TD, and the $\angle DLT$ will be $=$ the ⊙'s zenith diſt. at London on the given day, whoſe ſine and coſine denote by r and p, the ſine and coſine of ZP by s and c, thoſe of LP by a and b, the coſine of $\angle PZB$ ($= H$⊙) by v, and DL by x. Then $1 + x = DC$, and $\frac{rx}{p} = TD$; alſo (Euc. 47. 1.) $\sqrt{DC^2 + DT^2} = TC$: Moreover $TC : 1 :: TD :$ ſine $\angle DCT =$ ſine LB, whoſe coſine alſo is thence known; and (Simp. Trig. p. 55) coſine $PL \times$ coſ. $LB -$ ſine $PL \times$ ſine $LB =$ coſ. PB. Again, ſince TC is the ſecant of ZB; its coſine $= \frac{1}{TC}$, from whence its ſine alſo is determinable, and (per Anderſon's theo.) ſine $ZB \times$ ſine $ZP \times$ coſ. $\angle PZB +$ coſ. $ZB \times$ coſ. $ZP =$

coſ. PB. Theſe two values for the coſ. PB expreſſed in ſymbols, according to the preceding notation, and equated, give $b + \frac{x}{p} \times \overline{pb - ar} = sv\sqrt{2x + \frac{xx}{pp}} + c$; but $pb - ar$ and sv are each = the ſine of the ⊙'s declination, for which putting d, and $b - c = m$, the equation will be $\frac{m}{d} + \frac{x}{p} = \sqrt{2x + \frac{xx}{pp}}$; whence $x = \frac{pm^2}{2d \times dp - m} = 5{\cdot}9433341$. Hence TC and the $\angle TCD$ are readily found = 8·1928755, and 32° 4′; and thence the tower's height = 7·1928755 radii of the earth, and its latitude 19° 27′ north; and laſtly, the labourer's longitude = 95° 19′ eaſt.

Anſwer to the ſame by the ingenious Propoſer, Plus Minus.

In fig. 1. P repreſents the north pole, ⊙ that place of the earth to which the ſun was vertical when the ſhadow quitted its ſurface, Z the place of the tower, and M that of the man. It is evident that at the time aforeſaid the ſun muſt be in the horizon of M, and therefore $\odot M = 90°$; $\odot P$ is the comp. of declin. of the ſun, and the $\angle \odot PM = 101° \; 15'$: Hence the man's lat. will be found = 35° 27′ 46½″ north. Find the $\angle P\odot M$ and call its coſ. q, and let the ſine and coſ. of $\odot P$ be s and d reſpectively. Then, to find expreſſions for the ſine and coſine $\odot Z$ and the coſine of ZP, conſider fig. 2d, where A repreſents the top of the tower, Z its baſe, L London, P the north pole, C the earth's center, and M the man's place (which is here put in the meridian of L to prevent confuſion in the diagram): Draw AL cutting the axis CP in F, and let fall CD, $AE \perp$ to AL and CP, and join CL, CM, which call r, and AL, z; moreover, let the ſine and coſine of the ſun's merid. alt. at London be m and n (= LD and CD). Then $AC = \sqrt{zz + 2mz + rr}$, and the ſine of the $\angle CAM$ (= $\odot Z$) = $rr \div \sqrt{zz + 2mz + rr}$, and its coſine = $r\sqrt{zz + 2mz} \div \sqrt{zz + 2mz + rr}$. Now the $\angle$s FCD, FAE being equal





to the ſun's declination, $s : d :: n\,(CD) : \frac{dn}{s}\,(FD)$ $s : r :: n : \frac{rn}{s} = CF$, $r : d :: z + m - \frac{dn}{s}\,(AF) : \overline{dsz + dsm - ddn} \div rs\,(FE)$, and $FE + FC$ (or CE) $= \overline{dz + dm + sn} \div r$ (putting ss for $rr - dd$): Hence the coſine of $ZP = \overline{dz + dm + sn} \div \sqrt{zz + 2mz + rr}$, and (by means of an excellent theor. in the Lady's Diary 1732, pa. 8.) $qrrs \div \sqrt{zz + 2mz + rr} + drr\sqrt{zz + 2mz} \div \sqrt{zz + 2mz + rr} = rr \times \overline{dz + dm + sn} \div \sqrt{zz + 2mz + rr}$; whence $z = \overline{dm + sn - sq}\,\rceil^{2} \div 2ds \times \overline{q - n}$.——Hence the reſt will be found as follows, viz. The height of the tower $AZ =$ 7·370827 radii of the earth $=$ 29364·296 miles, its latitude $= 19°\,22'$ N. and the man's longitude from London $= 95°\,26'\,30''$ E. nearly.

This queſtion is alſo anſwered ingeniouſly by the Rev. Meſſ. *Crakelt* and *Wildbore*.

XV. QUESTION 626 *anſwered by Mr.* Geo. Coughron.

Put $AP = x$, $Pr\,(AQ) = y$, $AC = a$, $CD = b$, and 3·1416 $= p$ (vid. laſt year's fig.); then will $Pn^2 = aa - xx$, and, per prop. of the ellipſis, $QM^2 = b^2a^{-2} \times \overline{2ay - yy}$: Whence, by means of the ſim. △s APn, AQM, will be found $y = 2abbxx \div \overline{a^4 + ccxx}$ the equation of the curve, cc being $= bb - aa$; conſequently $py y\dot{x}$ (the flux. of the required ſolidity) $= 4paab^4x^4\dot{x} \div \overline{a^4 + ccxx}\,\rceil^{2}$, (and taking the flu.) the ſolidity itſelf, when $x = a$, will be either $2pa^3b^4c^{-5} \times \overline{\overline{3aac + 2c^3} \times \overline{aa + cc}\,\rceil^{-1} - 3a \times \text{cir. arc whoſe tan. is } \frac{c}{a} \text{ and rad. } 1}$, or $2pa^3b^4c^{-5} \times \overline{\overline{3aac - 2c^3} \times \overline{aa - cc}\,\rceil^{-1} - \frac{3}{2}a \times \text{h. log. of } \frac{a + c}{a - c}}$, according as b or a is the ſemi-tranſverſe, &c.

And in this manner, nearly, the anſwer is alſo given by Mr. *J. Addiſon*, the Rev. Mr. *Crakelt*, Meſſ. *J. Hellings*, *C. Hutton*, *E. Parnel*, and *S. Vince*.

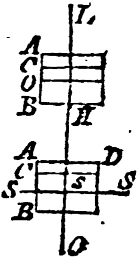
Anſwer to the ſame by the Rev. Mr. Wildbore.

The figure being drawn as directed in the queſt. erect the $\perp AO = AC^2 \div \sqrt{DC^2 - AC^2}$, with which, as rad. and cent. O, deſcribe the arc of a circle, and join O, P cutting it in s; let fall At, $tV \perp$ upon OP and AC reſpectively, then will AV (by ſim. triangles) $= AP . AO^2 . OP^{-2}$, which, becauſe $AO^2 + AP^2 = OP^2$, is evidently the fluent of $\dot{AP} . AO^2 . \overline{AO^2 + AP^2 - 2AP^2} . OP^{-4}$. Take any where a right line $M = 2AC . DC^2 . \overline{DC^2 - AC^2}^{\,-1} = 2AO^2 . DC^2 . AC^{-3}$; then, by conics, &c. $AM^2 = 2CD^2 . AQ . AC^{-1} - AQ^2 . AC^2 . AO^{-2}$, and, by means of ſimilar triangles, $M : AQ :: AO^2 + AP^2 : AP^2$; $\because AQ = Pr = M . AP^2 . OP^{-2}$, $Pr^2 (= M^2 . AP^4 . OP^{-4}) = M^2 \times \overline{\overline{AO^2 + AP^2}^{\,2} - \frac{3}{2} AO^2 . \overline{AO^2 + AP^2} + \frac{1}{2} AO^2 . \overline{AO^2 + AP^2 - 2AP^2}} \times \overline{AO^2 + AP^2}^{\,-2}$ and the ſolidity generated round AP, or the fluent of $p . \overline{rP}^2 . \dot{AP}$, is $p . M^2 \times \overline{AP - \frac{3}{2}\text{ the arc } As + \frac{1}{2} AV}$, which was required.

COROLLARY. Pr ($= M . AP^2 . OP^{-2}$ (being $= M \times \overline{1 - AO^2 . OP^{-2}}$), the flu. of Pr, $\dot{AP}$ is $M . AP -$ the arc As = the area of the curve ArP.

Plus Minus puts $AC = c$, $CD = t$, $AP = z$, and $Pr = v$, and then finds the equation of the curve ArG to be $v = 2cttzz \times \overline{c^4 \pm mmzz}^{\,-1}$ (ſee fig. in the laſt year's Diary), where $mm = tt - cc$, and is affirmative or negative according as CD is the longer or ſhorter ſemi-axis. In the former caſe, the curve ArG is the conchoidal hyperboliſm of an ellipſis with a diameter without a center, and that diameter is a line perpendicular to AC in the point A, and its aſymptote is parallel to AC; and, in the latter caſe, the curve is the hyperboliſm of an hyperbola with a diameter, its two parallel aſymptotes being perpendicular to AC and equidiſtant from A, and the 3d parallel to AC, which is the tangent at the vertex of the axis, viz. A.——If $tt = cc$, or $m = 0$, i. e. if the ſemi-ellipſis AMD becomes a ſemicircle, the curve degenerates into a common parabola, and AC is ſtill a tang. at A the vertex of the axis. The fluxion of the ſolid required is $4pc^2t^4z^4\dot{z} \times \overline{c^4 \pm mmzz}^{\,-2}$, and the fluent, or ſolid itſelf, is eaſily found from Emerſon's forms, &c.

The PRIZE QUEST. *anſwered by the Rev. Mr.* Wildbore (*the Propoſer*).



Suppoſe SS to be the ſurface of the water, L the land, O the bottom of the bucket when filled with water, B the bottom of it when the part AS is drawn out of the water, C the ſurface of the water in the bucket in this poſition, $LS = 348$ inches $= n$, $AB = SO = AD = 12 = a$, $1{\cdot}066105 = c$, $193 = d$, $s =$ the velocity per ſecond with which the bucket is drawn up, $AS = BO = r$, and $AC = x$. Then $\sqrt{d} : 2d :: \sqrt{\tfrac{1}{2}CS} : d^{\frac{1}{2}}\sqrt{2r-2x} =$ the velocity per ſecond of deſcent of a column of water whoſe height is CB and immerſed part SB, owing to its own preſſure, which increaſed by s gives $s + d^{\frac{1}{2}}\sqrt{2r-2x} =$ the real velocity per ſecond with which the water runs out of the orifice: Wherefore $c^2a^{-2} \times \overline{s + d^{\frac{1}{2}}\sqrt{2r-2x}} =$ the relative velocity per ſecond with which the ſurface of the water deſcends in the bucket at C, and $s - c^2a^{-2} \times \overline{s + d^{\frac{1}{2}}\sqrt{2r-2x}} =$ the abſolute velocity per ſecond, with which the ſurface of the water aſcends in the well. Whence $\dfrac{aa.\overline{\dot{r}-\dot{x}}}{\overline{aa-cc}.s - c^2d^{\frac{1}{2}}\sqrt{2r-2x}} =$ the flux. of the time of aſcent from S to C or from O to B; the fluent of which corrected by taking r and x each $= 0$, will, when $AS = BO = r = a$ (putting $\gamma = \dfrac{\overline{aa-cc}.s}{cc\sqrt{d}}$), give $\dfrac{aa}{cc\sqrt{d}} \times \gamma \times$ hyp. log. of $\overline{\dfrac{\gamma}{\gamma-\sqrt{2a-2x}} - \sqrt{2a-2x}} = \dfrac{a}{s}$, the time in which the bucket aſcends out of the water. ——When it has aſcended to H, ſuppoſe O the ſurface of the water therein, $b = a - x = CB$, and $y = CO$; then, in the ſame manner as before, will $\dfrac{cc}{aa} \times \overline{s + d^{\frac{1}{2}}\sqrt{2b-2y}} =$ the relative velocity per ſecond with which the ſurface of the water deſcends in the bucket at O, and the fluxion of the time of aſcent from S to H = that of deſcent in the bucket between thoſe two poſitions $= \frac{aa\dot{y}}{cc \times \overline{s + d^{\frac{1}{2}}\sqrt{2b - 2y}}}$; the fluent of which, when $y = b = a - x$, or when H arrives at L (putting $\frac{s}{\sqrt{d}} = e$), is $\frac{aa}{cc\sqrt{d}} \times :$ $\sqrt{2a - 2x} + e$. hyp. log. of $\frac{e}{e + \sqrt{2a - 2x}} = \frac{n}{s}$ = the time of aſcent of the bottom of the bucket from the ſurface of the water till it arrived at land: From which equation and that above x is found $= {}^{\cdot}4557$, $s = 12$, and $\frac{n + a}{s} =$ 30″ the time required.

COROLLARY. $\frac{AD^2\sqrt{2AB}}{\overline{{}^{\cdot}088842}|^2\sqrt{16{}^{\cdot}0853}} = 44{}^{\cdot}67768''$ is the time in which the bucket would be emptied at reſt.

SCHOLIUM. From the ſame principles may one part of the theory of the curious wheel for raiſing water upon the river Limat at Zurich in Switzerland be determined.*

Ingenious

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* This is the ſolution in which was propoſed that new principle of effluent water, which occaſioned that long altercation on the ſubject, in our Miſcellany, by two learned and ingenious gentlemen. This diſpute was carried on, by both parties, with that true gentlemen-like candour, openneſs, and modeſty, which has done them much honour. And of their ſeveral ingenious arguments, every reader is left to avail himſelf, and adopt that principle which to him ſeems to be the true one.

The above ſolution is given on the principle of the velocity of the effluent ſtream being increaſed by that of the veſſel itſelf. And we ſhall here ſubjoin the ſolution on the contrary principles, that every perſon may take which he pleaſes.

Let a, c, d, n, and s denote the ſame quantities as in the above original ſolution; alſo x = any variable alt. of the ſurface of the water in the bucket above that in the well, while the bucket is aſcending out of the water; and t the correſponding time. Then $\sqrt{2dx}$ = the velocity of the iſſuing ſtream, and ∴ $c^2a^{-2}\sqrt{2dx}$ = the velocity of the deſcending ſurface in the bucket; conſequently $s - c^2a^{-2}\sqrt{2dx}$ = velocity per ſec. with which the ſurface of

the

Ingenious anſwers to this queſtion have alſo been received from Meſſ. *J. Addiſon*, *T. Allen*, *T. Barker*, *G. Coughron*, the Rev. Mr. *Crakelt*, *C. Hutton*, and *Sam. Vince*; but the prize of 12 Diaries for the ſolution thereof is fallen to the lot of the Rev. Mr. *Wildbore* (the propoſer) and that of 8 to Mr. *Hutton*.

The Eclipſes calculated for 1771.

There will be four eclipſes this year, two of the ſun, and two of the moon, according to the following order.

The firſt is a lunar eclipſe, on Monday the 29th day of April; Beginning 0 h. 58 m. Middle 2 h. 4½ m. End 3 h. 11 m. and digits eclipſed 4° 18′.

The

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the water in the bucket aſcends above that in the well; $\because$ $s - c^2 a^{-2}\sqrt{2dx} : 1'' :: \dot{x} : \dot{x} \div \overline{s - c^2 a^{-2}\sqrt{2dx}} = \dot{t}$; hence t is $= \frac{-a^2}{dc^2}\sqrt{2dx} + \frac{a^4 s}{c^4 d} \times$ hyp. log. $\frac{s}{s - c^2 a^{-2}\sqrt{2dx}}$ which by the queſtion will be $= \frac{a}{s}$ when the bottom of the bucket arrives at the ſurface of the water in the well, or when the bucket is juſt totally emerged from the water in the well.

Now the remaining part of the time, or that in which x depth of water will iſſue from the bucket, is $\frac{a^2}{dc^2}\sqrt{2dx}$ by prob. 1 art. 1 Miſcel. and this muſt be $= \frac{n}{s}$ by the queſtion. Adding now this to the former time, we have $\frac{a^4 s}{c^4 d} \times$ h. l. $\frac{s}{s - c^2 a^{-2}\sqrt{2dx}}$ $= \frac{a+n}{s}$ the whole time of emptying. In this equation write $\frac{c^2 dn}{a^2 s}$ for $\sqrt{2dx}$ (its value by the equation $\frac{a^2}{c^2 d}\sqrt{2dx} = \frac{n}{s}$), &c. and we have $\frac{a+n}{a^4} \times c^4 d = s^2 \times$ hyp. log. $\frac{s^2}{s^2 - c^4 dna^{-4}}$; or, in numbers, $1{\cdot}879796 = s^2 \times$ com. log. $\frac{s^2}{s^2 - 4{\cdot}1841096}$. From hence we find $s = 7{\cdot}9667$. And $\because \frac{a+n}{s} = 45{\cdot}18809''$ the whole time required, on this principle.

The ſecond is of the ſun, on Tueſday the 14th day of May, about 8 m. after three o'clock in the afternoon. It is inviſible in Europe, but may be ſeen in the Atlantic ocean.

The third is of the moon, on Wedneſday the 23d day of October; part viſible at London, the moon riſing at 5 h. 5 m. in the afternoon. Eclipſed 3° 26', and ends 5 h. 55½ m.

The fourth and laſt is another inviſible eclipſe of the ſun, on Wedneſday the 6th day of November, about ſeven o'clock in the evening; but may be ſeen in North America.

ISAAC TARRATT.

New Queſtions.

I. QUESTION 627, *by Mr.* Tho. Sadler.

Young Hodge, a homely country ſwain,
Long courted Suſan of the plain,
But never could a method find
To bring the fair one to be kind.
Her pride is center'd in herſelf;
She calls him clown and country elf,
And mimics faſhion with an air:
For dreſſing few with her compare.
This charmer's name with eaſe you'll find
From what is underneath ſubjoin'd.*
O! tell the way to make her kind.

* Given $\left\{\begin{array}{l} x+y^2+z=201, \\ x^2+yz=37, \\ z^2+x^2y=130; \end{array}\right\}$ To find x, y, and z; their values expreſſing the places of the letters in the alphabet that compoſe her name.

II. QUESTION 628, *by Mr* Wm. Gawith.

Given $\overline{x^5+y^5+z^5}|^5+\overline{u^5+z^5}|^5=a$, $x^5+y^5+u^5+2z^5=b$, $\overline{u+z}|^2\times\overline{x^5+y^5+z^5}=c$, and $\overline{u^5+z^5}\times\overline{x+y}|^2=d$, to find x, u, y, and z by quadratics.

III. QUESTION 629, *by Mr.* G. Cetii.

If any plane triangle be circumſcribed by a circle, and have another circle inſcribed therein, and if from any one of the angular points a ſtraight line be drawn to the center of the inſcribed circle, and produced to meet the periphery of the circumſcribing one: Then will the ſum of the ſides which include the angle from whence the ſtraight line is drawn, be to the ſaid ſtraight line, ſo produced, as the the third ſide of the triangle is to that part of the ſtraight line which is produced beyond the center of the inſcribed circle. It is required to demonſtrate this geometrically.

IV. QUESTION 630, *by Mr*. R. Mayo.

To determine the locus of all the places on the earth which have the ſame altitude of the ſun with any given place, at a given inſtant of time.

V. QUESTION 631, *by Mr*. Wm. Spicer.

A ſhell was diſcharged from a mortar, which in its flight juſt touched the top of a ſteeple, and in four ſeconds of time after fell at the diſtance of $3262\frac{6}{7}$ feet from the bottom of the ſteeple: From whence the report of its fall was heard at the mortar juſt 12 ſeconds after the exploſion: Required the ſteeple's height.

VI. QUESTION 632, *by Mr* Paul Sharp.

Suppoſe a tub in the form of the fruſtum of a common parabolic conoid, whoſe top diameter = 4, bottom diameter = 2·52982, and depth = 3 feet, to be filled with water: Required the time of its emptying through a circular hole of 1 inch diameter in its bottom.

VII. QUESTION 633, *by Mr*. T. Moſs.

Having given the hypothenuſe of a right-angled plane triangle, and the part of the leg intercepted between the right angle and a point where a perpendicular on the middle of the hypothenuſe meets the ſaid leg, to determine the triangle.

VIII. QUESTION 634, *by Mr*. S. Clark.

Required to draw a right line *CB* from a given point *C* between two right lines *DE*, *DF* given in poſition to terminate in one of them (*DE*) ſo, that drawing another right line *BA* from the point of termination (*B*), parallel to another right line *DG* given, likewiſe, in poſition to terminate in the other of them (*DF*), the difference of the lines, *CB*, *BA*, ſo drawn may be equal to a given right line (*MN*).

IX. QUESTION 635, *by the Rev. Mr.* Crakelt.

Two of the ſides of a plane triangle, together with the line drawn from the angle they include to the center of the inſcribed circle being given, to determine the triangle by conſtruction.

X. QUESTION 636, *by Mr.* T. Moſs.

To draw a right line through the given point *E* in the ſide *B F* of the given plane triangle *B F C* meeting *B C* in *G* and *C F* produced in *H* ſo, that the ſum of the perpendiculars *G m*, *H n* drawn from the points of interſection *G*, *H* to the ſaid ſide *B F* may be equal to a given right line *P Q*.

XI. QUESTION 637, *by Mr.* J. Chipchaſe.

A gentleman has a direct ſouth-reclining dial in his garden. Its ſtyle is a ſtreight pin erected perpendicular to the plane of the dial, at the diſtance of 18 inches from its center, the ſhadow of which returns at a particular time of the day, for ſeveral days in ſummer. On the 21ſt of June the arch deſcribed by the returning ſhade contained 4° 24′ 40″, and the height of the ſun above the plane of the dial at 5 h. 17′ 57″ that afternoon, was = to his altitude above the horizon: Hence is required the latitude of the place and height of the pin that the top of its ſhade may ſhew the true hour of the day.

XII. QUESTION 638, *by the Rev. Mr.* Wildbore.

What right-angled plane triangle is that, the numerical value of whoſe double area being ſubtracted from that of each of its ſides ſeverally ſhall leave three ſquare numbers?

XIII. QUESTION 639, *by* Plus Minus.

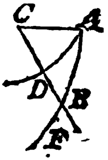
If any ſemidiameter of any conic ellipſis be lengthened in the ratio of the ſide of a ſquare to its diagonal, and from the end of that line two tangents be drawn to the ellipſis, the points of contact will be the vertices of two conjugate diameters: And moreover, the line joining thoſe vertices will be to the diameter which is parallel to it, in the ſame ratio of the ſide of a ſquare to its diagonal; the demonſtration is required.

XIV. QUESTION 640, *by Mr.* Wm. Wales.

Having given the repreſentation of any leſſer circle of the ſphere ſtereographically projected on a given plane, it is required to deſcribe the repreſentation of a great circle which touches the leſſer one, and has a given arc intercepted between the point of contact and its interſection with ſome other given great circle.

XV. QUESTION 641, *by Mr.* Tho. Allen.

Suppoſe *ABF* to be a certain ſpiral line deſcribed by a body in motion, beginning at *A*, and having the following property, viz. $x = \frac{N^{2z}+1}{2N^z}$, where $AC = CD = 1$, CB, any diſtance from the center C, $= x$, circ. arc $AD = z$, and N = the number whoſe hyp. log. $= 1$: Required the length of the ſaid ſpiral, or the ſpace deſcribed by the revolving body, when $x = 1000$.



The PRIZE QUESTION, *by the Rev. Mr.* Wildbore.

In the latitude of London what is the declination of that ſtar, whoſe azimuth increaſeth the leaſt poſſible in two hours after riſing;—and on what day of the year does the ſun's azimuth increaſe the moſt poſſible in two hours after riſing?

1772.

Queſtions anſwered.

I. QUESTION 627 *anſwered by Mr.* Hen. Clark.

IT is evident from the 1ſt equation that y cannot be greater than 14 nor leſs than 12 (becauſe x, y, and z are to be whole poſitive numbers by the nature of the queſt. not exceeding 24); therefore ſuppoſing $y = 14$, xx from the 2d equation becomes $= 37 - 14z$, which value ſubſtituted in the 3d equation it becomes $zz - 196z = -338$; whence, by compleating the ſquare, &c. $z = 2$, a whole number; conſequently $x = 3$, $y = 14$, and $z = 2$, anſwering to the letters in the alphabet *C O B*, the lady's name required.

It is alſo anſwered by Meſſ. *J. Bartlett*, *G. Cetii*, *J. Chipchaſe*, *S. Clarke*, *G. Coughron*, *R. Dening*, *Gemini*, *J. Haycock*, *J. Hellings*, *L. Ker*, *R. Mallock*, *T. Nicholſon*, *Pamphagus*, *E. Reed*, *W. Reynolds*, *T. Robinſon*, *A. Rowe*, *T. Sadler* (the propoſer), *J. Shadgett*, *P. Sharp*, *W. Spicer*, and *W. Wells*.

Mr. *Thomas Adcock* concludes his ſolution with the following conſolatory advice:

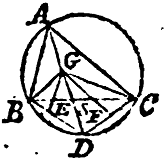
Friend Hodge, forbear to ſigh or ſob
For ſuch a one as Suſan COB;
Addreſs ſome other charming ſhe,
That is more complaiſant and free:
This method take; perhaps you'll find
The haughty fair will grow more kind.

II. QUESTION 628 *anſwered by Mr.* G. Coughron.

The two firſt given equations being the ſum and the ſum of the 5th powers of $x^5 + y^5 + z^5$ and $u^5 + z^5$, it will be found by queſt. 48 pa. 105 of Simp. Alg. 2d edit. that $x^5 + y^5 + z^5$ and $u^5 + z^5 = \frac{1}{2}b \pm \sqrt{\sqrt{\frac{1}{5}ab^{-1} + \frac{1}{20}b^4} - \frac{1}{4}bb}$, the upper of the double ſigns giving the value of the one, and the lower the other of them; theſe values being put $=$ m and n reſpectively, from the 3d and 4th equations, by diviſion, &c. will be had $u + z = c^{\frac{1}{2}} m^{-\frac{1}{2}}$, which call $2e$, and $x + y = d^{\frac{1}{2}} n^{-\frac{1}{2}}$ which call $2r$; and from thence, as above, will be found u and $z = e \pm \sqrt{\sqrt{\frac{1}{10}ne^{-1} + \frac{4}{5}e^4} - e^2}$; conſequently $x^5 + y^5 = m - z^5$ becomes known, which call s, and then from it and $x + y = 2r$, x and y come out $= r \pm \sqrt{\sqrt{\frac{1}{10}sr^{-1} + \frac{4}{5}r^4} - r^2}$, the affirmative ſign giving the value of the greater, and the negative ſign that of the leſſer of them.

Ingenious anſwers to this queſtion have alſo been received from Meſſ. *G. Cetii*, *J. Chipchaſe*, *Hen. Clark*, *S. Clarke*; the Rev. Mr. *Crakelt*, *Gemini*, Meſſ. *J. Hellings*, *L. Ker*, *E. Reed*, *W. Reynolds*, *Tho. Robinſon*, *P. Sharp*, and *W. Spicer*.

Qeustion 629 *anſwered by Mr.* Iſaac Dalby.



Demonstra. The line biſecting the $\angle A$ of the propoſed $\triangle BAC$ being produced to meet BC in S and the circumf. of its circumſcribing circ. in D, let the points B, D and C, D be joined, and right lines drawn from B, C to G the cent. of the inſcribed circle: Likewiſe draw GE, $GF \parallel AB$, AC meeting BD, CD in E and F; then (E. 3. 21.) the $\angle DBC$ being $= \angle DAC = \angle BAD$ and alſo $= \angle EGD$ (by conſtruction), the $\angle SBG = \angle GBA$ (Euc. 4. 4.) and the $\angle DGB =$ the $\angle$s GAB, GBA (Euc. 1. 32.) BD will be $= DC$ (Euc. 1. 6.) $= DG$, and the $\triangle$s BSD, DEG ſim. and equal; $\because GE = BS$: And in the ſame manner GF is proved to be $= CS$; and conſequently, the trapeziums $BACD$, $EGFD$ being ſim.l and alike ſituated, $AB + AC : AD :: EG + GF$ $(BS + SC = BC) : DG$.——Mr. *R. Mallock*'s demonſtration is the ſame as this exceeding nearly.

It is alſo demonſtrated in a curious manner by Meſſ. *G. Cetii* (the propoſer); *J. Chipchaſe*, *H. Clarke*, *S. Clarke*, *G. Coughron*, the Rev. Mr. *Crakelt*, *C. Hutton*, *Tho. Nicholſon*, *Tho. Robinſon*, *J. Turner*, and the Rev. Mr. *Wildbore*.

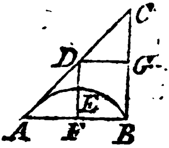
IV. Question 630 *anſwered by Mr.* G. Cetii.

As the ⊙'s alt. is conſt. and given, its complement, or the ⊙'s zenith diſt. will be conſt. and given alſo. Now, nothing can be plainer than that the ⊙'s zenith diſt. is always equal to the diſt. between the given place and that to which the ſun is vertical at the given time; whence this diſt. is given and conſtant, and therefore if the point to which the ⊙ is vertical at the given time be found, and round it, as a pole, a circle be deſcribed through the given place, it will be the locus required.

Corollary. Hence, if it was required to find the locus of all the places where the ſun riſes or ſets at the ſame time on a giving day, it is manifeſt it will be a great circle, whoſe pole is the point to which the ſun is then vertical.

Much after the ſame manner this queſtion is alſo anſwered by Meſſrs. *J. Chipchaſe*, *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Dalby*, *J. Haycock*, *C. Hutton*, *R. Mayo* (the propoſer), *Alex. Rowe* and the Rev. *Mr. Wildbore*.

V. QUESTION 631 *anſwered by Mr.* G. Coughron.



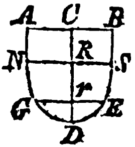
Let AEB repreſent the path of the ſhell, AC its direction at the commencement of motion, EF the the ſteeple; and let DG be parallel and BC perpendicular to the horizontal line AB. Put $FB\ (=DG) = 3262\frac{6}{7}$ feet $=a$, the time of its deſcription $=4'' = b$, the velocity of ſound per ſecond $=1142=c$, and the time of the ſhell's flight, or the time of its deſcribing AEB or $AC=x$; then will $b:a::x:ax\div b=AB$, and conſequently $x+ax\div bc=12''=t$; whence $x=\dfrac{bct}{a+bc}$ $=7''$, $AB=ax\div b=5710$ feet, $BC=7^2\times 16\frac{1}{12}=788\frac{1}{12}$ feet, $DE=3^2\times 16\frac{1}{12}=144\frac{2}{3}$ feet, $AF=\frac{3}{7}\times 5710=2447\frac{1}{7}$, $FD\ (=AF\times BC\div AB)=337\frac{3}{4}$, and the ſteeple's height $FE\ (=FD-DE)=193$ feet.

The ſame anſwered independent of Algebra, by Mr. Cha. Hutton.

The time in which ſound moves from B to F is $=FB\div 1142 = 3262\frac{6}{7}\div 1142 = 2\frac{6}{7}''$; conſequently the horizontal velocity of the ſhell is to that of ſound as $2\frac{6}{7}$ to 4. But, as $4+2\frac{6}{7}:4::12-4-2\frac{6}{7}:5\frac{1}{7}\times 4\div 6\frac{6}{7}=3''$, the time of the ſhell's flight from A to E or F, ſo that the whole time of its flight is $7''$. Alſo, $AF=\frac{3}{7}AB$, $FD=\frac{3}{7}BC$; and, by the law of falling bodies, $1^2:7^2::16\frac{1}{12}:16\frac{1}{12}\times 49=BC$, and $1^2:3^2::16\frac{1}{12}:16\frac{1}{12}\times 9=ED$; whence $FE\ (=FD-ED=\frac{3}{7}BC-ED=16\frac{1}{12}\times 21-16\frac{1}{12}\times 9=16\frac{1}{12}\times 12)=193$ feet, the height of the ſteeple required. ——And in this manner almoſt exactly the anſwer is alſo given by the Rev. Mr. *Wildbore*, without algebra.

It is alſo ingeniouſly anſwered by Mr. *G. Cetii*, the Rev. Mr. *Crakelt*, Meſſ. *Hen. Clark*, *S. Clark*, *J. Bartlet*, *J. Dalby*, *J. Haycock*, *J. Hellings*, *W. Reynolds*, *Alex. Rowe*, *P. Sharp*, *W. Spicer* (the propoſer), *J. Turner*, and *W. Wells*.

VI. QUESTION 632 *anſwered by Mr.* Iſaac Dalby.



Let *AGEB* repreſent the tub, and ſuppoſe the parabolic conoid to be compleated. Put the top diameter $(=4)=d$, the bottom diameter $(=2{\cdot}52982)=b$, the depth $(=3$ feet$)=h$, the part of the axis $Dr=n$, $\cdot7854=a$, $t=$ the time in which the tub's circumſcribing cylinder would empty itſelf thro' the ſame orifice with the firſt or greateſt velocity uniformly continued, x $(=rR)$ any variable alt. of the running water's ſurface above the bottom of the tub, and $y=$ the time fought. Then, by the nature of the curve, $NS^2=d^2\times\overline{n+h}|^{-1}\times\overline{n+x}$, and by reaſoning as in examples 4, 5 on pa. 139, 140, &c. of Emerſon's Fluxions, 2d edit. $\dot{y}=-\overline{n+h\times\sqrt{h}}|^{-1}\times\overline{tnx^{-\frac{1}{2}}\dot{x}+tx^{\frac{1}{2}}\dot{x}}$; the correct fluent whereof, when $x=0$, is $y=2t\times\frac{3n+h}{3n+3h}=14'\ 4{\cdot}3476''$ the time required.

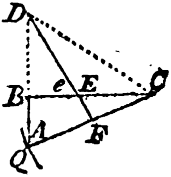
COROLLARY. The times of emptying the tub and its circumſcribing cylinder are as $\frac{3}{5}$ to 1; and if the tub was inverted, the time through an equal orifice would be $2t\times\frac{3n+2h}{3n+3h}=18'\ 45{\cdot}769''$.

The Rev. Mr. *Wildbore* anſwers it thus, viz. ' From the ' data are eaſily found $p=\frac{4}{3}$ the parameter of the genera' ting parabola, and the depth wanting to compleat it at ' bottom $=2$ feet $=a$. Let c $(=3$ feet$)=$ the depth of the ' veſſel, $bb=\frac{1}{4\times144}=\square$ of rad. of the orifice, $x=$ any ' variable height of the water above it, $y=$ the ord. cor' reſponding, $d=16\frac{1}{12}$ feet, and $t=$ the time required; ' then, by the laws of hydroſtatics, $\frac{bb}{yy}\sqrt{2dx}=\frac{bb\sqrt{2d}}{p}\times$ ' $\frac{\sqrt{x}}{a+x}=$ the velocity of deſcent along the axis of the ' veſſel; by which dividing $-\dot{x}$, it gives $\dot{t}$, whoſe corrected ' fluent, when $x=0$, is $t=\frac{p\sqrt{2c}}{bb\sqrt{d}}\times\overline{a+\frac{1}{3}c}=844{\cdot}3476$ ' $=14{\cdot}07246'$, the time required.'

For the ſolution of this queſt. Mr. *Hutton* refers to art. 1 of his New Mathematical Miſcellany, where the ſubject of ſuch exhauſtions is fully treated on.

Meſſ. *G. Cetii*, *J. Chipchaſe*, *Hen. Clark*, *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Hellings*, *J. Hitchcock*, *T. Robinſon*, *Alex. Rowe*, *P. Sharp* (the propoſer), and *W. Spicer* have likewiſe anſwered it.

VII. Question 633 *anſwered by Mr.* T. Moſs (*the Propoſer*).



Construction. On BE the given intercepted part of the leg produced, take EC (by prob. 18. pa. 107 of Simp. Geom. 2d edit.) ſuch that $BC \times EC$ may be = the rectangle under the given hypothenuſe and its half, and about C, as a center, with the given hypothenuſe as rad. deſcribe an arch of a circle cutting $BQ \perp CB$ in A; then, join the points A, C and the thing is done.

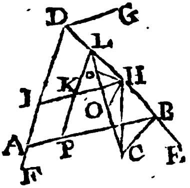
Demonstration. Upon the middle of AC erect the perpendicular Fe meeting BC in e; then, by ſim. triangles, $AC : BC :: eC : FC$, and conſequently (by Euc. 16. 6.) $AC \times FC = BC \times eC = BC \times EC$ (per conſtruction); whence $eC = EC$, and the points e, E coinciding, $Be = BE$ (the given intercepted part, per conſtruction), and ABC is a right angle, per conſtruction. *Q. E. D.*

Corollary. If AB, FE be produced to meet in D, and CD be drawn, it evidently appears that DE biſects the $\angle BDC$, and that the biſecting line ED is to the hypothenuſe CD in the given ratio of BE to FC ($\frac{1}{2}AC$); whence it manifeſtly follows that the preceding conſtruction is, in effect, the very ſame as that of a right-angled triangle having the hypothenuſe and a line biſecting one of its acute angles, and terminating in the oppoſite leg, given.

We are alſo favoured with ingenious conſtructions to this problem from Meſſ. *G. Cetii*, *J. Chipchaſe*, *Hen. Clark*, *S. Clark*, *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Dalby*, *Gemini*, *J. Haycock*, *J. Hellings*, *C. Hutton*, *L. Ker*, *R. Mallock*, *Tho. Nicholſon*, *E. Reed*, *Wm. Reynolds*, *Tho. Robinſon*, *Alex. Rowe*, *P. Sharp*, *W. Spicer*, and *J. Turner*.

VIII. Question 634 *anſwered by Mr.* G. Coughron.

Construction. Draw CH and $HI \parallel DF$ and DG. On HI take $IK =$ the given diff. MN, and through K draw $LKP \parallel DF$; join C, L, and with the rad. HK and cent. H, interſect CL in o, O, and $\parallel HO$ and DG draw CB and BA, which will be the lines required.

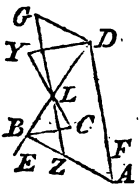


Demonstra. By ſim. $\triangle$s, $HK : BP\ (:: LH : LB) :: HO\ (= HK) : BC$; conſequently $BC = BP = AB - AP = AB - IK = AB - MN$, and therefore $AB - BC = MN$, as it ought. *Q. E. D.*

Corollary. The rad. HK cutting CL in two points ſhews that the prob. admits of two anſwers.——When MN is $\sqsubset HI$ the conſtruction is ſtill the ſame, only HI muſt be continued beyond H till it is $= MN$; and when CB is to be greateſt, MN muſt be ſet from I to the left hand, on HI produced, and then the reſt of the conſtruction will be the ſame as above.

Construction *to the ſame by Mr.* Cha. Hutton.

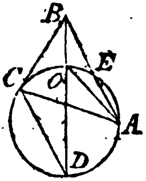
Take $DG = MN$, and draw $GZ \parallel DF$ cutting DE in L. Draw CLY ſo that $DY = DG$: then, drawing CB, $BA \parallel DY$ and DG, meeting DE and DF in B and A reſpectively, and the thing will be done.——For by ſim. $\triangle$s, $DL : LB :: DG : BZ :: DY : BC$; but $DG = DY$; ∵ $BZ = BC$, and $ZA = DG$.



This problem is alſo conſtructed in an ingenious and very elegant manner by Meſſ. *G. Cetii*, *S. Clark* (the propoſer), the Rev. Mr. *Crakelt*, *J. Dalby*, *J. Haycock*, *J. Hellings*, *R. Mallook*, *E. Reed*, *Tho. Robinſon*, *Alex. Rowe*, *P. Sharp*, *J. Turner*, and the Rev. Mr. *Wildbore*.

IX. QUEST. 635 *anſwered by the Rev. Mr.* Wildbore.

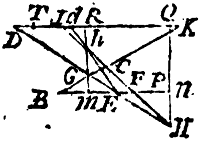
CONSTRUCTION. Upon BO the given biſecting line, produced, take BD a fourth proportional to BO and the two given ſides of the triangle, and on the diameter DO deſcribe a circle; to which from B ſet off the two given ſides BC, BA (the latter of them interſecting the circumf. ſuppoſe in E); join A, C, and ABC is the required triangle——For (per Euc. 3. 37.) DB is a fourth proportional to OB, AB, and EB, and (by conſtruc.) to OB, AB, and BC; therefore $EB = BC$, and $\angle CBO = \angle EBO$, and the ſides about the equal angles being (by conſtruc.) proportional, the $\triangle$s AOB, DCB (per Euc. 6. 6.) are ſimilar, and (per 3. 21.) $\angle CDO \, (= BAO) = \angle CAO$; ∴ (per 4. 4.) O is the center of the inſcribed circle, and the triangle is that required.—— And hence by way of corollary is eaſily deduced an elegant demonſtration to queſt. 3d.



The conſtruction by Meſſ. *Iſaac Dalby* and *Cha. Hutton* agree with the above in almoſt every particular; and it is alſo conſtructed in an ingenious and curious manner by Meſſ. *G. Cetii*, *Hen. Clark*, *S. Clark*, *G. Coughron*, the Rev. Mr. *Crakelt* (the propoſer), *J. Haycock*, *J. Hellings*, *R. Mallock*, *E. Reed*, *Tho. Robinſon*, *Alex. Rowe*, and *J. Turner*.

X. QUESTION 636 *anſwered by Mr.* G. Cetii.

CONSTRUCTION. At a diſtance from $BF = PQ$ and parallel thereto draw DK, and produce BC and FC to meet it in K and I. Make $IT = 2FE$, and take TD ſo, that $TD \times DK = BE \times EF$; then draw DE, meeting CF produced, in H, and the thing is done.

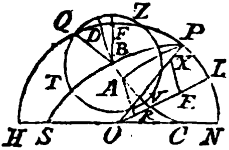


For $DT : EF :: BE : DK$ (by conſtruction), and (Simp Geom. 20. 4.) $Gm : PQ - Gm \, (GR) :: BE : DK$; alſo $PQ + nH : nH :: DT + 2EF \, (DI) : EF$, or by diviſion, $PQ - Hn : Hn :: DT : EF$; ∵ by equality, $PQ - Hn : Hn :: Gm : PQ - Gm$, or by compoſ. $PQ : Hn :: PQ : PQ - Gm$. Now the antecedents being here equal, the conſequents will be ſo alſo, and ∴ $PQ = Gm + Hn$.

But if FC be produced towards I, take Id ſo, that $Id \times Kd = BE \times EF$, and draw dE for the line required; the demonſtration of which being much more ſimple than that of the above caſe, needs not pointing out.

Very ingenious and curious conſtructions to this queſtion have alſo been received from Meſſ. *S. Clark*, *G. Coughron*, the Rev. Mr. *Crakèlt*, Meſſ. *J. Dalby*, *J. Haycock*, *J. Hellings*, *Cha. Hutton*, *T. Moſs* (the propoſer), *E. Reed*, *Tho. Robinſon*, *Alex. Rowe*, *P. Sharp*, and the Rev. Mr. *Wildbore*.

XI. QUESTION 637 *anſwered by Mr.* W. Wales.



PROJECTION. Let $HZPN$ be the meridian, P the elevated pole, and PO the ſemi-axis of the ſphere. Deſcribe DC, the parallel of the ☉'s declin. for the given day, and the great circle QAR to touch it, making the $\angle Q$, which it forms with the meridian, equal to half the arc the returning ſhade deſcribes, and Q will be the pole of the dial-plane, OL. Draw the hour-circle PBS, anſwering to 3h. 17m. 57s. and cutting DC in B, the place of the ſun at that time. Round B, as a pole, deſcribe, through Q, the leſs circle TQZ, cutting the meridian again in Z, which will be the zenith of the place required; and ∵ ZP is the complement of its latitude. Laſtly, make $OE = 18$ inches, and draw $EX \perp OL$, meeting PO in X, and EX will be the height of the pin, and the $\triangle OEX$ the proper gnomon of the dial.

DEMONSTRATION. It is plain that the motion of the ſhade will be direct until the ſun comes to A, i. e. into the vertical to the dial-plane, QAR, which touches the parallel of declination; after which it can go no farther weſtward, but will return; and when the ſun gets on the meridian at D, will have deſcribed an arc $=$ the meaſure of the $\angle AQP$, and which is therefore $=$ half the arc deſcribed by the returning ſhade. Again, Q and Z being the poles of the planes OL, HN, and $BQ = BZ$ by conſtruction, it is plain that the altitudes of the point B above thoſe planes are equal. Laſtly, OL being the plane of the dial, PO the ſemi-axis of the earth, and $OE = 18$ inches, EX muſt be the height of the pin; for its top muſt be in the right line drawn through the pole and the center of the dial.

Anſwer to the ſame by Mr. J. Chipchaſe (*the Propoſer*).

It is evident that the plane of this dial is parallel to the horizon of ſome place within the tropics, and that the time of the ſhadow's returning will be when the ſun's azimuth from the north is the greateſt poſſible at that place. Let LO be the horizon of the place, which is parallel to the plane of the dial (ſee fig. preceding), Q the zenith thereof, P the north pole, DC the parallel of declination on the 21ſt of June, A the place of the ſun when his azimuth from the north is the greateſt poſſible, and the $\angle QAP$ will therefore be a right one. Put the ſine of $PQ = x$, ſine of $AP = s$, its coſine $= c$, and then the ſine of the $\angle AQP$ or arch RL, will be $= \frac{s}{x}$, and the ſine of the amplitude $OV = \frac{c}{x}$, and its coſine, or the ſine of the arch VL, $= \sqrt{1 - \frac{cc}{xx}}$. The arch RV or $RL - VL$ deſcribed by the returning ſhade, being $= 4° 24' 40''$, call its ſine b, and then (per Emerſ. Trig. prop. 6. b. 1.) $\frac{sc}{xx} - \sqrt{\overline{1 - \frac{cc}{xx}} \times \overline{1 - \frac{ss}{xx}}} = b$; which reduced gives $x = \sqrt{\frac{1 - 2bsc}{1 - bb}} = {}$·9743693 the nat. ſine of 77°; therefore the height of the pole above the dial-plane is 13°; and rad. : 18 inches :: tang. 13° : 4·156 inches, the height of the pin required——Deſcribe a merid. to make an angle of 3 h. 17′ 57″ or 49° 29′ 15″ with HQP, cutting the parallel of declin. in B, and deſcribing $BF \perp QP$, make $FZ = FQ$, and Z will be the zenith of the place where the dial is fixed: For, by the queſt. at 3 h. 17′ 57″ the height of the ☉ above the plane of the dial was equal to his alt. above the horizon of the place where it ſtands; B is the ☉'s place, and QB, ZB are the complements of his alt. above the plane of the dial and horizon of the place where it is fixed, at that time, which are equal by conſtruction.—Laſtly, cotang. BP : rad. :: coſ. $\angle FPB$ (49° 29′ 15″) : tang. $FP = 56° 13' 30''$; whence $ZP = 35° 27'$, the comp. of the required latitude.

The Rev. Mr. *Crakelt*, Meſſ. *Iſaac Dalby*, *Cha. Hutton*, and the Rev. Mr. *Wildbore* have alſo favoured us with curious projections of this prob. and calculations to the ſame, deduced therefrom; and it is alſo anſwered in a very ingenious manner, without algebra, by Mr. *Geo. Coughton*.

XII. QUESTION 638 *anſwered by the Rev. Mr.* Crakelt.

Let the legs of the required triangle be repreſented by x and nx; then are $x-nxx$, $nx-nxx$ and $\sqrt{\overline{xx+nnxx}}-nxx$ to be ſquare numbers (per queſt.): Suppoſe $x-nxx=xx$; then will $nx=1-x$, by which means two of the conditions of the queſt. will be fulfilled, and therefore what remains is to make $\sqrt{\overline{xx+nnxx}}-nxx=\sqrt{\overline{1-2x+2xx}}+xx-x$ a ſquare number. Now, to effect this, imagine $1-2x+2xx=\overline{mx-1}|^2$, and x will be $=\frac{2m-2}{m^2-2}$, $\sqrt{\overline{1-2x+2xx}}+xx-x=\frac{m^4-4m^3+6m^2-4}{\overline{m^2-2}|^2}$, and m greater than 2: but, ſince $m^4-4m^3+6m^2-4$ is not a ſquare number, feign it $=\overline{r-1-m^2+2m}|^2$, and m will be $=\frac{2r-2+\sqrt{\overline{2r^3+2r+4}}}{2r}$, and r greater than 3; but $2r^3+2r+4$ not proving a ſquare number, ſuppoſe it $=\overline{2+2vr}|^2$, and r will be $=vv+\sqrt{\overline{v^4+4v-1}}$, and v greater than 1; let $4v-1=z$, and v^4+4v-1 will be $=\frac{z^4+4z^3+6z^2+260z+1}{256}$, which being ſuppoſed $=\overline{\frac{1+130z-zz}{16}}|^2$ we ſhall get $z=\frac{4223}{66}$, and thence $\frac{18465217}{20590417}$, $\frac{18325825}{20590417}$, and $\frac{2264592}{20590417}$ for the ſides of the triangle required.

Anſwer to the ſame by the Rev. Mr. Wildbore,
(*the Propoſer*).

Let x and $1-x=$ the legs of the triangle; then will $\sqrt{\overline{1-2x+2xx}}=$ the hypothenuſe, and $x-xx=$ the double area, which ſubtracted from theſe three values of the ſides, leaves xx, $1-2x+xx$, and $\sqrt{\overline{1-2x+2xx}}-x+xx$ for the three ſquare numbers required, the two firſt being neceſſarily ſquares; and in order to make the laſt a ſquare, put $v.\overline{1-x}+x=\sqrt{\overline{1-2x+2xx}}$, and the ſaid ſquare will become $\overline{1+4v^3-v^4}\times\overline{1+2v-vv}|^{-2}$; therefore, if $1+4v^3-v^4$ be a ſquare number leſs than unity

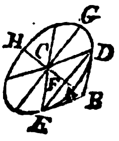
th,

the conditions of the queſtion will be anſwered. Make $v = ba^{-1}$, then muſt $a^4 + 4ab^3 - b^4 =$ a ſquare, or, putting $a = b + c$, $c^4 + 4bc^3 + 6bbcc + 8b^3c + 4b^4 =$ a ſquare; and in order to take off the two firſt terms and the laſt by tranſpoſition, make it = a ſquare whoſe ſide is $cc + 2bc - 2bb$, and we find by reduction, $3c = -8b$, or in the leaſt integers, $c = 8$ and $b = -3$: but b muſt not be a negative quantity; ∵ let $b = d - 3$ and $a = b + c = d + 5$, and ſubſtituting theſe values for a and b we obtain $4d^4 + 16d^3 + 24d^2 + 1040d + 4 =$ a ſquare; ſuppoſe its ſide $= 2dd - 260d - 2$ in order to deſtroy the firſt and two laſt terms by tranſpoſition, and we find $d = \frac{4223}{66}$, $b = \frac{4025}{66}$, $a = \frac{4553}{66}$, $v = \frac{4025}{4553}$, and the ſides of the triangle required = three fractions whoſe numerators are 2264592, 18325825, and 18465217, and com. denom. 20590417; which is the anſwer (though ſomewhat differently deduced) given by Ozanam at pa. 604 of his Nouveaux Elemens d'Algebre (which has been handed to me ſince I ſent the queſtion laſt year); and this being the only anſwer that the proceſs thus far admits of, has been ſaid to be the only one that could be given to the queſt.: But by artifices ſimilar to the preceding, v will next be found $= \frac{491050}{555466}$, $\frac{75846408}{1232216833}$, &c. and other values of the ſides of the triangle correſponding equal to three fractions whoſe numerators are 842654683 2, 68190394825, 6870907245, and com. denom. 76616941657, &c. &c.

Solutions to this queſtion have likewiſe been received from Meſſ. *G. Coughron*, *J. Dalby*, and *C. Hutton*.

XIII. QUESTION 639 *anſwered by the Rev. Mr.* Crakelt.

DEMONSTRATION. Produce any ſemi-diam. CA of the annexed ellipſis, till $CB : CA ::$ the diagonal of a ſquare : its ſide, and having drawn the tangents BD, BE, join the points C, D; D, E, and E, C; then, ſince by conics, DE will be biſected by CB in F, by the known prop. of tangents, $CF : CA :: CA : CB ::$ (by Euc. 1. 6.) $CA \times CB : CB^2 = 2CA^2$ (per queſt.) $:: CB : 2CA$, and conſequently $CB = 2CF$, or $CF = FB$; whence the △s CFE, BFD being (by Euc. 4. 1.) equal in all reſpects, CE will

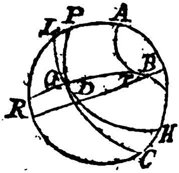


(by Euc. 27. 1.) be ∥ BD and CD ∥ BE, and thence CE and CD conjugate ſemi-diameters, which is the firſt thing to be proved.——Next, draw CG ∥ FD; then, by the common prop. of the ellipſe, $FD^2 : CG^2 :: CA^2 - CF^2 = CB \times CF - CF^2 = CF^2 : CA^2 = CF \times CB :: CF : CB :: CF \times CB = CA^2 : CB^2$, or $FD : CG$ ($:: 2FD : 2CG$) $:: CA : CB$. *Q.E.D.*

The Rev. Mr. *Wildbore* obſerves by way of corollary to his demonſtration (which is both very conciſe and curious) that from pa. 201 of Simp. Geom. beſt edit. it appears that $ADHE$ will be the greateſt ellipſis that can be inſcribed in the rhomboides formed by the interſections of the tangents BD, BE produced, and two others drawn parallel to them, reſpectively.

This property is alſo curiouſly demonſtrated by Meſſ. *G. Cetti*, *S. Clark*, *G. Coughron*, *J. Dalby*, *C. Hutton*, *Plus Minus* (the propoſer), *T. Robinſon*, and *Alex. Rowe*.

XIV. QUEST. 640 *anſwered by the Rev. Mr.* Wildbore.



PROJECTION. Deſcribe $ACRP$ the primitive, ATB the the given leſſer circ. POC the given great one, and the right circ. BDR touching ATB, ſuppoſe in B: From B ſet off BD = the given intercepted arc, and through D deſcribe OD ∥ to ATB, cutting the given circ. POC in O; then, through O deſcribe the great circ. OT, touching ATB in T, and it will be that required;—becauſe the two arches OT, BD, touching the leſſer circ. ATB, and being bounded by the ſame parallel OD, are equal.

Meſſ. *J. Chipchaſe*, *Geo. Coughron*, *J. Dalby*, *C. Hutton*, and *W. Wales* (the propoſer) have likewiſe favoured us with curious projections of this problem.

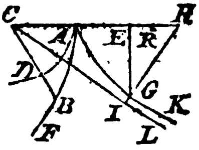
XV. QUESTION 641 *anſwered by the Rev. Mr.* Crakelt.

From the given equation ($x = \frac{N^{2z} + 1}{2N^z}$) by multiplying, compleating the ſquare, and taking the hyp. log. is had

= h. l. of $\overline{x+\sqrt{xx-1}}$. But 1 (= CD, ſee the laſt ig.) : x (CB) :: $\dot{x} \div \sqrt{xx-1}$ (the angular velocity dy at D) : $x\dot{x} \div \sqrt{xx-1}$ = the angular velocity ſame body at B; whence the fluxion of the curve's will be $= \dot{x} \times \sqrt{2xx-1} \div \sqrt{xx-1}$, and the gth itſelf = the arch of that equilat. hyperbola, hath two for each of its diameters and $x-1$ for iſſa = (when $x = 1000$) to 1414·2132088, &c.

nſwer to the ſame by the Rev. Mr. **Wildbore.**

CRIPTION. With the center C and vertex A deſcribe uilat. hyperbola AGK; draw ymp. CE and ord. EG. Take EG, AR = the hyp. log. of nd the circ. arc $AD = AR$; drawing a right line from C gh D, take upon it $CB = CE$, e point B will be in the ſpiral; ius may points be determined n *ad libitum.*



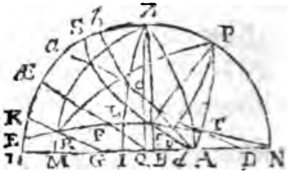
MONSTRATION. By conics, $CE = \frac{CH^2 + CA^2\,(1)}{2CH}$, $= CE + \sqrt{CE^2 - 1}$, and by conſtruc. $AR = AD$ = yp. log. of CH; ∵ $CH = N$ to the AD power, and quently $CE = CB = \frac{N^{2z}+1}{2N^z}$, as per queſtion.—— over, the flux. of the ſpiral AB is well known to be $\overline{\dot{CE}^2 + CE^2 . \dot{AD}^2}$, and $\dot{AG} = \sqrt{\dot{CE}^2 + \dot{EG}^2}$; viſe, $EG^2 = CE^2 - 1$, $EG . \dot{EG} = CE . \dot{CE}$, $\frac{CE . \dot{CE}}{EG}$) $+ \dot{CE} = \dot{CH}$ (becauſe $EG = EH$), $EG . \dot{CH} =$ $\dot{CE}$, $\frac{\dot{CH}}{CH} = \dot{AR} = \dot{AD} = \frac{\dot{CE}}{EG} = \frac{\dot{EG}}{CE}$; ∵ $CE^2 . \dot{AD}^2$ G^2, and $\dot{AB} = \dot{AG}$, and becauſe they both begin ther, $AB = AG$.

MPUTATION. Becauſe when $CE = 1000$, both it and are very great in compariſon of CA, the common ſeries approximations for AG are uſeleſs; but here the ſeries at p. 311 of Simp. Flux. may be applied to very great advantage: For, the diff. betwixt the part of the aſymp. beyond *I* and that of the curve beyond *G* being exceedingly ſmall, or quite inconſiderable in compariſon of the diff. betwixt *CI* and *AG*, $CI - AG =$ the ſum of that ſeries very nearly $= \cdot 32111$; $\because$ this being ſubtracted from $CI = CH \times \frac{1}{\sqrt{2}}$ (*HGI* being $\perp CL$) gives $AG = 1413 \cdot 8921 = AB$, the length of the curve required.

COROLLARY. The hyp. log. of *CH* (1999·9995) being $= 7 \cdot 6009024 = AD$, this divided by the whole circumference of the circle, ſhews that the body will have made 1·209725 revolutions round the center when its diſtance therefrom is 1000.

Meſſ. *T. Allen* (the propoſer), *G. Cetii*, *Hen. Clark*, *G. Coughron*, *J. Hellings*, and *Cha. Hutton* have likewiſe anſwered this queſtion according to the firſt of the preceding methods, very nearly.

The PRIZE QUESTION *anſwered by Mr.* Wm. Wales.



Let *HZPN* be the meridian, *HQN* the horizon, *ÆQ* the equator, *Z* the zenith, and *P* the pole. Make *QF* = the meaſure of the given interval, and, having deſcribed the circ. of perpetual apparition, *Nob*, deſcribe, through *F*, the great circle *EFD* to touch it. Then by prop. 13. b. 2. of Theodoſius's Spherics, the arcs of all circles ∥ *Nob*, or *ÆQ*, intercepted between the circles *HQN*, *EFD* are ſimilar and deſcribed in the ſame time with *QF*. Suppoſe, now, the circle *KRG* to be the parallel of that ſtar whoſe change in azim. *GM*, is a max. and *SCA* the parallel of that ſtar whoſe change in azim. *AB* is a min. in the given interval of time, *ZRM* and *ZCB* being verticals paſſing through *R* and *C*, the places of the two ſtars when ſituated in the circle *EFD*, and *ZLI* the vertical whoſe pole is *D*. By the above prop. of Theod. $QG = FR$; $\because$ *MG*, the change of azim. in the given time, is the diff. between *QM* and *FR*. and ſo $QM - FR$ is to be a max.; but when $QM - FR$ is a max. $IM - LR$ is a max. becauſe *QI* and *FL* are conſtant quantities. Now *IM* is the meaſure of the $\angle RZL$; therefore when the change in azim. is a max. the diff. between the $\angle RZL$ and its oppoſite ſide, RL, is a max. and conſequently its ſine is ſo too: But the $\angle L$ is right, and the ſide ZL given; alſo, by ſpherics, rad. : ſin. ZL :: tang. Z : tang. LR, and by compoſition, diviſion, and equality of ratios, rad. + ſin. ZL : rad. − ſin. ZL :: tang. Z + tang. LR : tang. Z − tang. LR :: ſin. $\overline{Z+LR}$: ſin. $\overline{Z-LR}$ (by prop. 4. p. 58. Simp. Trig.). Now, the two firſt terms of this analogy being conſtant, the laſt will manifeſtly be a max. when the third, or ſin. $\overline{Z+LR}$, is ſo, *i.e.* when $Z+LR=90°$; ∵ the ſine of their diff. $\overline{Z-LR}$, when a max. $=\frac{\text{rad.}-\text{ſin.}\,ZL}{\text{rad.}+\text{ſin.}\,ZL}$; and hence, having the ſum and diff. of Z and LR, thoſe arcs themſelves are known, and of courſe every thing elſe in the $\triangle RZL$; and, as the $\angle LZP$ is known by the conſtruc. the $\angle RZP$ will be known, and we ſhall have in the triangle RZP, the ſides RZ and ZP with the included angle, to find RP, the parallel's diſt. from the elevated pole, = (in the caſe given) 109° 27¾′, or its declin. 19° 27¼′ ſouth; and this parallel the ſun occupies Nov. 18th and Jan. 22d.

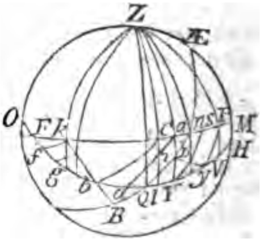
Again, becauſe $FC=\mathcal{Q}A$, the change in azim. $BA = FC - \mathcal{Q}B$, which is therefore to be a min.; or $LC-IB$ is to be a min.; or, which amounts to the ſame thing, $IB - LC$ muſt be a max. and hence the point C will be determined exactly in the ſame manner that the point R was, and we ſhall have in the $\triangle CZP$, the ſides CZ and ZP with the included angle to find CP, = 38° 55⅙′, and ſo the declin. required is 31° 4⅚′, north.

Scholium I. The declin. of the parallel SCA will, it is plain, be of the ſame name with the latitude of the place until the circ. EFD comes into the poſition aod, that is, to touch the circ. of perpetual apparition in the prime vertical, $Z\mathcal{Q}$: For then, the interſecants Nd, do being equal, $od+d\mathcal{Q}=90°$, which has been proved to be the caſe when the change in azim. is a max. or min. and conſequently the circle Nob is the parallel required, and the ſtar riſes due north or ſouth; after which the declin. of the parallel SCA becomes of a different name from the lat. of the place till the time given is 18h. when it becomes again of the ſame name therewith.

II. In like manner, the declin. of the parallel KRG will be of a diff. name until the given time be 6h. after which it becomes of the ſame name with the lat. until the circle EFG touches the circle of perpetual apparition in the weſtern prime vertical, and then it will be again of a diff. name from the latitude. And all theſe determ. hold equally true of a ſtar moving from the horizon at ſetting, only the declin. here ſaid to be of the ſame name with the lat. will there be of a contrary one, and *vice verſa*; for it is only conſidering them as ſtars riſing to the Antipodes.

III. In all theſe concluſions I have ſuppoſed the given lat. was not leſs than 45°; for if it be, the given time may be ſuch as to cauſe the circ. EFD to cut the merid. between the zenith and elevated pole: in which caſe the queſtion may admit of more anſwers than one; but they cannot always be exhibited by this, or, I believe, any other general method. It may, however, be eaſily done by others, adapted to the particular caſes where this fails; but your limits are much too narrow for ſo copious a ſubject.——I muſt, before I take leave, return thanks to the ingenious propoſer for bringing this queſtion again under conſideration, as I had paſſed it over in too haſty a manner before; the reaſons for which you are well acquainted with.

The ſame anſwered by the Rev. Mr. Wildbore, (*the Propoſer*).



The annexed ſcheme is an oblique repreſentation of the concavity of the eaſtern hemiſphere, bounded by the merid. $OZÆH$ of the place, HO the horizon, $ÆQ$ the equinoctial, and the given hour arch is [illegible]t off from Q to n; at n the $\angle anT$ is made $= TQ\Upsilon$, and thus is the great circ. FnM deſcribed. Then, by Gregory's Aſtron. b. 2. prop. 40. the arches of the equinoctial and its parallels intercepted between this circle and horizon are ſimilar, and $aT = T\Upsilon$, $an = Q\Upsilon$, $QT = Tn$, $an = dQ$, $ac = db$, $ak = df$, &c. Now, ſuppoſe the circle $FanM$ to be generated by the motion of a point both ways from a; then, when the ſun or ſtar riſes at d, its alteration of azim. in the time Qn will be $d\Upsilon$, and when at b it will be bl, and becauſe the $\angle$s at Υ, l, and a are right, Υl will be $\sqsubset ac$; but ac is $= bd$: $\because$ bl is $\sqsupset d\Upsilon$, or the alteration of azim. when the phœnom. riſes at b, will be $\sqsupset$ when it riſes at d, the diff. being $l\Upsilon - ca$: But in the right-angled iſoſceles ſpheric $\triangle F\Upsilon a$, $l\Upsilon$ is the decrem. of $F\Upsilon$, and ca of Fa; and if k be taken for any other poſition of the moving point, if in the right-angled ſpherical ∠ Fgk, the decrem. of Fk is $\sqsupset$ that of Fg, the ſaid alteration of azim. may ſtill decreaſe by taking the point k nearer F, for the ſame reaſon as at a: But if the decrem. of Fk be equal to that of Fg, it can decreaſe no further; for at the next point, the decrem. of Fk being greateſt, the ſaid quantity will be upon the increaſe; therefore, when the required increaſe of azim. is a min. the decrem. of Fk is = that of Fg.——And in the very ſame manner, it appears that when the point moves the other way from a (M being placed at the other interſection of the circle FaM with the horizon) the alteration of azim. as the declin. varies, will increaſe till the decrements of PM and VM are equal, and then it will be a max.——And hence, per ſpherics and pa. 280 of Simp. Flux. is eaſily found $\angle kFg = 18^\circ\ 32'$, $kg = 13^\circ\ 10'$, $\angle Fkg = 76^\circ\ 50'$, $Fg = kn = 44^\circ\ 14'$, $Fk = g\Upsilon = 45^\circ\ 46'$, $an = 11^\circ\ 51'$, $kn = 56^\circ\ 5'$, $kB = 31^\circ\ 5'$ the ſtar's declin N. required; $gd = 22^\circ\ 4'$, and thence $fg = 22^\circ\ 10'$, the min. azim. in 2 hours from riſing correſponding; $sP = 20^\circ\ 32'$, $yV = 25^\circ\ 14'$ the max. increaſe (or alteration) in 2 hours, and $Dy = 19^\circ\ 27\frac{2}{3}'$, S. the correſponding declination anſwering to Nov. 19 or Jan. 22.

Anſwer to the ſame by Mr. Cha. Hutton.

Let P be the north pole (ſee fig. 1ſt preceding), Z the zenith, HN the horizon, AC a great circle paſſing through the object at its riſing and 2 hours after; deſcribe the perpendicular circ. PU, and the circles PA, PC, ZA, and ZCB. Put a = ſine of 15° or ∠ APU, b = its tang. c = ſine of lat. or coſ. of ZP, d = its coſ. or ſine of ZP, x = s. declin. or coſ. of AP or PC, and z = coſ. of declin. or s. of AP. Then, in the right-angled ſpherical △ PUA, $1 : a :: z : az$ = s. AU or $\frac{1}{2}AC$, and hence $2az\sqrt{1-aazz} \div \overline{1-2aazz}$ = tang. AC; alſo, $1 : b :: x : bx$ = co-tang. PAU; and hence $\overline{1+bbxx}|^{-\frac{1}{2}}$ = ſine and $bx \times \overline{1+bbxx}|^{-\frac{1}{2}}$ = coſ. PAU. In the quadrantal △ AZP, $z : 1 :: c : \frac{c}{z}$ = coſine and $\frac{1}{z}\sqrt{zz-cc}$ = ſine PAZ: Whence $\overline{c - bx\sqrt{zz-cc}} \div z\sqrt{1+bbxx}$ = s. ZAC or coſ. CAB.——And in the right-angled ſpherical △ ABG, 1 : tang. AC :: coſ. CAB : tang. AB = $2a\sqrt{1-aazz} \times \overline{c-bx\sqrt{zz-cc}} \div \overline{1-2aazz} \times \sqrt{1+bbxx}$ = (ſince $\frac{a}{b}$ =

$= \sqrt{1-aa}$, and putting $e = \frac{1-2aa}{2aa}) \frac{1}{b} \times \frac{c-bx\sqrt{dd-xx}}{e+xx}$ $=$ a max. or min.——This in flux. &c. gives $2ex\sqrt{dd-xx}$ $= b\times\overline{dd+2e.xx-dde}$, or $x^4 - \cdot3780667xx = \cdot0296788$; and hence $x = \sqrt{\cdot2668459 \text{ or } \cdot1112207} = \cdot5165713$ or $\cdot3334977$, anſwering to 31° 6′ 15″ and 19° 28′ 52″, the two declinations required.

This queſtion is alſo ingeniouſly anſwered by Meſſ. *J. Chipchaſe*, *Hen. Clark*, *G. Coughron*, *Sam. A Day*, and *J. Haycook*; but the prize of 12 Diaries for the ſolution thereof is fallen to the lot of Mr. *Wm. Wales*, and that of 8 to Mr. *Geo. Coughron*.

The Eclipſes calculated for 1772.

This year will produce four eclipſes of the ſun, and two of the moon, as under:

1ſt eclipſe, of the ☉, will be on Friday the 3d day of April, about 5 in the morning; therefore inviſible in England; but will be ſeen in the northern parts of Aſia.

2d will be a total eclipſe of the ☽, on Friday the 17th day of April: Beginning 2 h. 16 m. 18 s. Middle 4 h. 3 m. 1 s. End 5 h. 49 m. 44 s. Dig. 15 deg. 20 m. apparent time at London, afternoon.

Note, This eclipſe will be over before the ☽ riſes at London.

3d will be an eclipſe of the ☉, on Saturday the 2d day of May; inviſible at London; but viſible in North America, at near our 10 at night.

4th will be an eclipſe of the ☉, on Sunday the 27th day of September; inviſible at London; but viſible near the ſouth pole, about our 1 o'clock in the morning.

5th is a viſible eclipſe of the ☽, on Sunday the 11th day of October: Beginning 3 h. 28 m. 40 s. Middle 5 h. 26 m. 2 s. End 7 h. 23 m. 24 s. Digits eclipſed 13 deg. 52 m. apparent time at London.

Note, The moon will riſe totally eclipſed at 5 h. 24 m. and continue ſo till about 16 m. after 6 o'clock; when the vertical limb of the ☽ will begin to emerge out of the ſhadow.

6th is a ſmall viſible eclipſe of the ☉, on Monday the 26th day of October: Begin. 8 h. 22 m. Middle 8 h. 30 m. 14 s. End 8 h. 40 m. Dig. 0 deg. 6½ m. apparent time, in the morning, at London.——By the late ingenious Gael Morris's Tables.

ISAAC TARRATT.

New Queſtions.

I. QUESTION 622, *by Mr.* John Shadgeſt.

In friendſhip two ſiſters together reſide,
With virtue replete; each a ſtranger to pride:
Maria for beauty with Venus may vie,
And Cloe for wiſdom Minerva defy:
Maria is prudent in ev'ry degree,
Whilſt Cloe is court'ous, good-natur'd, and free.
From what's under-written * their ages I aſk:
Reſolve it, dear ladies, nor think't a hard taſk.

* Given $\left\{\begin{array}{l} x^2+xy+y^2 = 1087, \\ x^4+x^2y^2+y^4 = 45777295; \end{array}\right\}$ to find the value of x the age of Maria, and that of y the age of Cloe.

II. QUESTION 643, *by Mr.* Wm. Spicer.

A piece of timber in the form of the fruſtum of a cone, whoſe girths at the ends are 10 and 3 feet, was meaſured by the cuſtomary way of taking ¼ of the girth in the middle for the ſide of the ſquare: Now, if the ſaid piece of timber be cut into two pieces at the diſtance of 14 feet from the ſmall end, and both the pieces meaſured according to the ſame method, they will amount to 7 ſolid feet more than the whole piece: Required the length of the ſaid piece of timber?

III. QUESTION 644, *by Mr* Paul Sharp.

Given the ſum of the tranſverſe and conjugate diameters of an ellipſis = 100, and the difference of its area and that of its greateſt inſcribed parabola = 845·73: Required the area of the ellipſis, and the axis of the ſaid parabola?

IV. QUESTION 645, *by Mr.* Steph. Hodges, *at the Right Hon. the Earl* Spencer's, *at* Althorp.

Having given a common parabola; it is required to draw a tangent thereto ſo, that a right line drawn from the point of contact to a given point within the ſame, may be the ſhorteſt poſſible?

V. QUESTION 646, *by the Rev. Mr.* Wildbore.

The difference betwixt the perpendicular and baſe, the difference of the ſegments of the baſe, and the difference of the angles at the baſe being given, to conſtruct the triangle?

VI. QUESTION 647, *by Mr.* T. Moſs.

From the given point E in the ſide DR produced, of the given rectangle $BDRF$, to draw a right line EA cutting the ſides RF, DB, thereof in a and C, and the ſide FB produced in A ſo, that the trapezium $CDRa$ may be to the $\triangle ABC$ ſo formed, in the given ratio of m to n.

VII. QUESTION 648, *by Mr.* Iſaac Dalby.

In the right-angled plane triangle, there is given one of the legs, alſo a line drawn parallel thereto and terminated by the other leg and hypothenuſe; to determine geometrically, the triangle ſo, that the rectangle under the hypothenuſe and a line drawn from the acute angle next the given leg, to the point where the ſaid parallel line meets that other leg, may be of a given magnitude.

VIII. QUESTION 649, *by Mr.* S. Ogle.

Two lines AB, AC being given in poſition, and a point P in one of them (AB); it is required to draw from thence a right line PH meeting the other given line AC in H ſo, that if another right line HI be drawn to meet AB and make a given angle therewith, the perimeter of the triangle PHI ſo formed may be the leaſt poſſible.

IX. QUESTION 650, *by Mr.* J. Chipchaſe.

On a certain day laſt ſummer, at a quarter paſt midnight, when the ſun was juſt riſing, a perſon ſet forward on a journey along a parallel of latitude, which he ended at ſunſet, having by his watch performed it in 24 hours; at noon the greateſt ſhadow of his walking-ſtick (which is 3¼ feet long) exceeded its ſhadow when placed perpendicular to the horizon by 1·522267 feet; from whence is required the latitude he was in, the day of performing the journey, and diſtance travelled?

. QUESTION 651, *by Mr.* W. Chartreux.

latitude of the place, and the poſition of two hour-
, with reſpect to the meridian, being given; it is re-
to determine what the declination of a ſtar muſt be,
in paſſing over the interval contained between thoſe
ircles, the change in altitude may be the greateſt
:.——This queſtion has been propoſed before, but
anſwered.

XI. QUESTION 652, *by Mr.* Cha. Hutton.

'ater runs through a pipe 1¼ inch diameter with a
at velocity of 6 feet per ſec. into an empty conical
having a hole of 1 inch diameter in its bottom: Re-
. the time when the ſurface of the water in the veſſel
e juſt 1 foot above the bottom of it; alſo, what will
greateſt height to which the water will riſe, and the
n which it will riſe to the ſaid greateſt height, the
ameter of the bottom of the veſſel being 3, the dia-
of the top 5, and the altitude 6 feet?

XII. QUESTION 653, *by* Aſtronomicus.

inveſtigate or demonſtrate the nature of the curve
a fixed ſtar by means of the abberration would appear
ſcribe, if the earth, inſtead of revolving in an ellipſis,
o move in a parabolic or an hyperbolic orbit.

te, This curious queſtion has been propoſed before,
s it was thought to have been neither inveſtigated nor
demonſtrated, we have been requeſted to re-propoſe it.

II. QUESTION 655, *by the Rev. Mr.* Wildbore.

ſtracting from refraction, at what time on the 21ſt of
at Salton in Norway, lat. 67° N. will the velocity of
adow of the ſummit of an erect object be the greateſt
le on an horizontal plane?

The PRIZE QUESTION, *by* Peter Puzzlem.

ppoſe a body to be faſtened to one end of a ſtring, the
end whereof is faſtened to a fixed point *c*, and ſuppoſe
be impelled from a certain given point *b* with a given
city in a direction at right angles to *bc*: To find the
and direction of the force which muſt continually act on
body ſo, that its velocity ſhall vary according to any
ſed law, and itſelf be always found in a circle (whoſe
er is *c*) revolving with any propoſed velocity about the
n fixed diameter *bcd*.

1773.

Queſtions anſwered.

I. QUESTION 642 *anſwered.*

PUT $xx + yy = v$, $xy = w$, $1087 = a$, and $45777295 = b$, and then the given equations become $v + w = a$ and $vv - 2w^2 + w^3 = b$, from the former of which $v = a - w$, and that value ſubſtituted in the latter of them, &c. gives $w^3 - w^2 - 2aw = b - aa$; whence $w = 357$, and conſequently from the firſt given equation, $xx + 2xy + yy = a + 357$ and $xx - 2xy + yy = a - 3 \times 357$; and thence x and $y = \frac{1}{2}\sqrt{a + 357} \pm \frac{1}{2}\sqrt{a - 3 \times 357} = 21$ and 17, or 17 and 21, the two ages required.——According to this method the anſwer is given by Meſſ. *Joſeph Cowley*, *William Dent*, *Pamphagus*, *William Reynolds*, *Alex. Rowe*, and *Michael Taylor*: but as x and y are concerned exactly alike in both the given equations, Meſſ. *John Aſpland* and *William Smart* put $s = \frac{1}{2}$ the ſum and $d = \frac{1}{2}$ the diff. of them, and then find, by an eaſy proceſs, $64d^6 - 52128d^4 + 4074476d^2 - 55467952 = 0$; whence $d = 2$, and the ages required come out the ſame as above.

Mr. *Geo. Coughron*, by tranſpoſing xy in the 1ſt equation, and ſquaring and taking it from the 2d, deduces the cubic equation $\overline{xy}|^3 - \overline{xy}|^2 - 2axy = b - aa$; from which $xy = 357$, and $x + y$ $(= \sqrt{xx + 2xy + yy}) = \sqrt{a + xy} = 38$; and thence x and y are found the ſame as before.

Meſſ. *Tho. Adcock*, *J. Bartlett*, *C. Burton*, *J. Dalby*, *R. Dening*, *Mark Elſtob*, *Edw. Fidler*, *Wm. King*, *Bewley Mellor*, *J. Parker*, *Wm. Richardſon*, *Abr. Robertſon*, *Tho. Robinſon*, *J. Shadgett* (the propoſer), *Tho. Smith*, *Wm. Spicer*, *Elean. Suggett*, and *Wm. Wilkin* have likewiſe anſwered it; and Mr. *Wm. Wells*'s anſwer is as follows, viz.

Friend Shadgett, I thought fit to try
If I could find your x and y:
Your x then wants three of a ſcore,
And y, I find, is juſt one more.*

* Viz. 17 and 21.

ESTION 643 *anſwered by Meſſ.* J. Aſpland, Mark
b, J. Turner, Wm. Smart, *and* Wm. Wilkin.

$=$ the required length; then by the nature of the
$: 7\ (= 10 - 3) :: 14 : \frac{7 \times 14}{x} = \frac{98}{x}$; and $\frac{1}{2} \times \overline{\frac{98}{x} + 6}$
$\frac{6x}{x}$ and $\frac{1}{2} \times \overline{\frac{98}{x} + 13} = \frac{98 + 13x}{2x}$ are the girths of
o places reſpectively; whence $\overline{\frac{98 + 6x}{8x}}\Big|^2 \times 14 +$
$\overline{\frac{3x}{}}^2 \times \overline{x - 14} = \overline{\frac{13}{8}}\Big|^2 \times x + 7$, by the queſtion;
d, $238x = 9604$, and $x = 40\frac{6}{17}$ feet, the length re-
.——Mr. *G. Coughron*, after his ſolution, which is
ngenious, refers to Mr. Hutton's Menſuration, where-
y be ſeen how to cut it ſo, as that the content of the
arts may amount to the greateſt quantity poſſible.

alſo ingeniouſly anſwered by Meſſ. *J. Bartlett*, *J.*
, the Rev. Mr. *Crakelt*, *J. Dalby*, *R. Dening*, *W.*
Edw. Fidler, *W. King*, *Bewley Mellor*, *J. Milbourn*,
ynolds, *Abr. Robertſon*, *Tho. Robinſon*, *Alex. Rowe*,
icer (the propoſer), and *Mich. Taylor*.

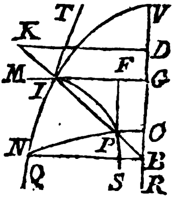
QUESTION 644 *anſwered by Mr.* Iſaac Dalby.

eaſily ſhewn that the axis of the greateſt parab. that
e inſcribed in any ellipſis is ¾ of its conj. diam. the
 being at the extremity thereof. Put $100 = 2a$,
$= b$, $\cdot 7854 = v$, and let $a + x$ and $a - x$ repreſent
anſverſe and conjugate diameters of the ellipſe; then
$\overline{a - 3x}$ is $=$ the axis of the parab. and $\frac{1}{4} \times \overline{a - x}$ the
between the center of the ellipſe and the parabola's
eſt ord. and by the prop. of the ellipſe, $a - x : a + x$
$\overline{\frac{1}{2} \times \overline{a - x}}\Big|^2 - \frac{1}{4} \times \overline{a - x}^2 : \overline{a + x}\sqrt{\tfrac{3}{16}} =$ the para-
s greateſt ord. and $\overline{aa - xx}\sqrt{\tfrac{3}{16}} =$ its area; whence
ueſt. $\overline{aa - xx}\sqrt{\tfrac{3}{16}} = \overline{aa - xx} \times v - b$; and by re-
on, $xx = aa - b \div \overline{v - \sqrt{\tfrac{3}{16}}} = 100$; conſequently x
, and the diameters of the ellipſis are 60 and 40, and
xis of the parab. 30.

COROLLARY. The area of every ellipſis is to that of its greateſt-inſcribed parab. as ·7854 is to $\sqrt{\frac{3}{16}}$, or as 3·1416 to $\sqrt{3}$.

The Rev. Mr. *Crakelt*'s ſolution is almoſt exactly the ſame with this; and it is likewiſe anſwered by Meſſ. *J. Aſpland*, *J. Bartlett*, *G. Goughron*, *J. Cowley*, *Wm. Dent*, *Wm. Reynolds*, *Tho. Robinſon*, *Alex. Rowe*, *Wm. Smart*, *P. Sharp* (the propoſer), *Wm. Spicer*, *Mich. Taylor*, *J. Turner*, and *Wm. Wilkin*.

IV. QUESTION 645 *anſwered by Mr.* Geo. Coughron.



Let *VQR* be the given parabola, *P* the given point, *I* the required point of contact, and let *IG* and *PC* be ⊥ to *VR*, and *PF* ∥ to the ſame. Then, ſince *PI* is to be a minimum, it manifeſtly muſt be ⊥ to the curve at *I*, and therefore, per ſim. △s, $BG : GI :: PF\ (VC - VG) : FI = GI - CP$; whence $BG \times \overline{GI - PC} = GI \times \overline{VC - VG}$; which equat. is general let the nature of the curve *VIQ* be what it will, and from which and the relation between *VG* and *GI*, the lines *VG*, *GI*, &c. may be determined; for both *VC* and *CP* are given, and *BG* is determinable from the equation of the curve.

When the curve is a parab. as given in the queſt. let $VC = a$, $CP = b$, the param. $= p$, $VG = x$, and $GI = y$; then, px being $= yy$ and $\frac{1}{2}p = BG$, the above equation becomes $\frac{1}{2}p \times \overline{y - b} = y \times \overline{a - x}$, or, by exterminating x, $y^3 + \overline{\frac{1}{2}p - a} \times py = \frac{1}{2}bpp$; from whence y may be determined.

Anſwer to the ſame by Mr. Burrow.

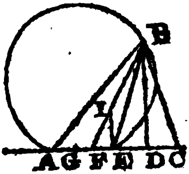
From *P*, the given point, draw *PC* ⊥ to the axis *VR*, and take *CD* therein = ½ the parameter, and draw *DK* ∥ to *CP*: with *DK* and *DB* as aſymptotes, deſcribe an hyperb. *MS* through *P*, cutting the parab. in *I*; draw the right line *PI*, and it will be the min. required.

DEMONSTRA. Draw *IG* ∥ to *PC*, and continue *IP* to meet *VR* and *DK* in *B* and *K*: then $IK = BP$ (by prop. 72. b. 2. of Emerſon's Conics), whence $DG = CB$; Conſequently $BG = CD =$ the ſemi-parameter, and therefore (by prop. 11. b. 3. of Emer. Conics aforeſaid) *BI* is the ſhorteſt line that can be drawn from B.——Alſo, PI is the ſhorteſt that can be drawn from P to the curve: for if not, let PN be it, and join B, N; then BN is $\sqsubset BI$ (by what has juſt been ſhewn), and $BP + PN$ ($\sqsubset$ than BN, by 20 Euc. 1.) is therefore much greater than BI; take away BP which is common, and there remains $PN \sqsubset PI$; but it was ſuppoſed $\sqsupset PI$; therefore PI is the line required; perpendicular to which draw IT, the tangent required.

Ingenious ſolutions to this queſtion have likewiſe been received from Meſſ. *J. Aſpland*, the Rev. Mr. *Crakelt*, *J. Dalby*, *S. Hodges* (the propoſer), *Tho. Robinſon*, *Alex. Rowe*, *Wm. Smart*, *Michael Taylor*, *J. Turner*, and *Wm. Wilkin*.

V. QUESTION 646 *anſwered by Mr.* J. Chipchaſe.

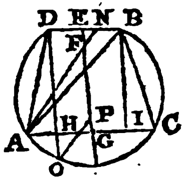
CONSTRUCTION. On AE the given diff. of the ſegments of the baſe, deſcribe the ſegment of a circle capable of containing an angle = the given diff. of the angles at the baſe, and take AG = the given diff. of the baſe and perpendicular, and biſecting GE in F, erect the perpendicular $EI = EG$, and draw the right line FIB cutting the circumference in B; from whence upon AE produced, let fall the perpendic. BD, and having taken $DC = DE$, let the points A, B and B, C be joined, and ABC will be the triangle required.



DEMONSTRATION. Join the points E, B; then the angles BEC, BCE are equal, and the $\angle ABE$ is the diff. between the angles at the baſe BAC, BCA (BEC) by 32 Euc. 1. which is of the given quantity by conſtruction. Alſo, becauſe $DC = DE$ by conſtruc. AE is the diff. of the ſegments of the baſe, which by hypoth. is of the given mag. likewiſe. Laſtly, EI being $= EG = 2EF$ (by conſtruc.) DB will be $= 2DF = 2DE + 2EF = CE + EG = CG$; whence the diff. of the baſe AC and perpendic. DB is = AG, the given diff. by conſtruc.——In this ſolution the baſe is ſuppoſed to be greater than the perpendicular; but if the perpendicular be greateſt, the conſtruction will be the ſame, only AG muſt be ſet off the contrary way on the baſe produced.

In this manner exceedingly near it is alſo conſtructed by Meſſ. *Geo. Coughron*, *H. Curtis*, *J. Turner*, *W. Wales*, and the Rev. Mr. *Wildbore* (the propoſer); and Meſſ. *J. Aſpland*, *Wm. Reynolds*, *Tho. Robinſon*, *Alex. Rowe*, *Mich. Taylor*, and *Wm. Wilkin* have anſwered it algebraically.

Mr. *Iſaac Dalby*'s conſtruction is conſiderably different, and truly ingenious; and the Rev. Mr. *Crakelt*'s conſtruction is as follows, viz.

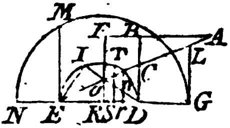


Upon DB, equal to the given diff. of the ſegments of the baſe, deſcribe the ſegment of a circle capable of containing the given diff. of the angles at the baſe; and having taken thereon and on an indefinite perpendicular biſecting it, EN and EF reſpectively equal to the half and the whole of the diff. betwixt the perpendicular and the baſe, through N and F draw a right line to meet the circular arc in A: then draw $AGC \parallel$ to BD, and join the points A, B; C, B, and ABC will be the triangle required.

DEMONSTRATION. On AC let fall the perpendiculars DH, BI, and draw DA: then ſince, by 29 Euc. 1. and 29 of 3d, $AG = GC$, and HG (by 34 Euc. 1.) $= DE = EB$ (by conſtruction) $= GI$, therefore will $AH = IC$, and $AI - IC = HI = DB$. Moreover, by ſim. triangles, $FE : EN :: FG : GA$; but by conſtruction FE is $=$ twice EN; wherefore FG will be $=$ twice $AG = AC$; and conſequently $BI - AC = EG - AC = EF$; and the diff. of the angles BCA, BAC is manifeſtly $=$ the $\angle BAD$.

VI. QUESTION 647 *anſwered by* Mr. T. Moſs (*the Propoſer*).

CONSTRUCTION. Biſect DR in r, and take $rG = Er$; alſo, in GE produced, take EN a fourth proportional to m, n and DR, and upon GN deſcribe a ſemicircle, and erect the perpendicular EM meeting its circumference in M: then in FB produced, take $BA = EM$; join E, A and the thing is done.



DEMONSTRA. Draw $GL \perp GE$ meeting AE in L: then, by the prop. of the circle and conſtruc. the rectangle $NEG = EM^2 = AB^2$, and therefore $EG : AB :: AB : EN$;

but by ſim. Δs, $EG : AB :: GL : BC$; whence, by equality, $AB : GL :: EN : BC$; but by conſtruction, $Er = rG$, $Rr = rD$, and $GL \parallel rn$; wherefore $GL = 2rn$, and conſequently the rectangles contained by AB, BC, and EN, $2rn$, and alſo their halves, are equal: whence m being to n (:: $DR : EN$ by conſtruc.) :: $DR \times rn$ (= trapez. $CDRa$) : $EN \times rn$ $(= \frac{1}{2} AB \times BC = \Delta\, ABC)$.

Mr. G. Coughron

Conſtructs it by taking BA ſo, that m may be to n :: $ED^2 - ER^2 : BA^2$; then drawing EA, the thing required will be done.——For, by ſimilarity of Δs, the $\Delta\, EDC : \Delta\, ERa :: ED^2 : ER^2$, and dividedly, the trapez. $CDRa : ED^2 - ER^2 :: \Delta\, EDC : ED^2 :: \Delta\, ABC : AB^2$, and alternately, the trapez. $CDRa : \Delta\, ABC$ $(:: ED^2 - ER^2 : AB^2)$ :: $m : n$, per conſtruction.

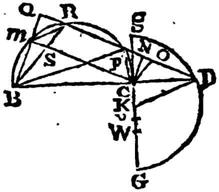
Anſwer to the ſame by the Rev. Mr. Crakelt.

CONSTRUCTION. Upon ED deſcribe a ſemicircle, and inſcribe therein $EI = ER$, and divide ED in S in the given ratio of m to n; then take BA (in FB produced) = a fourth proportional to ES, ST ($\perp$ to ED and meeting the circumference in T) and DI, and draw $ACaE$ for the line required.

DEMONSTRATION. By Euc. 6. 19, the $\Delta\, EDC : \Delta\, ERa :: ED^2 : ER^2$; $\because$ by diviſion. permutation, &c. the trapezium $CDRa : ED^2 - ER^2 :: \Delta\, EDC : ED^2 :: \Delta\, ABC : BA^2$; whence by permutation, &c. the trapez. $CDRa : \Delta\, ABC :: ED^2 - ER^2$ or $ED^2 - EI^2 =$ (Euc. 1. 47.) $DI^2 : BA^2$ :: (by conſtruc.) $ES^2 : ST^2 =$ (by prop. of the circle) $ES \times SD$ :: (Euc. 6. 1.) $ES : SD :: m : n$, by conſtruction.

We have likewiſe been favoured with ingenious and elegant conſtructions to this queſtion from Meſſ. *R. Burrow*, *J. Chipchaſe*, *J. Dalby*, *J. Turner*, and the Rev. Mr. *Wildbore*; and algebraic ſolutions from Meſſ. *J. Aſpland*, *Wm. Reynolds*, *Tho. Robinſon*, *Alex. Rowe*, *Mich. Taylor*, and *Wm. Wilkin*.

VII. QUESTION 648 *anſwered by Mr.* Iſaac Dalby.



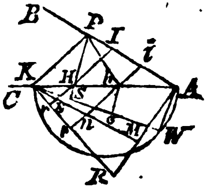
CONSTRUCTION. At right angles to the given baſe Cv take CB ſo, that their rectangle may be = the given magnitude. Upon BC let a ſemicircle be deſcribed, and produce BC ſo, that BD may be a fourth proportional to the given parallel line, the baſe Cv and BC; alſo, produce Cv to make vW a fourth proportional to BD, CD, and Cv: Biſect CW in K, and draw the line DK; with which as rad. and cent. K, deſcribe another ſemicircle meeting CW produced both ways in G and g; from D to the former ſemicirc. on BC, apply $DP = CG$, producing it to meet the circumference again in R, and on DR produced let fall the perpendic. BQ, cutting the ſaid ſemicircle in m; join B, R, and BQR will be the triangle required.

DEMONSTRATION. Join Rm, mC, CP, BP, and draw CN, $CO \parallel BQ$, BR reſpectively. By Euc. 3. 35, $gC : CD :: CD : CG$ ($= DP$, by conſtruc.), and by ſim. $\triangle$s, $DO : DC :: DC : DP$; $\because$ (5. 9) $DO = gC$; but, becauſe Kg, KG are equal, and CW is biſected in K (by conſtruc.), WG is $= Cg$; $\because OP = CW$. Moreover, the $\angle$s BmC, BQP being right ones, Cm is $\parallel PQ$; conſequently the arcs mR, CP, and their chords mR, CP are equal to each other reſpectively, and thence $QR = PN$: but, by ſim. triangles, $BD : CD :: BQ : CN :: QR\ (PN) : NO$, and by conſtruc. $BD : CD :: Cv : vW$; whence by equality, $PN : NO :: Cv : vW$, and, compounding, $OP : CW :: NO : vW$; but we have already proved that $OP = CW$; $\because$ (5. 9.) $NO = vW$, and conſequently $PN\ (= QR) = Cv$, the baſe. Again, by ſimilar triangles, $Bm : BQ$ ($:: mS : QR$) $:: BC : BD$; $\because mS$ is = the given parallel line, by conſtruction. Laſtly, by ſim. triangles, $BC : CP\ (mR) :: BR : QR$; whence $mR \times BR = BC \times QR$ (Cv), the given magnitude, by conſtruction.

This conſtruction, though leſs conciſe than ſome others, may perhaps be found acceptable to ſome of our readers, as it is derived from firſt principles only, without either tranſforming it to another prob. or calling in the aſſiſtance of any other prob. already known in order to its ſolution, &c. whereby it appears to be an original, independent problem.

Meſſ. *G. Coughron*, the Rev. Mr. *Crakelt*, *S. Ogle*, *J. Turner*, *W. Wales*, and the Rev. Mr. *Wildbore* have conſtructed this problem in a neat, elegant, and conciſe manner; and Meſſ. *J. Aſpland*, *T. Robinſon*, *Alex. Rowe*, *Mich. Taylor*, and *Wm. Wilkin* have given algebraic ſolutions to it.

VIII. QUESTION 649 *anſwered by Mr.* BURROW.



CONSTRUCTION. Draw PK making with the given line AB, the given angle APK, and deſcribing on AK a ſemicirc. take AW = the diff. between AP and PK, and apply it therein from A, and join K, W: then draw $PS \perp$ to KW, cutting AK in H, the point required; for drawing $HI \parallel$ to PK, PHI will be the triangle whoſe perimeter is a minimum.

DEMONSTRATION. Draw $AR \parallel$ to PK and $= AP$, and join K, R, and draw $KM \parallel AP$, meeting AR in M, and continue IH to meet KM in N and KR in r: then I ſay the perimeter of the $\triangle PHI$ is always $= PH + HS + IN$; for, $KH : KA :: PI : PA :: Hr : AR$; but $PA = AR$ (by conſtruc.); $\because PI = Hr$, and HS is always $= Nr$; for ſince HS and AW are each $\perp$ to KW, $AW : HS :: AK : HK :: AP : IP :: MK : NK :: MR : Nr$; but $AW = MR$, each being by conſtruc. $= AP - PK$: whence $HS = Nr$; therefore $HS + HN = PI$, and $PH + HS + IN =$ the perimeter of the $\triangle PHI$, as was affirmed. Now, the ſame holds true wherever the point H is taken betwixt A and K; but of the three lines which conſtitute the perim. IN is always the ſame, and therefore the perim. will be a min. when the ſum of the two variable lines PH and HS is ſo, and that will evidently be when they make one ſtraight line $PHS \perp$ to KW.

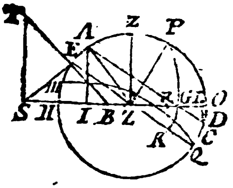
The conſtructions by Meſſ. *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Dalby*, *S. Ogle* (the propoſer), *J. Turner*, *W. Wales*, and the Rev. Mr. *Wildbore*, are likewiſe exceedingly elegant and ingenious; from which, by way of corollary and remark, they have, in a curious manner, pointed out various limitations, &c. to it, but our great want of room at preſent obliges us to omit them.

Meſſ. *J. Aſpland*, *Tho. Robinſon*, *Alex. Rowe*, *Wm. Smart*, *Mich. Taylor*, and *Wm. Wilkin* have ſolved it algebraically.

IX. QUES-

IX. QUESTION 650 *anſwered by the Rev. Mr.* Crakelt.

It will readily appear that the ſtaff will caſt the greateſt ſhad. at the given time, when the ſun's rays fall upon it in a perpendicular direction; therefore, if TSB and SAZ be ſuppoſed two right-angled triangles, having ST (the given ſtaff $= 3\frac{1}{2}$ feet) perpendic. to the horiz. SO, and $= SA$ ſo inclined to it as to make its ſhad. SZ a max. as likewiſe $TB \parallel$ to AZ, they will be ſim. and equal, and of courſe $SZ - SB = BZ = 1{\cdot}522267$ feet $= TB - SB$, and thence the $\angle TBS$ or $AZS = 47^\circ\ 0'\ 42''$ = the ſun's merid. altitude.——Now, with ZA rad. deſcribe the circumference of a circle interſecting SZ in H, and when produced in O, and having let fall $AI \perp$ on HZ, drawn $zZ \parallel$ thereto, and made $OF =$ the verſed ſine of $3^\circ\ 45''$ the given time of the ☉'s riſing after midnight, perpendicularly to HO apply to the circumf. of the ſaid circ. LD a fourth proportional to HF, FO, and AI; then draw AD, cutting HO in G, and perpendic. thereto ZP, and PO will be = the latitude required.

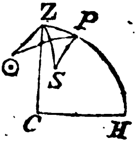


For, if PG be the repreſentation of a merid. arc paſſing through G and meeting EQ drawn through the center $Z \parallel$ to AD, in the point K, by the nature of the orthograph. project. ſim. triangles, and conſtruc. it will be $EK : KQ :: AG : GD :: AI : LD :: HF : FO$, or, by compoſition, $EQ : KQ :: HO : FO$, and conſequently, KQ (correſponding to the ſemi-nocturnal arc GD) $= FO$, anſwering to $3^\circ\ 45'$ as it ought.

The calculation will from hence be very eaſy; for, ſince ZK and KQ are reſpectively = the coſine and verſed ſine of $3^\circ\ 45'$, and $AI =$ ſine of $47^\circ\ 0'\ 42''$ the ☉'s merid. alt. by ſuppoſing ZA or $ZE = 1$, will be had $DL\ (= KQ \times AI \div EK) = {\cdot}0007839$, the nat. ſine of $0^\circ\ 2'\ 42''$ the ☉'s depreſſ. at midnight; therefore by the nat. of the ſphere, the comp. of the required lat. HE or $zP\ (= \frac{1}{2} \times \overline{HA + OD}) = 23^\circ\ 31'\ 42''$, and the ☉'s declin. $= EA\ (= \frac{1}{2} \times \overline{HA - OD}) = 23^\circ\ 29'$, giving June 21ſt for the day on which the traveller performed his journey.——Laſtly, if a degree on the equator be ſuppoſed to meaſure $69\frac{1}{2}$ miles, we ſhall, by ſim. ſectors, have $1 :$ ſine of $23^\circ\ 31'\ 42'' :: 69\frac{1}{2} \times 7\frac{1}{2} : 208{\cdot}0843$, &c. miles, the length of the required journey.

In this manner nearly the ſolution is alſo given by Meſſ. *J. Chipchaſe* (the propoſer), *G. Coughron*, *J. Dalby*, *J. Turner*, *W. Wales*, and the Rev. Mr. *Wildbore*. And Meſſ. *Edw. Fidler*, *Tho. Robinſon*, *Alex. Rowe*, *Elea. Suggett*, *Mich. Taylor*, and *W. Wilkin* have ſolved it algebraically.

X. QUESTION 651 *anſwered by the Rev. Mr.* Crakelt.



When the change in alt. or $ZS - Z\odot$, in the given time is a max. the ∠s $Z\odot P$, ZSP will, by art. 293 of my tranſlation of Mauduit's Trigonometry, be equal; therefore, if h and h' be put for the hour angles $ZP\odot$ and ZPS, L for the lat. or comp. of ZP, and D for the required declin. or comp. of $P\odot = PS$, we ſhall have, by art. 214, ſin. h ÷ $\overline{\text{coſ. } D \times \text{tang. } L - \text{ſin. } D \times \text{coſ. } h}$ = tang. $Z\odot P$ = tan. ZSP = ſin. h' ÷ $\overline{\text{coſ. } D \times \text{tan. } L \mp \text{ſin. } D \times \text{coſ. } h'}$, according as the ∠ ZPS is acute or obtuſe; or, by multiplying, dividing by coſ. D, ſubſtituting tang. for $\frac{\text{ſin.}}{\text{coſ.}}$, and tranſpoſition, ſin. h' × coſ. h × tang. D ∓ coſ. h' × ſin. h × tang. D = (by art. 85.) ſin. $\overline{h' \mp h}$ × tang. D = $\overline{\text{ſin. } h' - \text{ſin. } h}$ × tang. L; and therefore, by art. 87 and 92, coſ. ½ × $\overline{h' - h}$: coſ. ½ × $\overline{h' + h}$: : tang. L : tang. D, and ſin. ½ × $\overline{h' + h}$: ſin. ½ × $\overline{h' - h}$: : tang. L : tang. D; which are the two theorems given on p. 189 of the above-quoted book.

We are alſo favoured with ingenious ſolutions to this queſtion from Meſſ. *G. Coughron*, *J. Dalby*, *Wm. Smart*, *W. Wales*, and the Rev. Mr. *Wildbore*.

XI. QUESTION 652 *anſwered by Mr.* Hutton, (*the Propoſer*).

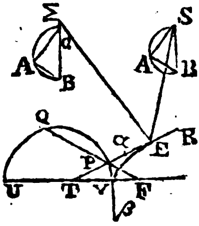
Let x denote any variable height of the fluid in the veſſel, and z the correſponding time, $m = 32\frac{1}{6}$, p = the area of the ſection of the pipe, b = that of the hole in the veſſel, and a = the velocity of the water in the pipe.——Then $b\sqrt{mx}$ = the quantity running out of the veſſel per ſecond (ſee p. 4 of my Math. Miſcel.), and ap = the quantity running in per ſecond: wherefore $ap - b\sqrt{mx}$ = the rate of the veſſel's filling per ſec. and conſequently $ap\dot{z} - b\dot{z}\sqrt{mx}$ will be the increaſe in $\dot{z}$ time, and is therefore = the flux.

of the quantity in the veſſel. Now, the veſſel being given, by the rules of menſuration, the content of the part whoſe height is x, will be found $= \frac{1}{27} \times \overline{xx + 27x + 243} \times {\cdot}7854x$, whoſe flux. or $\frac{1}{9} \times \overline{xx + 18x + 81} \times {\cdot}7854\dot{x}$, muſt therefore be $= ap\dot{z} - b\dot{z}\sqrt{mx}$: whence $\dot{z} = \frac{xx + 18x + 81}{2s - 2r\sqrt{x}} \times \dot{x}$, by putting the given numbers inſtead of b, m, a, and p, and making $r = \frac{\sqrt{32\frac{1}{6}}}{32}$, and $s = \frac{75}{256}$; or $\dot{z} = \frac{v^5 + 18v^3 + 81v}{s - rv} \times \dot{v}$, by putting vv for x: the fluent of which is $z = -\frac{v^5}{5r} - \frac{sv^4}{4rr} - \frac{ss + 18rr}{3r^3}.v^3 - \frac{ss + 18rr}{2r^4}.svv - \frac{\overline{ss + 9rr}^2}{r^5}.v + \frac{\overline{ss + 9rr}^2}{r^6}.s \times$ hyp. log. of $\frac{s}{s - rv}$; which is a general expreſſion of the time for any alt. vv (x).——Now, when $x = 1$ foot, v is $= 1$ alſo, and then the above expreſſion brings out 4′ 47·555″ for the time when the water will be one foot high in the veſſel.

Alſo, ſince the greateſt height is evidently when the water runs as faſt out of the veſſel as it runs in by the pipe, $s - rv$ muſt, in that caſe, be $= 0$; $\because v = \frac{s}{r} = 1{\cdot}652983$, and x $(= vv) = 2{\cdot}732352$ feet, the greateſt height of the fluid. But when $s - rv$ vaniſhes, the expreſſion $\frac{s}{s - rv}$ becomes infinite, and the value of z infinite alſo of conſequence; ſo that the water will be an infinite time in riſing to the ſaid greateſt height 2·732352, that height being the limit to which it continually approaches from nothing, but to which it can never attain in any given finite time.

The ſolution by the Rev. Mr. *Wildbore* is very ingenious, and founded on a new principle, which our narrow bounds cannot contain. Mr. *G. Coughron*, Mr. *J. Dalby*, and the Rev. Mr. *Crakelt* have alſo favoured us with ingenious anſwers to it.

XII. QUESTION 653 *anſwered by Mr.* W. Wales; *at at whoſe Requeſt it was re-propoſed.*



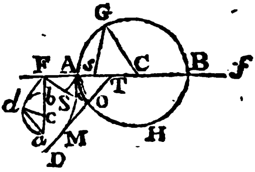
Let VER repreſent the conic-ſection in which the earth is ſuppoſed to revolve, F the focus, V the place of the perihelion, E the place of the earth at any aſſigned time, and S that of a ſtar: Draw the tangent ET, which will give the direction of the earth's motion at E, take $E\alpha$ to ES, as the velocity of the earth when at E, is to the velocity of light, and $V\beta$, $\perp VF$, as the velocity when at V. From S, on a plane paſſing through the ſtar parallel to that of the earth's orbit, draw SB and SA, $=$ and $\parallel$ to $V\beta$ and $E\alpha$, and A and B will be the apparent places of the ſtar in that plane, when the earth is at E and V, on account of aberration. Let fall on ET the perpendicular FP, which, it is well known, will be reciprocally as $E\alpha$, or its equal SA; moreover, FV will be reciprocally as SB. Now, if the trajectory VER be a parabola, the tang. which is drawn parallel to the axe will be at an infinite diſtance therefrom, and the perpend. thereon from F will be ſo likewiſe; its reciprocal therefore is $= 0$, and ſo the path in queſtion paſſes through the points S, A, and B. Draw AB. By a prop. of the parab. $FV : FP :: FP : FT$, and becauſe $FV \times SB = FP \times SA$, $FV : FP :: SA : SB$, and by equality, $FP : FT :: SA : SB$; whence, the $\angle$s F and S being equal by conſtruction, the $\triangle$s FPT and SAB are ſimilar; and conſequently, P being a right angle, A will be a right angle alſo, and that whereever the point A may fall; the curve SAB (or path in this caſe, required) is therefore a circle.

But if the earth's orbit VER be an hyperbola, and Σ the true place of the ſtar, deſcribe on the tranſverſe axe UV the circ. $UQPV$, &c. then it is well known that all perpendiculars from the focus F will meet the tangents to this orb on which they are demitted, in the circumf. of this circ. and if FP be produced to meet it again in Q, by the prop. of the circ. FP and FV will be reciprocally as FQ and FU; but they are alſo reciprocally as $A\Sigma$ and $B\Sigma$; therefore $FU : FQ :: B\Sigma : A\Sigma$; and as the $\angle$s QFU and $A\Sigma B$ are equal, and that whereſoever A and Q fall, it is evident that the path of the point *A* will be ſimilar to that of the point Q; but that of the point Q is known to be a circle; conſequently that of the point *A* (or the curve apparently deſcribed by the ſtar on account of the abberration, in this caſe) is alſo a circle.

SCHOL. I. Becauſe the apparent path of the ſtar will not be on the plane above ſpecified, unleſs its true place be the pole of the ecliptic, but on one which is perpendicular to the line joining the eye and ſtar, it follows from the nat. of the orthographic projection, that the apparent path will be in every other caſe an ellipſis, excepting that when the ſtar is in the ecliptic, and then it will be a right line.

II. Becauſe the aſymptote becomes a tang. to the hyperb. when both are produced out to an infinite diſt. it follows that the perpendic. from the focus on the aſymp. will be greateſt; therefore its reciprocal will be leaſt, and the ſtar can apparently approach no nearer to its true place than by that diſt. Moreover, becauſe the aſymp. paſſes through the center of the circ. $UQPV$, the ſaid perpendic. will be a tang. to it; therefore if from Σ, ∥ to the aſymp. a line be drawn = to the ſaid reciprocal, it will be a tang. to the path at its neareſt approach to the ſtar's true place; and therefore if the circ. was compleated, the true place of the ſtar would be without it.

Anſwer to the ſame by the Rev. Mr. Wildbore.



Let AMD repreſent the earth's orbit, AB its tranſverſe axis when an hyperbola, F, f its foci, &c. MT a tang. to it at M, FO a perpendic. let fall from F upon MT, aF another upon Ff, and $AGBH$ the circumference of a circ. deſcribed about the diameter AB; and ſuppoſe, when the earth is at A, that F and a are the real and apparent places of the ſtar reduced to the plane of the ecliptic. From F ∥ to the tang. MT, take $Fd : Fa :: FA : FO$, and from a upon Fa take $ca = dc$, with which as rad. deſcribe the ſemicircle adb, and the arc ad will be the apparent locus of the ſtar reduced to the ecliptic, whilſt the earth moves from A to M in an hyperbola; but when the orbit is a parabola, ca the rad. of the circ. muſt be taken $= Fc = \frac{1}{2}Fa$; which circ.

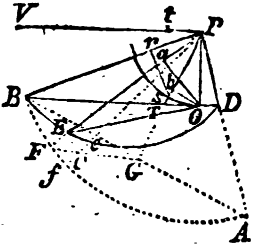
will likewiſe be the locus in this caſe.——For, by the nature of aberration and central motion (vide prop. 2. pa. 3, of Simp. Eſſays), d is the apparent place of the ſtar, reduced to the ecliptic, when the earth is at M as by conſtruc.), becauſe the velocity of the earth at M is to the velocity when at A, as FA is to FO, per cor. 2d to prop. 1ſt on pa. 24. of Simp. Eſſays aforeſaid, and (drawing AS to meet FO ſo that AS and SO may be equal, and joining the points A, O and a, d) the $\angle$s dFa, AFO being equal, and their including ſides proportional (by conſtruc.) the $\triangle$s AFO, dFa, and dca, ASO will be ſimilar, and conſequently $Fa : Fc :: FO : FS$; but the ratio of FO to FS is always the ſame (being in the conſtant ratio of FB to FC, as is eaſily demonſtrated, the circumf. $AGBH$ being the locus of the point O, per prop. 21. part 3. of Steel's Conic-Sections), and therefore the ratio of Fa and Fc will be always the ſame likewiſe, and Fa being conſtant, Fc and ca will be always the ſame, and the point d be always found in the circumf. of the ſame circ. adb, as per conſtruction.——But in the caſe of a parabolic orbit, AC being infinite, the locus of the point O becomes a right line $\perp$ to FA, and as the triangle FAO may be then circumſcribed by a ſemicircle, $AS = SO$ will be $= FS$, and conſequently $Fc = ca$.

COROLLARY. The proportion $FB : FC :: FO : FS :: Fa : Fc$ alſo holds when the orbit is an ellipſis; only AC being then greater than FC, SO muſt be greater than FS, and ca than Fc; wherefore the circ arc ad will in this caſe fall partly beyond the axis, and be compleated in one revolution of the earth. But in the caſe of the hyperbola above, it is evident that the point d, in moving along the arc ad, can never arrive at the poſition wherein Fd would be $\parallel$ to the aſymptote, nor at the point F when the orbit is a parabola.

We have alſo received very ingenious, elegant geometrical ſolutions to it from Meſſ. *Burrow*, *Caughron*, and *Dalby*; and *Peter Puzzlem* has obliged us with a curious analytical ſolution, but through great want of room, we have been forced, with much regret, to leave it out.

XIII. QUEST. 654 *anſwered by the Rev. Mr.* Wildbore, (*the Propoſer*).

Suppoſe PO to repreſent the object, BE the path of the ſhad. of P from midnight to the time required: draw PS $\parallel$ to the earth's axis, till it cuts the tranſverſe diameter BD. With PO (1) rad. deſcribe the arc OS cutting the axis in S, and having joined PD, PB, PE, and OE, let fall $Oa \perp$ to PE, Ob to PS, and Sr to PB.



Then 'tis well known that rS is the ſine of the co-declination BPS, Ob that of the co-lat. OPS, aO the ſine, EO the tang. PE the ſecant of the co-alt. EPO, and BOE the azim. from the north at the time required. Moreover, the diſt. deſcribed at E, during the infinitely ſmall inſtant of time that the velocity can be conſidered as uniform, is, it is well known, $=\sqrt{E\dot{O}^2+EO^2\times\angle B\dot{O}E^2}$. But, by Simp. Flux. p. 282, as $B\dot{O}E : E\dot{P}O = \frac{E\dot{O}}{EP^2}$ :: (in the oblique ſpherical triangle terminated by P the pole, Z the zenith, and $\odot$ the ſun, *ſee fig.* Q. 651) the co-tang. $\angle\odot : \text{s.}\ Z\odot$ (the co-alt. $= EPO) = aO = \frac{EO}{EP}$; conſequently $B\dot{O}E = \frac{E\dot{O}.\text{co-t.}\odot}{EO.PE}$: Moreover, as $\text{s.}\ P\odot\times\text{coſ.}\ \odot : \text{s.}\ Z\odot$ :: $\angle\dot{Z} : \dot{P}$ the fluxion of the time $= \frac{\dot{Z}\times\text{s.}\ Z\odot}{\text{s.}\ P\odot\times\text{coſ.}\ \odot} = \frac{B\dot{O}E.aO}{rS.\text{coſ.}\ \odot} = \frac{E\dot{O}.\text{co-t.}\odot}{rS.PE^2.\text{coſ.}\ \odot}$. But, by the laws of motion, the diſtance $\sqrt{E\dot{O}^2+EO^2\times BOE^2} = E\dot{O}\times\sqrt{1+\frac{\text{co-t.}\odot^2}{PE^2}}$ ÷ by the time $\frac{E\dot{O}}{rS.PE^2.\text{s.}\odot}$ is = to the velocity of the uniform motion during that time $= rS.PE\sqrt{PE^2.\text{s.}\odot^2+\text{coſ.}\odot^2}$, which by the queſt. is to be a max. or, becauſe $PE^2=1+EO^2$, $\text{s.}\odot^2+\text{coſ.}\odot^2=1$, $PE\sqrt{1+EO^2.\text{s.}\odot^2} = PE\sqrt{1+EO^2-EO^2.\text{coſ.}\odot^2}$, $PE^2\times\overline{PE^2-EO\ .\text{coſ.}\odot^2}$ is a max. But it is proved by writers on ſpherics; that the $\text{coſ.}\ \angle\odot\times\text{s.}\ Z\odot\times\text{s.}\ P\odot = \text{coſ.}\ ZP-\text{coſ.}\ Z\odot\times\text{coſ.}\ P\odot = \text{coſ.}\ \odot\times aO\times rS = Pb-rP.aP$; but $aO=\frac{EO}{EP}$, and $aP=\frac{1}{EP}(\frac{PO^2}{EP})$, $\therefore$ $\text{coſ.}\ \odot\times EO = \overline{Pb.EP-RP}\times rS^{-1}$: Conſequently

$PE^2 \times \overline{PE^2 - Pb.PE - rP}|^2 : rS^{-2} = PE \times \overline{PE + \overline{PE.Pb - rP}.rS^{-1}} \times PE \times \overline{PE - \overline{PE.PB - rP}.rS^{-1}}$ is a maximum, or $PE \times \overline{PE.\overline{rS + Pb} - rP} \times PE \times \overline{PE.\overline{rS - Pb} + rP}$ a max. take any where, $Pt = \frac{rP}{Pb + rS}$ and $PV = \frac{rP}{Pb - rS}$, and $PE \times \overline{PE - Pt} \times PE \times \overline{PV - PE}$ is a max. wherefore, per Euc. II. 5. and v. 4. $\overline{2PE - Pt} \times PE \times \overline{PV - PE} = \overline{2PE - PV} \times PE \times \overline{PE - Pt}$; whence PE may be readily found by a geometrical conſtruction, or otherwiſe, $= 89{\cdot}79712 =$ the co-ſecant of 38° 43″, the ☉'s alt. at the time required.*

Ingenious anſwers to this queſtion have alſo been received from Meſſ. *G. Coughron*, *J. Dalby*, and *W. Wales*.

The

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* As ſeveral of our correſpondents have complained of the obſcurity of the ſolution above given to this queſt. we ſhall here, at their requeſt, ſubjoin another ſolution a little different from it, which may perhaps be better underſtood by ſome readers than the above original one.

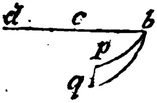
In order to this, let the following additions, in dotted lines, be made to the figure above, viz. complete the cone PBA, and produce its axe PS to cut BD in T and the diam. AB in the center G of its baſe; alſo produce PE to the circumf at F, and draw another line Pef indefinitely near it, on which demit the ⊥ Ec; and, laſtly, join G, F.

Then, conceiving the right-angled △ PGB to revolve with an uniform motion about the axe PG till it comes to the poſition PGF, it is evident that BGF is the angle of time correſponding to the ∠ BOE of azimuth and PKO of altitude; alſo Ff is as the conſtant flux. of the time or arc BF, Ee the flux. of the arc BE deſcribed by the ſhadow of P on the horizontal plane BED, ec the flux. of PE, and Ec the flux. of an arc whoſe rad. is PE. And when the ſhadow of P moves quickeſt, then Ee will be greateſt in reſpect of Ff, and therefore we are to find $Ee \div Ff$ a max.

Now put a and b for the ſine and coſine of the lat. PTO or BTG, c and d for the ſine and coſine of the declin. PBG, and z the ſine of the alt. PEO, to the rad. 1. Then, in the ſpherical △ PZ☉ or PZS in the fig. to the ſolution of queſt. 651, all the ſides are expreſſed, to find the ∠ P of the time, whoſe coſine therefore, or the coſine of BGF in our figure, will, by ſpherics, be found $= \overline{ac - z} \div bd$, and therefore the flux. of BGF, or of

the

The PRIZE QUESTION *anſwered by* Peter Puzzlem, (*the Propoſer*).



Let *bp*, *bq* be circular arcs deſcribed from the cent. *c* in different planes *cbp*, *cbq*; and, the ∠ *pbq* being ſuppoſed indefinitely ſmall, let *pq* be an indefinitely ſmall particle of the curve deſcribed by the body. Put *r*, *y*, and *s* to denote the ſines of the arcs *bp*, *bq*, and the angle *bpq* reſpectively; alſo, put *u* for the angular velocity of the circ. in which the body is always to be found, about *bcd*, meaſured at a diſtance from *c* = to the rad. *cb*, and *v* for the velocity of the body along the ſaid circle. Call the ſaid rad. *a*, and conſidering the required force as compounded of two forces, one acting at right angles to the plane of the circ. in which the body is always found, and the other in the direct. of the tang. to the ſaid circ. at the point where the body at any time is; let *f* and *g* denote thoſe two forces reſpectively.

Then $a : u :: y : w \left(= \frac{uy}{a}\right)$, the velocity at *q* at right angles to *bq* being denoted by *w*. Alſo, $y : s :: r\ r\ \frac{rs}{y} =$ ſine

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the arc whoſe coſine is this quantity and radius 1, is $\dot{z} \div \sqrt{bbdd - \overline{at - z}|^2}$, which drawn into the rad. *BG*, or *m*, gives $Ff = m\dot{z} \div \sqrt{bbdd - \overline{ac - z}|^2}$. Again, $d : 1$ (rad.) $:: m$ $(BG) : m \div d = PB = PF$, and $z : 1 :: n\,(PO) : n \div z = PE$; hence $(PF : PE :: Ff :)\ Ee = nd\dot{z} \div z\sqrt{bbdd - \overline{ac - z}|^2}$; alſo $ce = P\dot{E} = -n\dot{z}z^{-2}$; ∵ $Ee \left(= \sqrt{Ec^2 + ce^2}\right) = \frac{n\dot{z}}{zz}\sqrt{\frac{bbdd - aacc + 2acz - cczz}{bbdd - \overline{ac - z}|^2}}$; conſequently $Ee \div Ff = \frac{n}{mzz}\sqrt{bbdd - aacc + 2acz - cczz}$ a minimum; whoſe flux. being equated to 0, we obtain $z = \frac{3a - \sqrt{8dd + aa}}{2c}$ = the ſine of 38° 17′, ſuppoſing the declin. to be 23° 29′. [But if the true preſent declin. 23° 28′ be uſed, *z* will come out = the ſine of 36° 58″ for the alt. when the ſhadow of the ſummit moves faſteſt.—— And hence the time from midnight is eaſily found, its coſine being $= \overline{ac - z} \div bd$.

ſine of bqp, and $\frac{rs}{y} : w :: a : \frac{awy}{rs}$ the abſolute velocity of the body at q along the curve pq; which would be invariable if no force were to act on the body but the tenſion of the ſtring. Therefore in that caſe, a, r, and s being conſidered as invariable whilſt w and y vary, $\dot{w}y + w\dot{y}$ would would be $= 0$, and $\dot{w} = -\frac{w\dot{y}}{y} = -\frac{u\dot{y}}{a}$. But the force f continuing to act on the body, $\dot{w}$ will be $= \frac{\dot{u}y}{a} + \frac{u\dot{y}}{a}$ conſequently $\frac{\dot{u}y}{a} + \frac{2u\dot{y}}{a}$, the exceſs of $\frac{\dot{u}y}{a} + \frac{u\dot{y}}{a}$ above $-\frac{u\dot{y}}{a}$, will be the flux. of the velocity of the body at right angles to bq, occaſioned by the action of the force f; and therefore, the fluxion of the time being manifeſtly $= \frac{a\dot{y}}{v\sqrt{aa-yy}}$, $f \times \frac{a\dot{y}}{v\sqrt{aa-yy}}$ will be $= \frac{\dot{u}y}{a} + \frac{2u\dot{y}}{a}$, and $f = \frac{v\sqrt{aa-yy}}{aa} \times \frac{\dot{u}v + 2u\dot{y}}{\dot{y}}$.——Moreover $\sqrt{vv+ww}$, the velocity in the curve pq, would be invariable, if the forces f and g were to ceaſe acting; therefore $v\dot{v} + w\dot{w}$ would in ſuch caſe be $= 0$, and $\dot{v} = -\frac{w\dot{w}}{v} = \frac{uuy\dot{y}}{aav}$: conſequently we have $g \times \frac{a\dot{y}}{v\sqrt{aa-yy}} = \dot{v} - \frac{uuy\dot{y}}{aav} = \frac{aav\dot{v} - uuy\dot{y}}{aav}$, and $g = \frac{\sqrt{aa-yy}}{a^3} \times \frac{aav\dot{v} - uuy\dot{y}}{\dot{y}}$.

Thus is the queſtion ſolved generally, the required force being compounded of the forces f and g found above; from whence its quantity and direction may be known, and the ſolution may be eaſily adapted to particular caſes. I ſhall here only take notice of one particular caſe, and that is when u is invariable, which is the ſame as lem. 2. p. 3. of the late Mr. Simpſon's Miſcell. Tracts. In which caſe $\dot{u}$ being $= 0$, f appears to be $= \frac{2vu\sqrt{aa-yy}}{aa}$, and its greateſt value (when $y = 0$) $= \frac{2vu}{a}$. The gent. I have juſt now mentioned has conſidered v as invariable, without taking any notice that it will not be ſo, unleſs the body be acted on by a force $g = \mp \frac{uuy\sqrt{aa-yy}}{a^3}$. Indeed he has conſidered the velocity u as very ſmall, and then g will be very ſmall, but not abſolutely $= 0$, nor yet indefinitely ſmall, for u (though ſmall) being finite, g will be alſo finite. The ſolution here given is true whether the velocity u be ſmall or great.——If, u being invariable, g be $= 0$, $aav\dot{v}$ will be $= uuy\dot{y}$, and $v = \frac{\sqrt{aaee+uuyy}}{a}$, e being put for the velocity of the body at the commencement of the motion. Therefore it appears that f will in that caſe be $= \frac{2u\sqrt{aa-yy}\times\sqrt{aaee+uuyy}}{a^3}$; which, when u is very ſmall, is nearly $= \frac{2eu\sqrt{aa-yy}}{a^2}$, the value of f as computed by Mr. *Simpſon*.

If u be invariable, and only one force act on the body, that force muſt be $= \frac{\sqrt{4aavv+uuyy}\times\sqrt{aauu-uuyy}}{a^3}$; and the direction in which it muſt act, inclined to the plane of the circle in which the body is, in an angle whoſe ſine is to rad. as $2v$ to $\frac{\sqrt{4aavv+uuyy}}{a}$.——Mr. Simpſon ſeems not to have been aware that the force g muſt neceſſarily act to keep the velocity v from varying!

Mr *De la Lande*, in his Aſtronomy (art. 3457) propoſing to explain Mr. Simpſon's ſolution, has obſerved, that, only the force f acting, v will not be invariable when u is ſo. ——But, without computing the neceſſary force g, or the exact value of the force f when g is $= 0$, he neglects a part of the force f, and entirely neglects the force g, as being what he calls *infiniments petits du troiſiéme ordre*; whereas they are not generally *infiniments petits* of any order whatever, being aſſignable quantities which may be conſiderable, and therefore ſhould not be neglected without firſt computing their values, and ſhewing that, in the caſe in queſtion, they are inconſiderable.

The conſequence is obvious, when a number of bodies, kept from flying from the center c by any attractive force, follow one another in the revolving circ. from the point b, each having the ſame velocity (e) when at that point.—— The force f being computed as above, if only that force acts on each, they may be always found in one and the ſame circle, forming a kind of a ring, but will not revolve uniformly therein, unleſs each be alſo acted on by the force g.

The Rev. Mr. *Wildbore* has alſo favoured us with an ingenious ſolution to this queſtion, and the prizes of 8 and 12 Diaries for the ſolution of it are the reſpective claims of that gentleman and the propoſer *Peter Puzzlem.*

The Eclipſes calculated for 1773.

This year will preſent us with two eclipſes of the ſun and two of the moon, according to Mayer's Tables.

The firſt will be of the ſun, on March the 23d, in the morning, inviſible in England, but viſible in Aſia.

The ſecond will be of the moon, on April the 7th, in the morning, inviſible at London, but viſible in the pacific ocean.

The third will be of the ſun, on September the 16th, in the afternoon, inviſible at London, but viſible in South America.

The fourth will be of the moon, on September the 30th, in the evening, part viſible at London: Beginning 4h. 33m. 25s. Middle 6h. 2m. 44s. End 7h. 32m. 3s. Digits eclipſed 7° 58′ apparent time. ISAAC TARRATT.

New Queſtions.

I. QUESTION 655, *by Mr.* John Shadgett.

Ye ladies, court'ous, kind, and fair,
Who oft myſterious truths declare,
From hence * the name of him you'll find,
Whoſe heav'n-born muſe charm'd all mankind.
Be quick, conciſe, correct, and true;
Next year I'll do as much for you.

* Given $\left\{\begin{array}{l} y^5z + x + xy^3u = 4505639, \\ xu^2z^2 + yx^2 = 81690, \\ ux^2 + zxy^2 = 18690, \\ x^3y = 35535 + u^2z^2; \end{array}\right\}$ to find the values of u, x, y, and z, which denote the places of the letters in the alphabet which compoſe the poet's name.

II. QUESTION 656, *by Mr.* Wm. Spicer.

Given the legs of a plane triangle equal to 90 and 100, and the rectangle contained by the baſe and the line biſecting the vertical angle and terminating in the baſe a maximum, to determine the triangle.

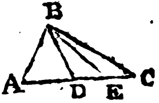
III. QUESTION 657, *by Mr.* Wm. Wilkin.

Required the number of ale gallons contained in a caſk compoſed of two equal conic fruſtums of the leaſt reſiſtance, lying with its axis parallel to the horizon and the liquor juſt covering its ends; the bung diameter being 7 and the length 11 feet?

IV. QUESTION 658, *by Mr.* Moſs.

To determine all the different ways it is poſſible to pay 50l. with piſtoles at 17s. each, guineas, moidores, and ſix-and-thirties.

V. QUESTION 659, *by Mr.* Wm. Wilkin.



In the plane triangle *ABC*, there is given the angle at *C*, and the parts or ſegments of the baſe *AD*, *AE*, to conſtruct the triangle ſo, that, if *BD* be drawn, the ∠ *ABD* may be a maximum, and *BC* to *EC* as *m* to *n*.

VI. QUESTION 660, *by Mr.* Steph. Hodges, *at the Right Hon. the Earl* Spencer's, *at* Althorp.

Of all the plane triangles having the ſame given baſe and perpendicular, to determine geometrically, that whoſe vertical angle ſhall be the greateſt.

VII. QUESTION 661, *by Mr.* Moſs.

The difference of the ſides including a known angle of a plane triangle being given, and alſo the ſum of one of thoſe ſides and that oppoſite the given angle, to conſtruct the triangle.

VIII. QUESTION 662, *by Mr.* Iſaac Dalby.

Given the vertical angle, the line biſecting it, and the difference of the ſegments of the baſe made thereby, to conſtruct the triangle.

IX. QUESTION 663, *by Mr.* Steph. Ogle.

While a given circular wheel is trundling on an horizontal plane, it is required to determine its point of contact therewith ſuch, that the ſum of the altitudes of any four given points in its circumference above the ſaid plane, may be equal to a given quantity M; and to aſcertain the limits within which the ſolution is poſſible.

X. QUESTION 664, *by the Rev. Mr.* Crakelt.

Through the point of interſection of two given circles it is required to draw a right line in ſuch a manner, that the ſum of the reſpective rectangles under the parts thereof intercepted between the ſaid point and their peripheries, and given lines M and N, may be equal to a given ſquare, R^2.

XI. QUESTION 665, *by the Rev. Mr.* Lawſon.

Having given the baſe and vertical angle of a plane triangle, it is required to find the locus of the extremity of the line that continually biſects the ſaid vertical angle, and is an arithmetical mean between the two ſides comprehending the ſame.

XII. QUESTION 666, *by Mr.* Todd.

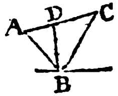
To determine the nature of the curve which will cut at right angles, any number of parabolas having the ſame vertex and axis.

XIII. QUESTION 667, *by the Rev. Mr.* Wildbore.

To redintegrate (or find the whole fluent of) the expreſſion $\frac{\dot{x}^n}{1-xx}$; where n is any whole poſitive number whatever.

The PRIZE QUESTION, *by* Peter Puzzlem.

Let *ABC* be a given triangle, and let the line *BD* meet *AC* in *D*, ſo that *BC* being greater than *AB*, *AD* may be greater than *CD*: It is required to find with what velocity the ſaid triangle, conſidered as a very thin plate of heavy metal, muſt revolve about *BD* as an axis, that, whilſt it is ſo revolving, the ſaid line *BD* ſhall (if poſſible) always keep in an upright poſition, with the angular point *B* reſting on an horizontal plane?



AN APPENDIX:

CONTAINING

Additional Solutions to ſome of the Queſtions, and the Corrections of ſuch material Errata as have been diſcovered in the three preceding Volumes.

VOL. I.

PAGE 30, line 5, for ♂ read ♀.——Pa. 71, l. 19, for z read az.

Pa. 74. We ſhall here ſupply a ſolution to Queſt. 22 on the principles of Mercator's Sailing. In order to which let $a = BD = 150$; $b =$ the latitude, in minutes, of A the place ſailed from; $m = 3437\frac{3}{4}$ the reciprocal of the length of an arc of 1 min. to the rad. 1; $c = m \times : b + \frac{1}{6}b^3 + \frac{1}{24}b^5 + \frac{61}{5040}b^7$ &c. = the merid. parts in the lat. b, or c might be taken from a table of merid. parts; $d = 60$, the dif. between the dif. of lat. and longitude; alſo put $x =$ the lat. of C, the place arrived at.

Then $m \times : x + \frac{1}{6}x^3 + \frac{1}{24}x^5 + \frac{61}{5040}x^7$ &c. = the merid. parts in the lat. x.

Hence $x - b = AB$, the proper dif. of lat. and $-c + m \times : x + \frac{1}{6}x^3 + \frac{1}{24}x^5$ &c. = the merid. dif. of lat.

Now $AD = \sqrt{\overline{x-b}|^2 - a^2} : DB :: (AB : BC ::)$ merid. dif. of latitude : dif. of long. $= -\frac{ac}{\sqrt{\overline{x-b}|^2 - a^2}} + \frac{ma}{\sqrt{\overline{x-b}|^2 - a^2}} \times : x + \frac{1}{6}x^3$ &c. But the ſame dif. of long. is $= AB + 60 = x - b + d$. Then this value being equated to the other, there will reſult an equation from which the value of x may be found, and thence every thing elſe.

Pa. 102, l. 28, for $5x^3$ read $\overline{5x}|^3$.——Pa. 226, l. 16, for $z =$, read $x =$ ——Pa. 337, as alſo, ſome other places, for *Geo. Anderſon*, read *Geo. Auderſon*.

VOL II.

P. 11, l. 6, for — *diminitive*, read 8128 *perfect*. Alſo dele all the numbers but 6, 28, 496, and 8128, which are the only perfect numbers.——Pa. 12, l. 9, for *exponent* read *number of terms*. Alſo l. 19, for 8123 read 8128. And dele all the reſt of the line after, as alſo part of the next line to the number 2196128 incluſive.

In the ſolution of the former part of the prize queſt. given at pa. 17 by the Editor, it muſt be noted that it is adapted to that caſe in which the cyl. is of no ſenſible thickneſs, or a mathematical line; and therefore after the word *cylinder* in the 9th line from the bottom, add the words *when conſidered as of no thickneſs*. Alſo inſtead of the laſt6 lines of the ſolu. add as here follows: viz. $\frac{c+z}{z}\sqrt{bb+zz} = AF$, *which muſt be a minimum. This put into fluxions, &c. we get* $z^3 - bbc = 0$; *hence* $z = \sqrt[3]{bbc} = AD$. *And therefore the length of the rod or line AF in this caſe is* $\frac{c+bq}{q}\sqrt{1+qq}$, *where q is* $= \sqrt[3]{\frac{c}{b}}$.

Pa. 34, l. 14, for $= \frac{18m+4}{19}$ read *or* $\frac{18m+4}{19}$. Alſo l. 20, for $= \frac{14n+18}{15}$ read *or* $\frac{14n+18}{15}$.

Pa. 65, l. 5 from the bottom, for $\frac{x}{2}$ read $\frac{1}{2}\dot{x} : x$.

Pa. 148, l. 22, before y^2 write $=$.

Pa. 151, l. 24, 25, and 26, for $\cdot 2^2$, $\cdot 3^3$, and $\cdot 4^4$, read $\cdot 2^{\cdot 2}$, $\cdot 3^{\cdot 3}$, and $\cdot 4^{\cdot 4}$.

QUESTION 262 *anſwered by Mr.* J. Landen, *F. R. S.*

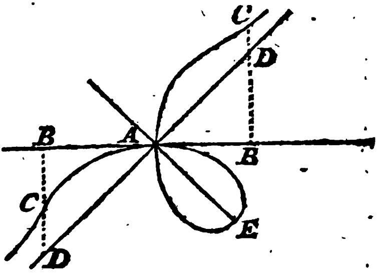
Of $x^5 - y^5 + axy = 0$, the equation of the curve, take the fluxions; and for $\dot{x}$ and $\dot{y}$ put the invariable quantities m and n reſpectively: repeating the operation till you get an equation conſiſting only of invariable quantities. By ſo doing you will get

$$5mx^4 - 5ny^4 + amy + anx = 0,$$
$$10m^2x^3 - 10n^2y^3 + amn = 0,$$
$$m^3x^2 - n^3y^2 = 0,$$
$$m^4x - n^4y = 0,$$
$$m^5 - n^5 = 0:$$

each of which equations, ſhewing the relation of x and y, will (as appears by chap. IX of my *Reſid. Anal.*) expreſs a line which will be an aſymptote to the curve expreſſed by the given equation, when $\frac{m}{n}$ is determined by the final equation $m^5 - n^5 = 0$.

Now by that final equation $\frac{m}{n}$ is $= 1$: therefore each of the equations $5x^4 - 5y^4 + ay + ax = 0$, $10x^3 - 10y^3 + a = 0$, $xx - yy = 0$, $x - y = 0$, is an aſymptotic equation with reſpect to the propoſed curve.

The 4th aſymptotic equation correſponds to a right line, paſſing through the point where x begins, making an angle of 45° with the baſe on which x is meaſured. The 3d aſymptotic equation correſponds to two right lines; one of which is the ſame as that expreſſed by the 4th aſymptotic equation; and the other, expreſſed by $x + y = 0$, is (by what is ſaid chap. X of my *Reſid. Anal.*) a diameter of the propoſed curve; which diameter cuts the aſymptote at right angles. The 2d aſymptotic equation correſponds to a line of the 3d order (which is Sir Iſ. Newton's 45th ſpecies) having the ſame aſymptote as the propoſed curve. The 1ſt aſymptotic equation likewiſe appertains to a line of the 3d order (of the ſpecies juſt now mentioned), having the ſame aſymptote as the propoſed curve, and to the right line which I have obſerved is the diameter of that curve. And the form of the propoſed curve, with its rectilineal aſymptote and diameter, is (by chap. VIII, IX, and X of my *Reſid. Anal.*) as in the marginal figure: x being meaſured on AB (a tangent at the node A) to which the ordinate BC (y) is perpendicular; AD the aſymptote; AE the diameter.



☞ Mr. *Farrer's* Anſwer is very erroneous!

Pi.

Pa. 233. If any perſon be not ſatisfied with the ſolutions to the Prize Q. for 1746, here given, and in the *Miſcel. Curios.* therein referred to; he may conſult Mr. *Landen's Lucubrations*, where the ſubject is very fully handled.

P. 248, l. 8, for $\frac{4a}{3}$ read $\frac{4}{3}$. Ibid. l. 13, f. $+xx$, r. $-xx$. Ib. l. 14, f. $+4mx$, r. $-4nx$; alſo here and in the following parts of the ſolution, for *m* write *n*. Line 15, for $+4nx$ and $+xx$, r. $-4nx$ and $-xx$. Line 16, f. $\sqrt{mx+2xx}$, r. $\sqrt{nx}$. Line 18 and 19, f. $8\sqrt{4x+4xx}$, r. $8\sqrt{nx}$. Alſo in l. 19, for $+4nx$, r. $-4nx$.——It may alſo be remarked that the ſolidity required in this queſtion, may be found, without an infinite ſeries, by the rules at p. 335, &c. of my *Menſur.* And uſing the numbers given in this queſtion, the ſolidity by thoſe rules will become $= 6{\cdot}48510957$ very accurately.

Pa. 253, l. 19 and 21, for $4hhxx$, read $4bbxx$. And l. 21, f. $bbtt$, r. $3bbtt$. Alſo the *N. B.* near the bottom is a miſtake; for when $x = TL$, then the fluxion of the ſegment *DTH* is $n\dot{x} \times bb \times \frac{tx - xx}{tt}$.

But it ſeems the principle of the ſolution is not accurately true, but only an approximation, as will appear from the following

Solution of Queſtion 293, *by the Rev. Mr.* Wildbore.

In the original ſolution to this queſtion, it is ſaid that the greater the vacuity, the greater will be the quantity of liquor, and conſequently the ſum of both; therefore when the ullage is a max. the whole caſk will be alſo a max. But this method of reaſoning is not altogether to be depended on: for, retaining the notation there given, viz. $x = Dm = Gt$, $d = BD$, and $c = Bt$; then the conjugate ſemiaxes of the ſurface of the liquor, being the ſemiordinate paſſing through *t* of the circle whoſe diam. is *BG*, muſt be $= \sqrt{cx}$; whence, ($x - c$ being the head, $x + c$ the bung diam. $2\sqrt{dd - xx} =$ the length of the caſk, and $p = 3{\cdot}14159$) by Hutton's Menſuration, pa. 278 and 287, we have $\frac{1}{6}p \times \overline{3xx + 2cx + 3aa}\sqrt{dd - xx} =$ the content of the whole caſk, and $\frac{1}{6}p \times \frac{3ccx + c^3}{\sqrt{cx}}\sqrt{dd - xx} =$ that of the empty part; conſequently the diff. of theſe two muſt be the ullage.

But theſe expreſſions are ſo very different, that it is evident at firſt ſight that the one will not be a max. when the other is ſo; and conſequently their diff. cannot be a max. when either of them is ſo. But the queſtion expreſsly requires the ullage to be a max. and though this, by reaſon of the ſmallneſs of the vacuity in this particular caſe, be nearly a max. when the caſk is a max. it will not be exactly ſo: and caſes may be put in which they will be very different: and in the preſent, though there be but little difference in the content, the form of the caſk will come out conſiderably different, when the ullage is a max. from what it does when the caſk is ſo. Thus, putting $cx = vv$, when the ullage is a max. then $\overline{\frac{3v^4}{cc} + 2vv + 3cc - 3cv - \frac{c^3}{v}} \times \sqrt{dd - \frac{v^4}{cc}}$ is ſo; the fluxion, reduced, gives $v^9 + \frac{4c^2v^7}{9} - \frac{c^3v^6}{2} - \frac{2d^2c^2 - c^4}{3}v^5 - \frac{c^5v^4}{18} - \frac{2d^2c^4v^3}{9} + \frac{d^2c^5v^2}{6} - \frac{c^7d^2}{18} = 0$; in numbers, $v^9 + 36v^7 - 364\frac{1}{2}v^6 - 161163v^5 - 3280\frac{1}{2}v^4 - 4410450v^3 + 3307837\frac{1}{2}v^2 - 89356612\frac{1}{2} = 0$. Whence $v = 20{\cdot}1424$; $x = 45{\cdot}07958$, head-diam. $= 36{\cdot}07958$, bung-diam. $54{\cdot}07958$, length $= 63{\cdot}01844$, and ullage $= 384{\cdot}4266$ ale gallons.

But if the vacuity be a max. the form of the caſk will be ſtill more different, and the content ſtill leſs. For $\overline{3x + c} \times \sqrt{\frac{dd}{x} - x}$ being a max. its fluxion, reduced, is $3ddx - 9x^3 - cd^2 - cx^2 = 0$; in which caſe therefore x is only $= 29{\cdot}625$. C. W.

Pa. 270, l. 27, for 350, read 250.——Pa. 274, l. 11, for $1 + z$, r. $v + y$.——Pa. 275, l. 31, f. *retarding*, r. *accelerating*. Line 32, f. *accelerating*, r. *retarding*. And to the note at the bottom add, *Alſo in Turner's Exerciſes, pa.* 77 *No.* 3.

QUESTION 307 *anſwered by Mr.* J. Landen, *F. R. S.*

Let b be the angular velocity of the plane per ſecond, meaſured on an arc of a circle whoſe radius is 1; y the ſine of the inclination of the plane to the horizon, to the ſame radius; z the arc whoſe ſine is y; and x the diſtance of the ball from the axis about which the plane revolves.

Then bx will be the circulatory, or paracentric, velocity of the ball; and bbx will be its centrifugal force.

Moreover, denoting $16\frac{1}{12}$ feet by g, $2gy$ will be the force of gravity urging the ball down the plane. Therefore $bbx + 2gy$

$+ 2gy$ will be the whole force urging the ball from the axis of motion. Suppoſe its velocity from that axis to be v: then then will $\overline{bbx + 2gy} \times \frac{\dot z}{b}$ be $= \dot v$; and $\frac{\dot z}{b} = \frac{\dot x}{v}$, or $v = \frac{b\dot x}{\dot z}$. Hence, $\dot v$ being $= \frac{b\ddot x}{\dot z}$ when $\dot z$ is ſuppoſed invariable, we get by ſubſtitution $\overline{bbx + 2gy} \times \frac{\dot z}{b} = \frac{b\ddot x}{\dot z}$. It appears therefore that $\frac{2g}{bb}y\dot z^2$ is $= \ddot x - x\dot z^2$: whereof the equation of the fluents is $n^z - n^{-z} = \frac{2bbx}{g} + 2y$, n being the number whoſe *hyp. log.* is 1; as appears by what follows.

Multiplying each ſide of the equation $\frac{2g}{bb}y\dot z^2 = \ddot x - x\dot z^2$ by n^z, we have $\frac{2g}{bb}yn^z\dot z^2 = n^z\ddot x - xn^z\dot z^2$: whence, by taking the fluents, we find the fluent of $\frac{2g}{bb}yn^z\dot z^2 = n^z\dot x - xn^z\dot z$.

Now, denoting the coſine $\sqrt{1 - yy}$ by w, $\dot w$ is $= -y\dot z$, and $\dot y = w\dot z$; therefore $y\dot z$ is $= -\dot w - w\dot z + \dot y$, and it is obvious that, by adding $y\dot z$ on each ſide and multiplying by $\frac{gn^z\dot z}{bb}$, we ſhall get $\frac{2g}{bb}yn^z\dot z^2 = \frac{g}{bb} \times \overline{yn^z\dot z^2 + \dot y\dot z n^z - wn^z\dot z^2 - \dot w\dot z n^z}$. Whence it appears that the fluent of $\frac{2g}{bb}yn^z\dot z^2$ is $= \frac{g}{bb} \times \overline{yn^z\dot z - wn^z\dot z + \dot z}$ $= n^z\dot x - xn^z\dot z$ by what is ſaid above. Hence, by diviſion, we have $n^{-z}\dot x - xn^{-z}\dot z = \frac{g}{bb} \times \overline{n^{-2z}\dot z + yn^{-z}\dot z - wn^{-z}\dot z}$ $= \frac{g}{bb} \times \overline{n^{-2z}\dot z + yn^{-z}\dot z - n^{-z}\dot y}$: from which equation, by again taking the fluents, we get $xn^{-z} = \frac{g}{bb} \times \overline{\frac{1}{2} - \frac{1}{2}n^{-2z} - yn^{-z}}$; and conſequently $n^z - n^{-z} = \frac{2bbx}{g}$ $+ 2y$, as expreſſed above.

By means of this equation of the curve deſcribed by the ball, the velocity $v\ (=\frac{b\dot{x}}{\dot{z}})$ is found $=\frac{g}{2b}\times\overline{n^{z}+n^{-z}-2w}$.

To find the preſſure againſt the plane: let p be the perpendicular from the center of motion to the tangent to the curve deſcribed by the ball, the ray from that center to the point of contact being x; and let q be the abſolute velocity of the ball in the curve at that point. If then the gravity ceaſed to act on the ball, it would proceed in the direction of the ſaid tangent, and (p and q being in that caſe invariable) the fluxion of $(\frac{pq}{x})$ its circulatory velocity would be $=\frac{-pq\dot{x}}{xx}$; which expreſſion is $=-b\dot{x}$, $\frac{pq}{x}$ being $=bx$. But the gravity acting, and the ball preſſing againſt the plane, the fluxion of (bx) its circulatory velocity is $=b\dot{x}$. Therefore $2b\dot{x}=2v\dot{z}$ is the fluxion of the circulatory velocity ariſing from $(2gw-P)$ the exceſs of the force of gravity at right angles to the ray x above P the preſſure againſt the plane. Conſequently $2v\dot{z}$ is $=\overline{2gw-P}\times\frac{\dot{z}}{b}$: whence $P=2gw-2bv=g\times\overline{4w-n^{z}-n^{-z}}$.

When the ball quits the plane P is $=0$, and $4w=n^{z}+n^{-z}$; from which equation z is found to be, at that inſtant, equal to (·823766) an arc of 47° 11′ 54″. Mr. *Simpſon*, finding the fluents by infinite ſeries, makes the angle 47° 9′.

Remark. If, inſtead of the plane, a *tube* be ſubſtituted, continued both ways from the axis of motion, juſt capable of receiving the ball ſo that it may move freely therein; and, the inſtant the tube begins to revolve from a horizontal poſition, the ball be made to move therein from the ſaid axis, along the aſcending part of the tube, with a velocity $=\frac{g}{b}$; the fluxionary equation adapted to this caſe will be $\frac{2g}{bb}y\dot{z}^{2}=x\dot{z}^{2}-\ddot{x}$: and, by correcting the fluents accordingly, x will from thence be found equal to the ſimple expreſſion $\frac{gy}{bb}$! Which equation ſuggeſts this *very remarkable inference*: The ball being at firſt put in motion with the velocity $\frac{g}{b}$, it will revolve uniformly in a circle (whoſe diameter is $\frac{g}{bb}$) touching the horizontal line with which the tube at firſt coincides! and it will continue ſo to revolve (moving up and down alternately in the different branches of the tube) ſo long as the motion of the tube is continued, making two complete revolutions whilſt the tube makes one revolution! and the uniform velocity ($\frac{g}{b}$) wherewith the ball ſo revolves in the circle will be to its velocity along the tube every where as 1 to w. J. L.

A ſolution to Queſt. 311 may be ſeen in Ozanam's Dictionary, and in Euler's Algebra. We ſhall alſo here inſert the following ſolutions to it, viz.

I. *By Mr.* J. Landen, *F.R.S.*

To find three ſuch numbers, that the ſum and difference of any two of them ſhall be ſquare numbers. Suppoſing x, y, and z to be the required numbers; we may aſſume

1ſt, $x = \frac{1}{2} \times \overline{f^4g^4 + g^4 + f^4 + 1}$,
$y = \frac{1}{2} \times \overline{f^4g^4 - g^4 - f^4 + 1} + 2ffgg$,
$z = \frac{1}{2} \times \overline{f^4g^4 - g^4 - f^4 + 1} - 2ffgg$;

whence the values of $x+y$, $x-y$, $x+z$, $x-z$, and $y-z$, are manifeſtly ſquares. Therefore, to ſolve the queſtion, it only remains to find $f^4g^4 - g^4 - f^4 + 1$ (the value of $y+z$) a ſquare.

Or 2dly we may aſſume
$x = ffgg + 1$,
$y = ff + gg$,
$z = 2fg$;

where the values of $x+z$, $x-z$, $y+z$, $y-z$, are manifeſtly ſquares. Therefore, after this aſſumption, we have to find $ffgg + gg + ff + 1$ (the value of $x+y$) a ſquare, and $ffgg - gg - ff + 1$ (the value of $x-y$) a ſquare.

It is obvious, that, according to this aſſumption, the value of the expreſſion $f^4g^4 - g^4 - f^4 + 1$ $(= \overline{x+y} \times \overline{x-y})$ muſt be a ſquare as in the 1ſt aſſumption where it is $= y+z$.——To find that expreſſion a ſquare, ſubſtitute $f+r$ inſtead of g; it will then bcome $\overline{f^4-1}|^2 \times \overline{1 + \frac{4f^3r + 6ffrr + 4fr^3 + r^4}{f^4-1}}$. It appears therefore that $1+\frac{4f^3r+6ffrr+4fr^3+r^4}{f^4-1}$ muſt be a ſquare: ſuppoſing it $=\overline{1+\frac{2f^3r}{f^4-1}+\frac{\overline{f^6-3ff}\times rr}{\overline{f^4-1}|^2}}\Big|^2$, from that equation r will be found $=\frac{4f\times\overline{f^8-1}}{1+6f^4-3f^8}$, and $g\ (=f+r)$ $=\frac{f\times\overline{f^8+6f^4-3}}{1+6f^4-3f^8}$. Therefore f being any number whatever in the 1ſt aſſumption, and g as here found, the queſtion will be anſwered.

It is plain that, $f^4g^4-g^4-f^4+1$ being a ſquare, its factors $ffgg+gg+ff+1$ and $ffgg-gg-ff+1$ will be ſquares if $\frac{gg\pm1}{ff\pm1}$ be a ſquare. Now, taking g equal to its value found above, it will appear that the value of $\frac{gg\pm1}{ff\pm1}$ is $=\overline{\frac{f^8\pm4f^6-6f^4\pm4f^2+1}{1+6f^4-3f^8}}\Big|^2$. Conſequently taking f any number whatever, and g as juſt now mentioned, the queſtion will be anſwered by the 2d aſſumption in much lower terms than by the 1ſt aſſumption.——If f be taken $=2$, the numbers given in the Diary 1750 will be obtained.

There is another way to find $f^4g^4-g^4-f^4+1$ ($=\overline{f^4-1}\times\overline{g^4-1}$) a ſquare. Aſſume it $=\overline{f^4-1}|^2\times\overline{g^2+1}|^2$, or $=\overline{f^4-1}|^2\times\overline{g^2-1}|^2$: whence $g=\frac{ff}{\sqrt{2-f^4}}$, or $=\frac{ff}{\sqrt{f^4-2}}$. Conſequently, if, in the 1ſt aſſumption, f be ſo taken that $2-f^4$, or f^4-2, be a ſquare, and $g=\frac{ff}{\sqrt{2-f^4}}$, or $\frac{ff}{\sqrt{f^4-2}}$ reſpectively, the queſt. will be anſwered.

To find $2-f^4$ a ſquare, I write $1-d$ for f: by which means the expreſſion becomes $1+4d-6d^2+4d^3-d^4$, which being ſuppoſed $=\overline{1+2d-5dd}|^2$, we find $d=\frac{12}{13}$, $f\ (=1-d)=\frac{1}{13}$, and $g=\frac{1}{239}$. Therefore, if, in the 1ſt aſſumption, f be $=\frac{1}{13}$ and $g=\frac{1}{239}$, or $f=13$ and $g=239$, the queſt. will be anſwered.

To find $f^4 - 2$ a ſquare, I ſuppoſe it $= \overline{ff - bb}|^2$; whence $ff = \frac{4 + 2b^4}{4bb}$. Therefore $4 + 2b^2$ muſt be a ſquare.

To find it ſo, I ſuppoſe it $= \overline{2 + bb.bb}|^2$; and, by this ſuppoſition, I find $bb = \frac{4bb}{2 - b^4}$; where, if b be $= 1$, bb will be $= 4$; and conſequently $f = \frac{3}{2}$, $g = \frac{9}{7}$.

Having found $2 - f^4$ a ſquare when f is $= \frac{1}{13}$, it is obvious that bb (and conſequently $f^4 - 2$) will be found a ſquare by taking $b = \frac{1}{11}$. And other values of f and g may be found in like manner.

II. *By the Rev. Mr.* Wildbore.

Suppoſe $2abxy$, $aaxx + bbyy$, and $aayy + bbxx$ to be the three numbers required. Then the ſums and differences $aaxx \pm 2abxy + bbyy$ and $aayy \pm 2abxy + bbxx$ being neceſſarily ſquares, it only remains to make $aaxx + bbyy + aayy + bbxx$ and $aaxx + bbyy - aayy - bbxx$ ſquares; their product therefore $= a^4x^4 - b^4x^4 - a^4y^4 + b^4y^4$ muſt be a ſquare: make $x = z - \frac{ay}{b}$; then ſubſtituting this value for x in the laſt expreſſion, it will thence appear that $a^8y^4 - 2a^4b^4y^4 + b^8y^4 - 4a^7by^3z + 4a^3b^5y^3z + 6a^6bbyyzz - 6aab^6yyzz - 4a^5b^3yz^3 + 4ab^7yz^3 + a^4b^4z^4 - b\ z^4$ muſt be a ſquare. Suppoſe its ſide $= \overline{a^4 - b^4}.yy - 2a^3byz + \frac{a^4 - 3b^4}{a^4 - b^4}aabbzz$; this ſquared and made equal to the other, will give, by reduc. $-4a^9y + 4ab^8y = -3a^8bz + 6a^4b^5z + b^9z$; whence $z = 4a^9 - 4ab^8$, and $y = 3a^8b - 6a^4b^5 - b^9$, and conſequently $x = a^9 + 6a^5b^4 - 3ab^8$. Theſe values being ſubſtituted in the aſſumed expreſſions, they will become reſpectively

$$6a^{18}b^2 + 24a^{14}b^6 - 92a^{10}b^{10} + 24a^6b^{14} + 6a^2b^{18},$$
$$a^{20} + 21a^{16}b^4 - 6a^{12}b^8 - 6a^8b^{12} + 21a^4b^{16} + b^{20},$$
$$10a^{18}b^2 - 24a^{14}b^6 + 60a^{10}b^{10} - 24a^6b^{14} + 10a^2b^{18}.$$

Which are general theorems for infinite anſwers, where a and b may be taken any numbers at pleaſure; the only limitation being that $6a^4b^4 + b^8$ muſt be leſs than $3a^8$. And when $a = 2$, and $b = 1$, the three numbers anſwering the queſtion are 2288168, 2399057, and 1873432, as put down at page 284.

III. *By the* Editor.

Three numbers anſwering the conditions may be found by the following eaſy proceſs.

Aſſume $4x$, $4 + xx$, and $1 + 4xx$ for the three numbers; where the ſum and diff. of the firſt and each of the other two being all four ſquares, we have only to make the ſum and diff. of the two latter to be ſquares, viz. $5xx + 5$, and $3xx - 3$ each a ſquare. Their product $15x^4 - 15$ will therefore be a ſquare. Which it will evidently be when $x = 2$; for then it is $15^2 = 225$. And then the three aſſumed numbers become 8, 8, and 17, which anſwer the conditions. But as two of the numbers are equal to each other, make $x = z - 2$; then $15x^4 - 15$ is $= 225 - 480z + 360z^2 - 120z^3 + 15z^4 =$ a ſquare $=$ ſuppoſe $\overline{15 - az + bzz}|^2 = 225 - 30az + \left.\begin{matrix}30b\\+aa\end{matrix}\right\} z^2 - 2abz^3 + b^2z^4$. Here equate the 2d term of the one of theſe to the 2d of the other, and a is found $= 16$; then equate the 3d to the 3d, and there reſults $b = \frac{52}{15}$; and finally the laſt two terms of the one equated to the laſt terms of the other, we have $z = \frac{2040}{671}$. Hence $x = z - 2 = \frac{698}{671}$; which ſubſtituted in the aſſumed expreſſions, they become $4 \times \frac{698}{671}$, $4 + \frac{698^2}{671^2}$, and $1 + \frac{4.698^2}{671^2}$; or, by multiplying each by 671^2, they become 1873432, 2288168, and 2399057, the very ſame numbers as before.

Pa. 289, in Queſt. 316, when the given dividing line is conſidered as making given angles with the two ſides of the triangle about a given vertical angle, it is conſtructed by theor. 8 pa. 199 Simpſon's Geom. And when the area is a given quantity inſtead of a min. it is conſtructed in prob. 5 pa. 214.

QUESTION 318 *anſwered by the Rev. Mr.* Wildbore.

As no notice whatever is taken of the given breadth of the viſto, in the original ſolution, it cannot be right. For, though the Δs GeE, oHF in this particular example be ſmall in reſpect of the $\square eoFE$, the breadth of the viſto may be ſuppoſed increaſed till they even infinitely exceed it. Theſe three things (the breadth and two Δs) muſt not therefore be omitted in the ſolution, which may be thus.

The area of the viſto is evidently equal to the ſum of theſe two Δs and $\square$. And Ge being, by ſim. triangles, $= \frac{ED . Ee}{FD}$, and $Ho = \frac{FD . Ee}{ED}$; the viſto $= \frac{ED . Ee^2}{2FD} + \frac{FD . Ee^2}{2ED} + FE . Ee$ a minimum, or $\frac{Ee}{2} \times \overline{\frac{ED}{FD} + \frac{FD}{ED}} + \sqrt{FD^2 + ED^2}$ a min. At B erect $BS \perp FD$ meeting FE in S. Then, by ſim. Δs, $SB = ED \times \frac{FD - BD}{FD}$ and, by conics, $SB = \frac{Tt^2}{ED}$; hence $ED^2 = \frac{FD . Tt^2}{FB} = Tt^2 + \frac{BD . Tt^2}{FB}$.

Make now $FB = u$, $BD = 2b$, $Tt = c$, and $Ee = d$. Then $FD = 2b + u$, $ED = c\sqrt{\frac{2b+u}{u}}$, $\frac{ED}{FD} = \frac{c}{\sqrt{2bu+uu}}$, and $\frac{dc}{2\sqrt{2bu+uu}} + \frac{d\sqrt{2bu+uu}}{2c} + \sqrt{\overline{2b+u}^2 + cc . \frac{2b+u}{u}}$ is a min. The flux. made $= 0$, &c. there reſults $d \times \frac{b+u}{2c} \times \frac{2bu+uu-cc}{2b+u} = \frac{bcc - uu . \overline{2b+u}}{\sqrt{2bu+uu+cc}}$. Whence u may be found by an equation of the 8th power, or by the well-known rule of poſition; and thence the reſt. Thus in the preſent caſe $u = 24{\cdot}863187$. Whereas when EF only is a min. u is $= 25{\cdot}26757$, $EF = 153{\cdot}702$, and the mean length $= 157{\cdot}552$. But the mean length determined as above is $= 157{\cdot}545$, and the quantity of land in the viſto ·02545 parts of a ſquare pole leſs than when EF is a minimum.

C. W.

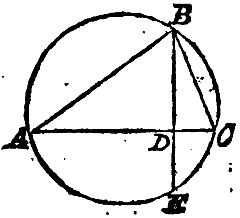
Pa. 332, l. 16, f. $2c + 4d$, r. $\frac{2c+4d}{3}$.

Pa. 335, l. 3, f. HI, r. HF. Line 5, f. $2aabb$, r. $2abbxx$]

Pa. 336, l. 7, in the value of $2v^2$, after $2bac$ write $+ ss$ in the denominator.

QUESTION 340 *Conſtructed by the Rev. Mr.* Wildbore.

Having made two rectangles equal to the given ones, ſo that the given perpendicular may be the ſhorteſt ſide of the greater and the longeſt ſide of the leſs, upon the longeſt ſide of the greater as a diam. deſcribe the circ. ABC, any where in which apply $BE =$ the ſum of the two ſides of the leſs rectangle, on which take $BD =$ the given perpendicular; through D draw $AC \perp BE$, cutting the circle in A and C; then, AB, BC being drawn, ABC is the required triangle.



For, by Simpſon's Geom. III. 25, calling the diam. of the circle D, the rectangle $D \times DB = AB \times BC =$ the given one by conſtruc. and, by III. 21, $AD \times DC = BD \times DE =$ the leſs given one by conſtr. and BD being the given perp. ABC muſt be the triangle required.

COROL. Since $D \times DB = AB \times BC$, and $DE \times BD = AD \times DC$, the two given rectangles are in the ratio of D to DE; ∴ when, inſtead of the perpendicular, the vertical angle is given; AC being aſſumed at pleaſure, and the circle deſcribed to contain thereon the given angle; and DE taken to D in the given ratio of the rectangles, and continued to B; ABC will be a triangle ſimilar to the required one.
C. W.

Pa. 343, l. 17 and 18, for ·243 read ·226.——Pa. 387, l. 17, f. p, r. q; and f. q, r. r.——Pa. 390, l. 8, f. $aa + aa$, r. $aa + cc$.

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Pa. 16, l. 13, f. $PP + 1$, r. $P.\overline{P+1}$.——Pa. 34, l. 8, f. $x^5 + x^5$, r. $u^5 + x^5$.——Pa. 45, l. 1, f. is the time, r. is as the time. Line 26, f. ABD, r. ADB.——Pa. 61, l. 2, f. $4ab$, r. $4abs$.——Pa. 67, l. 12, f. $\surd$&c. r. $y\surd$&c.——Pa. 95, l. 13, f. 10°, r. 19°.——Pa. 96, l. ult. f. $4pxx$, r. $4px$.——Pa. 123, l. 3, after *conſequently* put $y =$.——Line 4, for *we*, r. *as*.——Pa. 160, l. 19, f. 298, r. 289. Lines 21 and 25, for the decimal point write the ſign of multiplication.

As a ſecond *Corol.* to the ſolution to queſt. 511 at pa. 184, it may be remarked that, as td is $= DC$ in the fig. pa. 186, therefore the area or illuminated ſurface tdd becomes $d \times DC$; that is, the diam. of the cyl. drawn into the alt. of the illuminated ſurface. Which agrees with the rule proved in corol. 1, p. 148 of my Menſuration, from whence the ſurface might have been at firſt eaſily found.

QUESTION 579 *anſwered by Mr.* J. Landen, *F. R. S.*

If $\overline{1+x}|^4 \times \overline{1+x^2}|^4 \times \overline{1+x^3}|^4\ (9) \times \overline{1+x^{10}}|^{16}$ be actually involved, the coefficient of that power of x whoſe exponent is 15 will be the number ſought.

To find the coefficient with facility, obſerve that the above expreſſion is $= \overline{1+x}|^4 \times \overline{1+x^2}|^4 \times \overline{1+x^4}|^4 \times \overline{1+x^8}|^4 \times \overline{1+x^3}|^4 \times \overline{1+x^6}|^4 \times \overline{1+x^5}|^4 \times \overline{1+x^7}|^4 \times \overline{1+x^9}|^4 \times \overline{1+x^{10}}|^{16} = \overline{1+x+x^2+x^3 \ldots\ldots x^{15}}|^4 \times \overline{1+x^3+x^6+x^9}|^4 \times \overline{1+x^5}|^4 \times \overline{1+x^7}|^4 \times \overline{1+x^9}|^4 \times \overline{1+x^{10}}|^{16}$.

Now $\overline{1+x+x^2+x^3 \ldots\ldots x^{15}}|^4$ being $= \overline{\frac{1-x^{16}}{1-x}}|^4 = \overline{1-4x^{16}+\frac{4.3}{1.2}x^{32}\ \&c.} \times \overline{1+4x+\frac{4.5}{1.2}x^2+\frac{4.5.6}{1.2.3}x^3\ \&c.}$ it appears, that, in the value of $\overline{1+x+x^2+x^3 \ldots\ldots x^{15}}|^4$ the terms wherein the exponents of the powers of x are not greater than 15 are $1+4x+\frac{4.5}{1.2}x^2+\frac{4.5.6}{1.2.3}x^3$ &c. $(16) = \frac{16.17.18}{1.2.3}x^{15}+\frac{15.16.17}{1.2.3}x^{14}+\frac{14.15.16}{1.2.3}x^{13} \ldots\ldots\ldots 4x+1$. It appears alſo (by an eaſy involution) that, in the value of $\overline{1+x^3+x^6+x^9}|^4 \times \overline{1+x^5}|^4 \times \overline{1+x^7}|^4 \times \overline{1+x^9}|^4 \times \overline{1+x^{10}}|^{16}$, the terms wherein the exponents of the powers of x are not greater than 15 are $1+4x^3+4x^5+10x^6+4x^7+16x^8+24x^9+38x^{10}+40x^{11}+63x^{12}+128x^{13}+102x^{14}+212x^{15}$. It is obvious therefore, that the coefficient ſought is $= 1 \times \frac{16.17.18}{1.2.3} + 4 \times \frac{13.14.15}{1.2.3} + 4 \times \frac{11.12.13}{1.2.3} + 10 \times \frac{10.11.12}{1.2.3}$ &c. $(13) = 816+1820+1144+2200+660+1920+2016+2128+1400+1260+1280+408+212 = 17264$.

Pa. 340. Concerning Mr. Wildbore's ſolution there given to the prize queſt. for the year 1770, he 'deſires it may 'be obſerved that in the ſolu. to that queſt. the motion of 'the bucket is ſuppoſed to commence when the water there-'in, and that in the reſervoir, are both at reſt, with their 'ſurfaces on the ſame horizontal level.'

Question 653 *anſwered by Mr.* J. Landen, *F. R. S.*

Lemma 1. If $\frac{2d}{1 \backsim ee}$ and $\frac{2d}{\sqrt{1 \backsim ee}}$ be the axes of a conic ſection, the curve will be a circle if e be $= 0$; and, according as e is leſs, equal to, or greater than 1, the curve will be an ellipſis, a parabola, or a hyperbola.

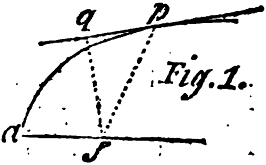

Fig. 1.

The diſtance of the focus f from the vertex a, of any conic ſection, will be $= \frac{d}{1+e}$: and, y being put to denote the ray fp from the ſaid focus to any point p of the curve, the perpendicular fq, from the ſame focus upon the tangent pq, will be $= d\sqrt{\frac{y}{2d+\overline{ee-1}\,.\,y}}$. Moreover $\sqrt{\frac{2dy+\overline{ee-1}\,.\,yy-dd}{2dy+\overline{ee-1}\,.\,yy}}$ will be the ſine of the angle pfq, radius being 1; and, that ſine being always to the ſine of the angle afq in the conſtant ratio of e to 1, the ſine of the laſt-mentioned angle will be $= \frac{1}{e}\sqrt{\frac{2dy+\overline{ee-1}\,.\,yy-dd}{2dy+\overline{ee-1}\,.\,yy}}$.

Lemma 2. The earth being ſuppoſed to deſcribe a conic ſection whoſe axes are as expreſſed in the preceding lemma, let its velocity at the vertex a be denoted by $b \times \overline{1+e}$: then will its velocity at any other point p of its trajectory be $= b\sqrt{\frac{2d+\overline{ee-1}\,.\,y}{y}}$, being reciprocally as the perpendicular from the focus upon the tangent.

PROPOSITION.

The velocity of the earth in its trajectory being to the velocity of light as $b\sqrt{\frac{2d+\overline{ee-1}\cdot y}{y}}$ to c; it is propoſed to determine the linear aberration of a ſtar from its true place, meaſuring ſuch aberration on a plane paſſing through the ſtar parallel to the plane of the ſaid trajectory?

Let s be the ſtar, and p the earth in its trajectory moving from the vertex a; and, qkp being a tangent to the trajectory, let pk be to pb as the velocity of the earth at p to the velocity of light, *i. e.* as $b\sqrt{\frac{2d+\overline{ee-1}\cdot y}{y}}$ to c: then, pks being the line joining the earth and the ſtar, if the angle spt be equal to the angle pbk, st parallel to pk will be the linear aberration ſought, at the time the earth is at p, the ſtar appearing to be at t, inſtead of its true place s.

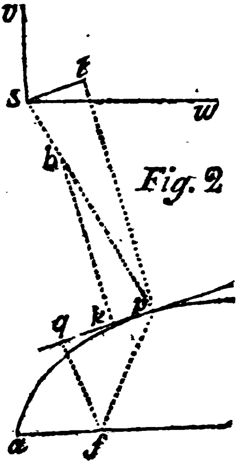

Fig. 2

Hence it follows that ps (the diſtance of the ſtar from the earth) being denoted by D, and st by x, x will be $=\frac{bD}{c}\sqrt{\frac{2d+\overline{ee-1}\cdot y}{y}}$ $=r\sqrt{\frac{2d+\overline{ee-1}\cdot y}{y}}$, r being put for $\frac{bD}{c}$.

Moreover, if sw be parallel to the axis af, and sv perpendicular to sw; the angle tsv will, it is obvious, be equal to the angle afq made by af and the perpendicular fq on the tangent pq.——Let the ſine of the angle tsv (or afq) be denoted by v: then, by lemma 1, v will be $=\frac{1}{e}\sqrt{\frac{2dy+\overline{ee-1}\cdot yy-dd}{2dy+\overline{ee-1}\cdot yy}}$; whence $y=\frac{d}{1-ee}\mp\frac{de\sqrt{1-vv}}{\overline{1-ee}\cdot\sqrt{1-eevv}}$.

But from the equation $z = r\sqrt{\frac{2d + \overline{ee - 1}.y}{y}}$ (found above) we get $y = \frac{2drr}{zz + \overline{1 - ee}.rr}$. From which two values of y it appears that z will be $= r\sqrt{1 - eevv} \pm er\sqrt{1 - vv}$.

Let AEB be a ſemicircle whoſe center is C; and in the diameter ACB (or the continuation thereof) take any point D; moreover let CE, DE be drawn and denoted by r and z reſpectively: then, the ſine of the angle CDE being denoted by v, and the diſtance CD by m, $z\sqrt{1 - vv} \mp \sqrt{rr - vvzz}$ will be $= m$. Hence $z = r\sqrt{1 - \frac{mmvv}{rr}} \pm m\sqrt{1 - vv}$. Comparing this equation with that which we juſt now found above, we may obſerve that they will be exactly alike if m be equal to er.——It follows therefore, that, s being the true place of the ſtar, sw parallel to the axis (af) of the earth's trajectory, sv perpendicular to sw, and st ſo drawn that the ſine of the $\angle tsv$ be equal to v: if sc be taken (upon sv) equal to er $(= \frac{beD}{c})$; and from the center c, with a radius equal to r $(= \frac{bD}{c})$, an arc be deſcribed interſecting st in t; the point t ſo determined will be the apparent place of the ſtar in the plane paſſing through the ſtar parallel to the earth's trajectory, when the earth is at ſuch a point p thereof that the ſine of the angle afq (fig. 2) be $= v$.

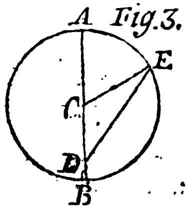

Fig. 3.

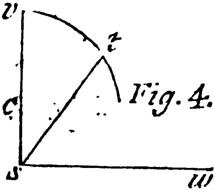

Fig. 4.

If D be invariable, (which can only be when the earth's trajectory is a circle, and the ſtar is ſituated in a line erected perpendicular to the plane of the orbit from the center thereof) the curve of aberration, on our ſaid plane parallel to the trajectory, will be a circle whoſe center is c, e being then $= 0$.

If the trajectory be an ellipſis not very eccentric, and the diſtance of the ſtar from the earth be at any time *very great* in compariſon with the diſtance of the earth from the focus; D may be conſidered as continuing of *nearly* the ſame value whilſt the earth deſcribes its whole orbit: our curve of aberration will therefore, in ſuch caſe, be *nearly* (but not exactly) a circle.

Whether *D* be conſidered as invariable or variable, *z* (or the ſtar's apparent place) will be truly determined by our conſtruction; and by means thereof the ſtar's true place may be readily found from its obſerved apparent place.

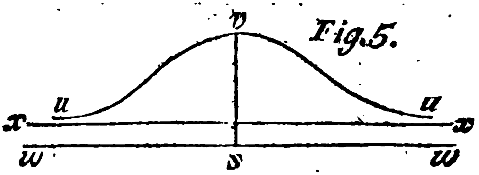
Fig. 5.

Suppoſing the trajectory to be a parabola, and *g* the diſtance of the ſtar from the vertex thereof; let *s* be the ſtar's true place, and $sv = \frac{2bg}{c}$: then will *sw*, at right angles to *sv*, be parallel to an aſymptote (*xx*) to the curve of aberration *uvu*, which will always have two branches extending ad infinitum from the point *v*.——The diſtance of the aſymptote from *s* depends on the ſituation of the ſtar with reſpect to the earth's trajectory.

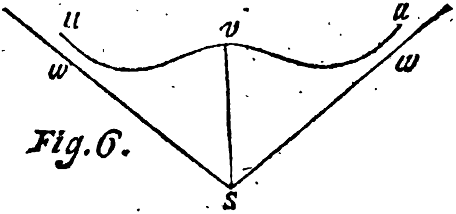
Fig. 6.

If the trajectory be a hyperbola; and, *s* being the ſtar's true place, sv be $= \frac{bg}{c} \times \overline{1+e}$; the right lines *sw*, *sw*, ſo drawn that the ſine of the angle *vsw* be $= \frac{1}{e}$, will be aſymptotes to the curve of aberration *uvu*.

The two branches of the curve of aberration excurring ad infinitum from the point *v* (fig. 5 and 6) will be ſimilar when the ſtar is ſituated any where in a plane at right angles to the plane of the earth's trajectory, the common ſection of the two planes being the principal axis of the trajectory. ——If the ſtar be not ſo ſituated, the two branches will take different forms; but their aſymptotes will make equal angles with the line sv on different ſides thereof, and form one continued right line (as in fig. 5) or interſect each other at s (as in fig. 6).

Theſe concluſions reſpecting the curve of aberration are ſo obvious, upon conſidering the value of st and its poſition in fig. 4, that it is unneceſſary to be more explicit on that head. But it may be worth while to add a remark or two concerning *the angle of aberration*, i. e. the angle under which the line of aberration above found would appear to an obſerver on the earth; which, from what is done above, may be readily computed when the ſituation of the earth with reſpect to the ſtar is known.

It is obſervable, that, in a parabola, as the earth recedes from the vertex, the angle of aberration will after ſome time (if not immediately) continually decreaſe, and at length may become leſs than any aſſignable angle: and it will decreaſe in like manner when the trajectory is a hyperbola, if c be greater than $b\sqrt{ee-1}$. But if c be $= b\sqrt{ee-1}$, the angle of aberration may, as the earth ſo recedes, increaſe nearly to 90°; and if c be leſs than $b\sqrt{ee-1}$, that angle may increaſe ſtill more.

The Rev. Mr. *Wildbore* ſent alſo a corrected ſolution on the ſuppoſition that the parallax is ſenſible, &c.

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